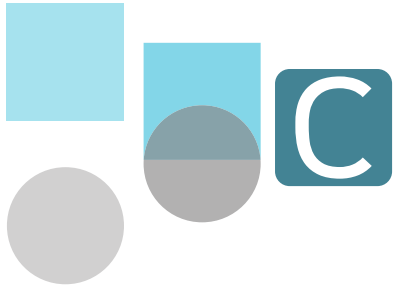



A Circuit-Based Approach to Efficient Enumeration

Antoine Amarilli[1], Pierre Bourhis[2], Louis Jachiet[3], Stefan Mengel[4]

December 6th, 2017

[1]Télécom ParisTech

[2]CNRS CRIStAL

[3]Université Grenoble-Alpes

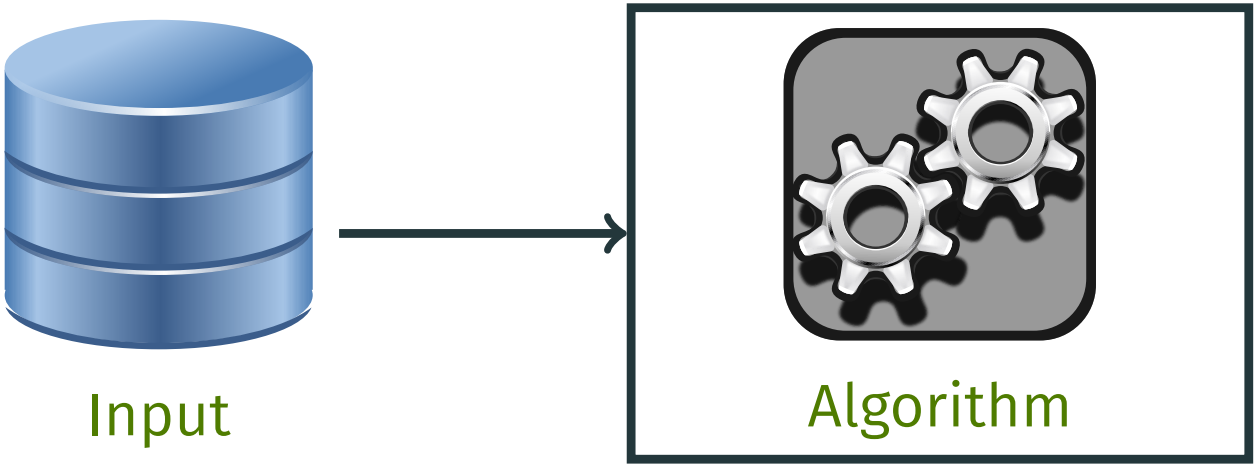
[4]CNRS CRIL

Problem statement

Problem: Enumerating large result sets

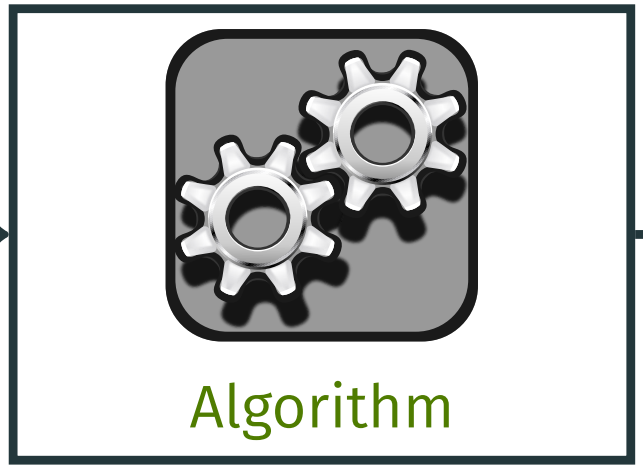
Input

Problem: Enumerating large result sets



Problem: Enumerating large result sets

Input

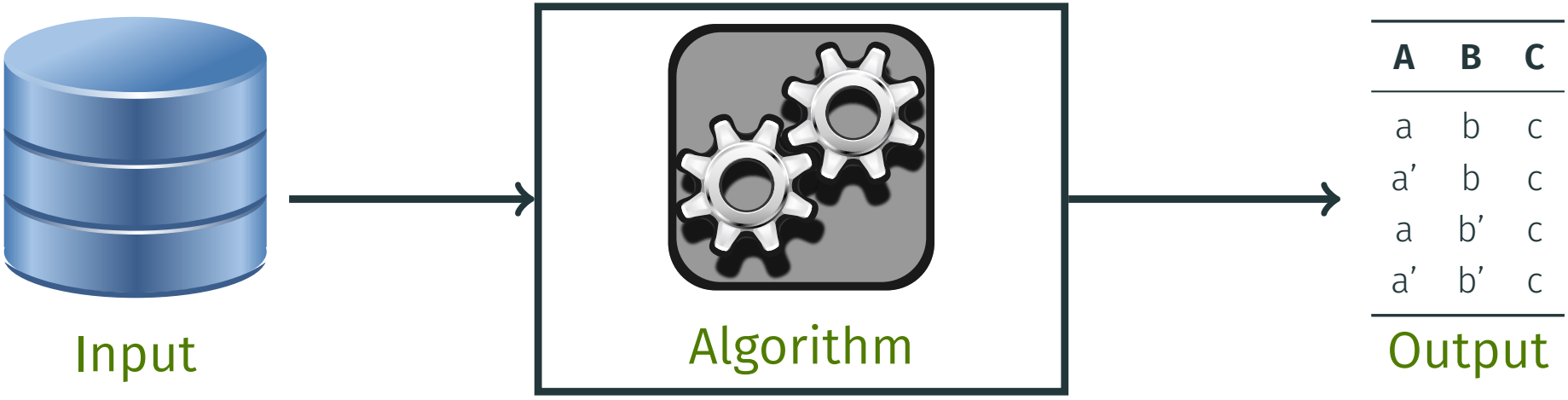


Algorithm

A	B	C
a	b	c
a’	b	c
a	b’	c
a’	b’	c

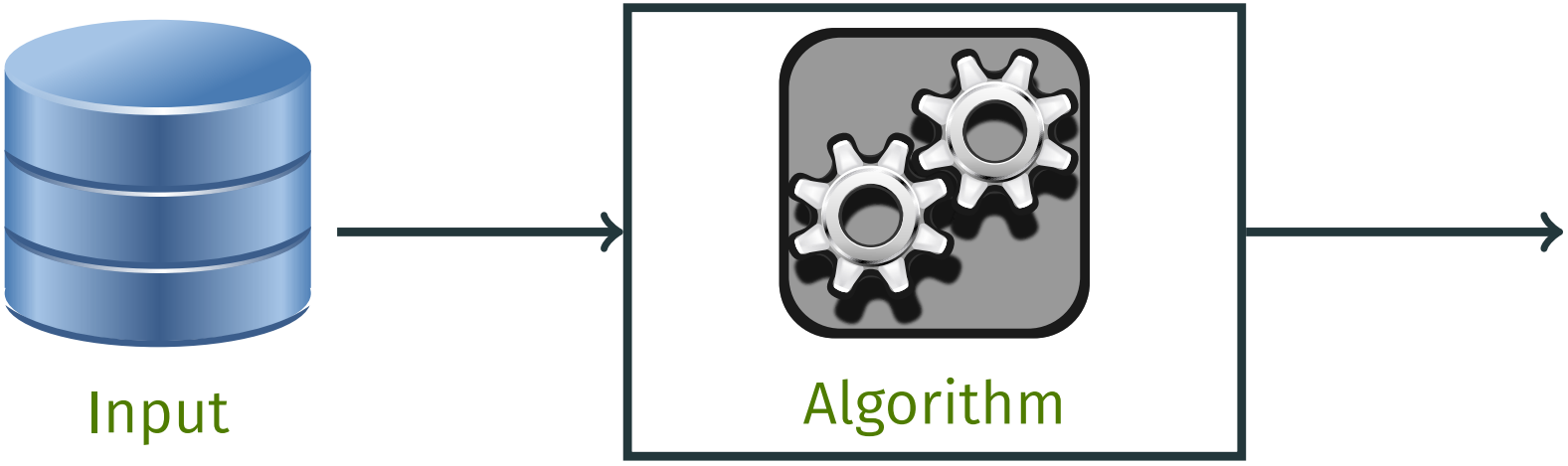
Output

Problem: Enumerating large result sets



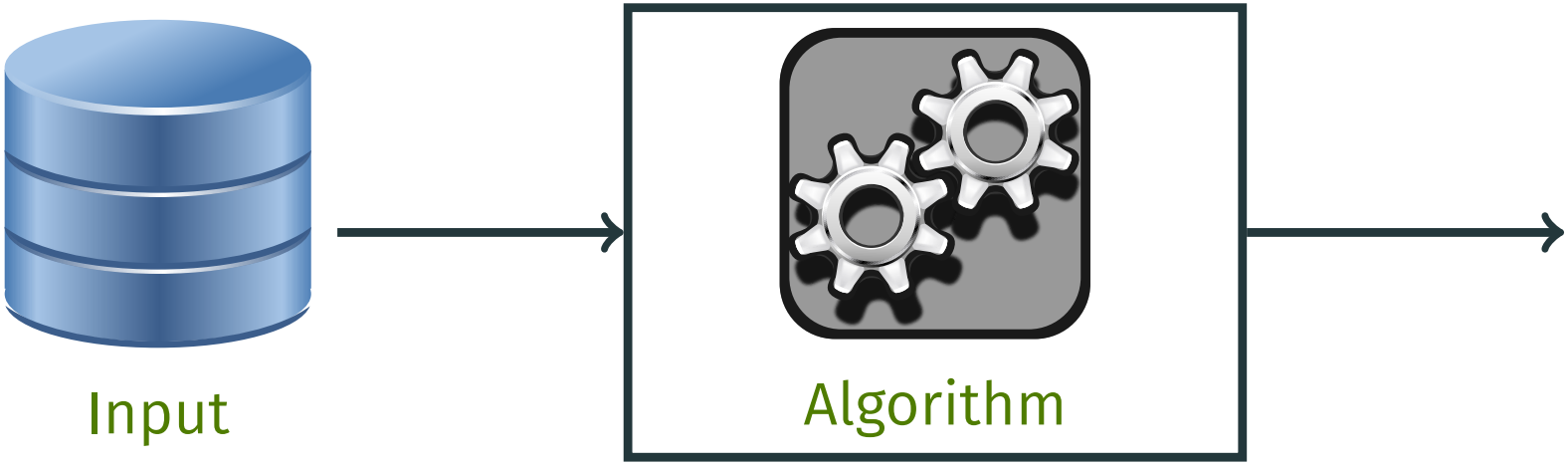
- Problem: The output may be too large to compute efficiently

Problem: Enumerating large result sets



- Problem: The output may be too large to compute efficiently

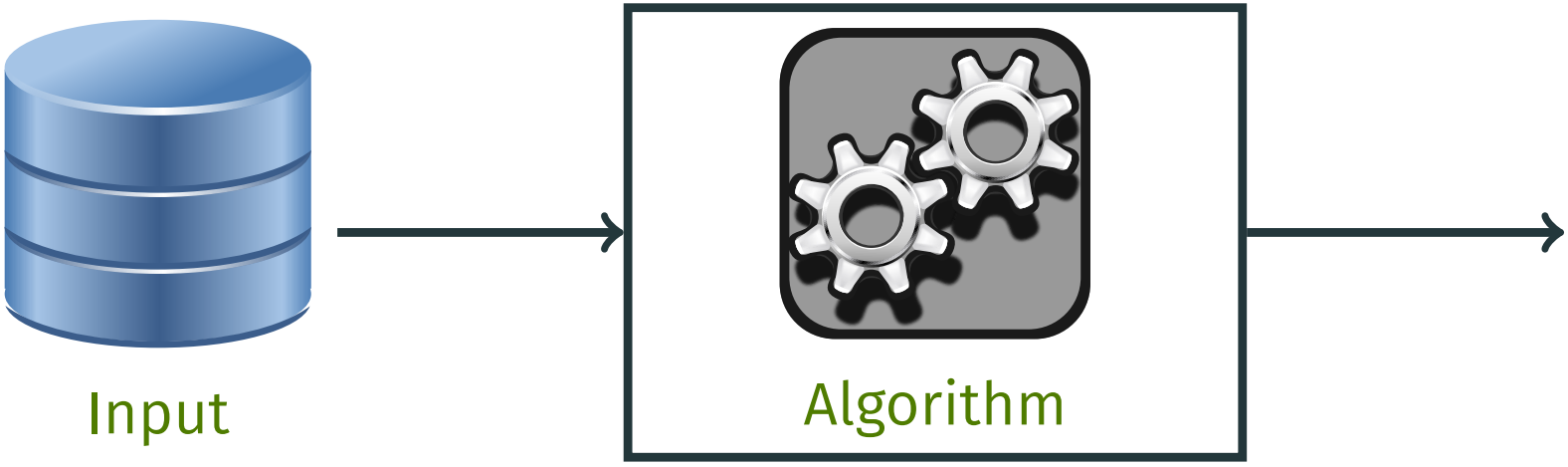
Problem: Enumerating large result sets



- Problem: The output may be too large to compute efficiently

Results **1 - 20** of **10,514**

Problem: Enumerating large result sets



- Problem: The output may be too large to compute efficiently

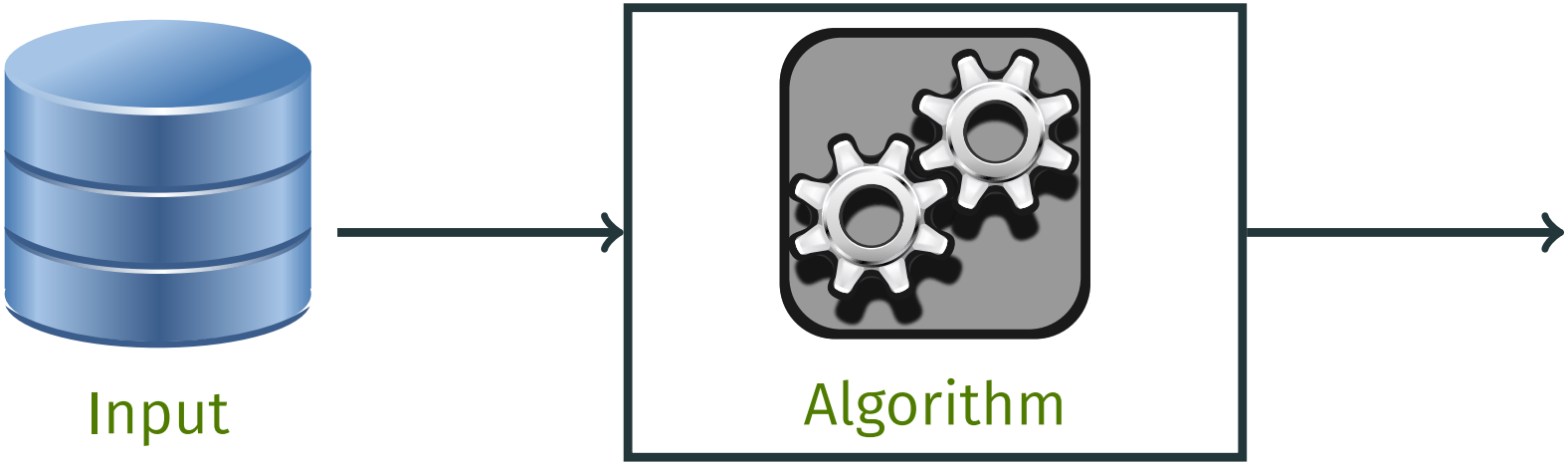
knowledge compilation

Search

Results **1 - 20** of **10,514**

...

Problem: Enumerating large result sets



- Problem: The output may be too large to compute efficiently

knowledge compilation

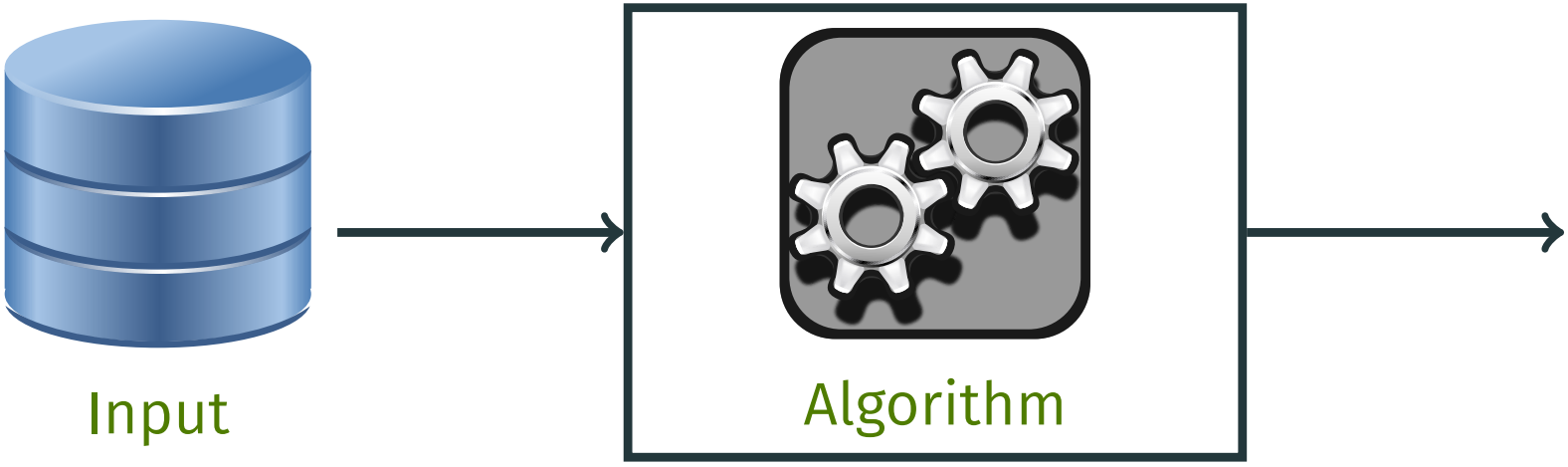
Search

Results **1 - 20** of **10,514**

...

View (previous 20 | next 20) (20 | 50 | 100 | 250 | 500)

Problem: Enumerating large result sets



- Problem: The output may be too large to compute efficiently

knowledge compilation

Search

Results **1 - 20** of **10,514**

...

View (previous 20 | next 20) (20 | 50 | 100 | 250 | 500)

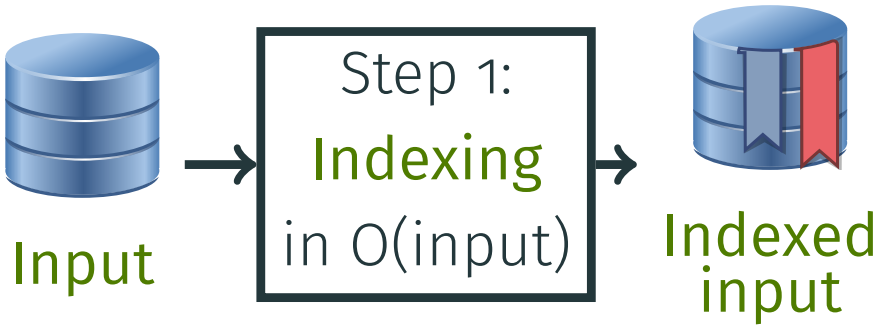
→ Solution: Enumerate solutions one after the other

Enumeration algorithm

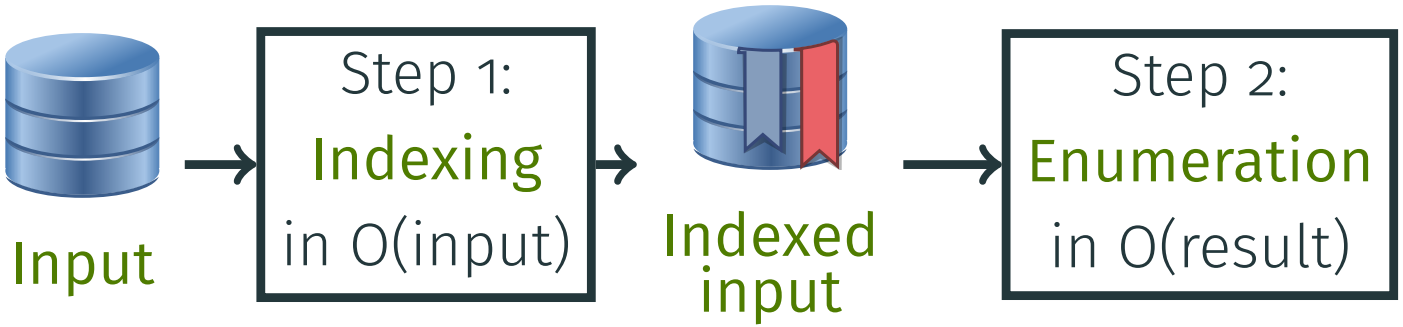
Input

Enumeration algorithm

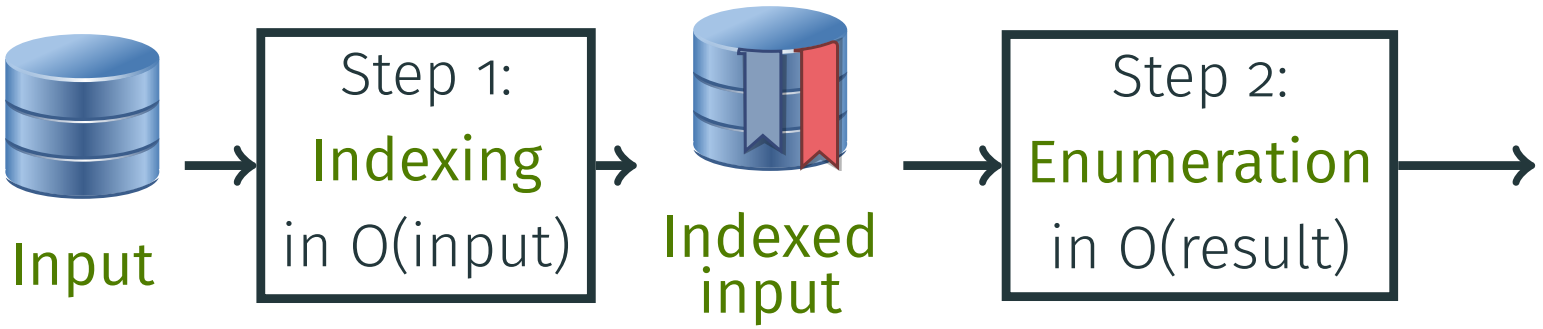
Enumeration algorithm



Enumeration algorithm



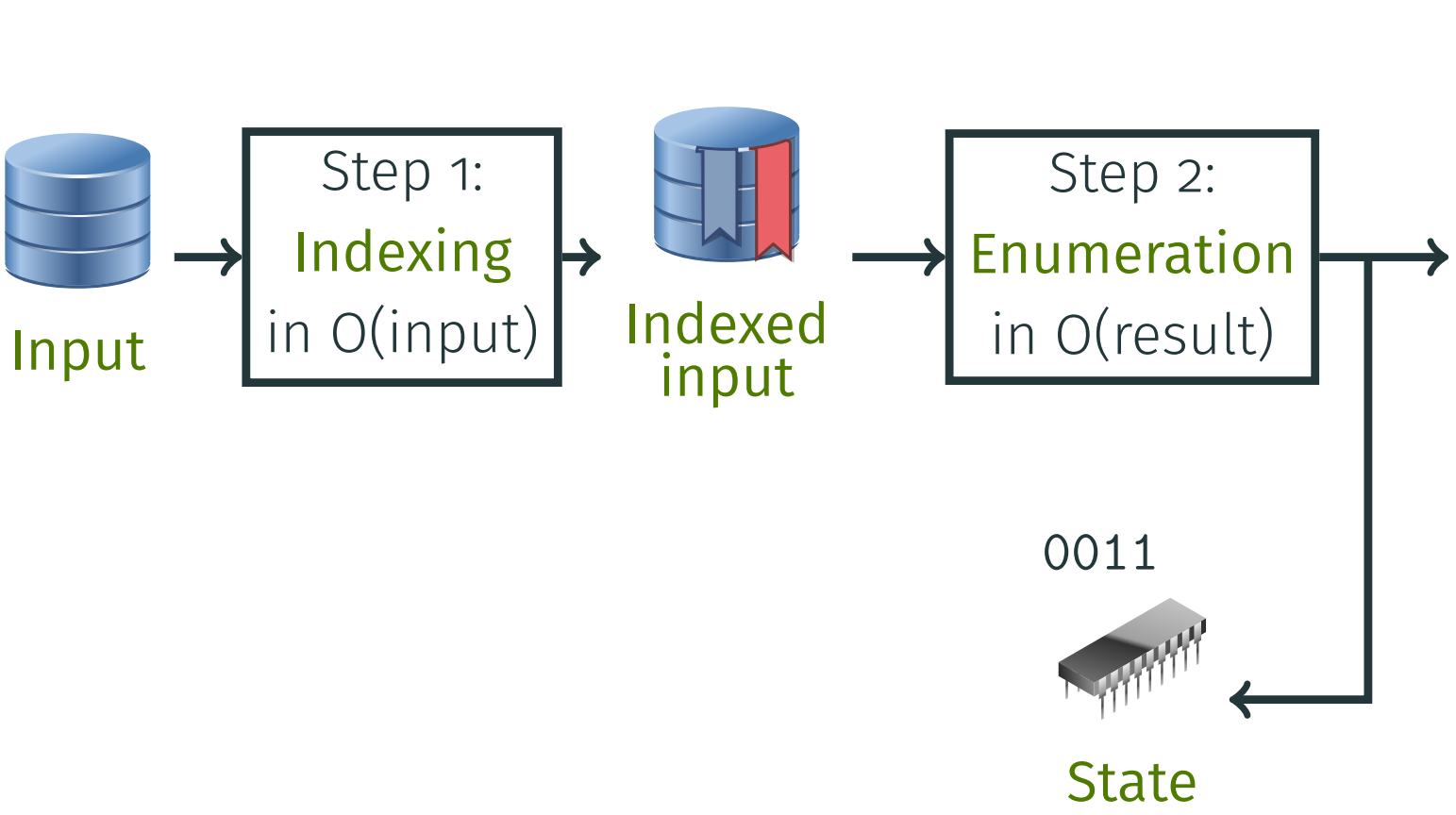
Enumeration algorithm



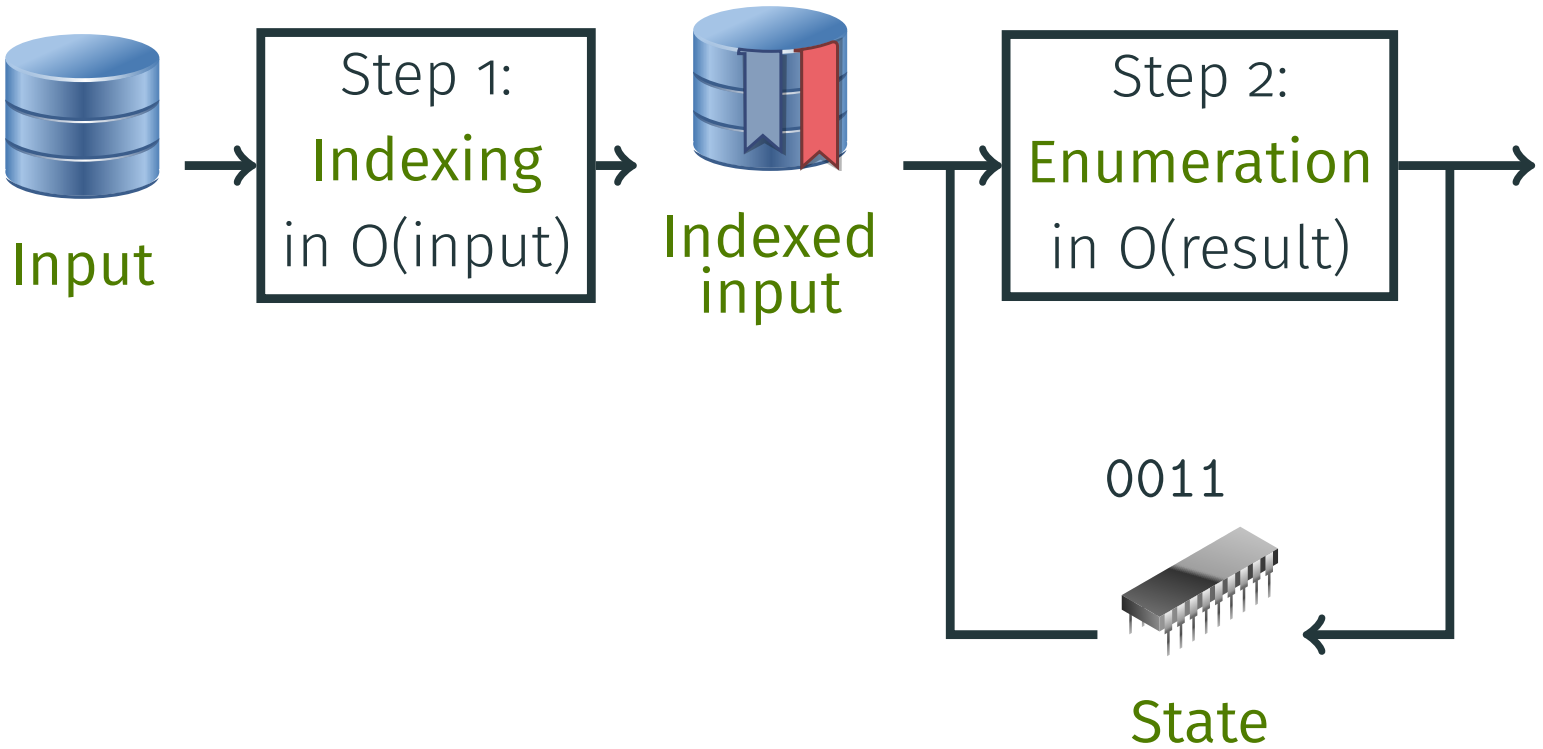
A	B	C
a	b	c

Results

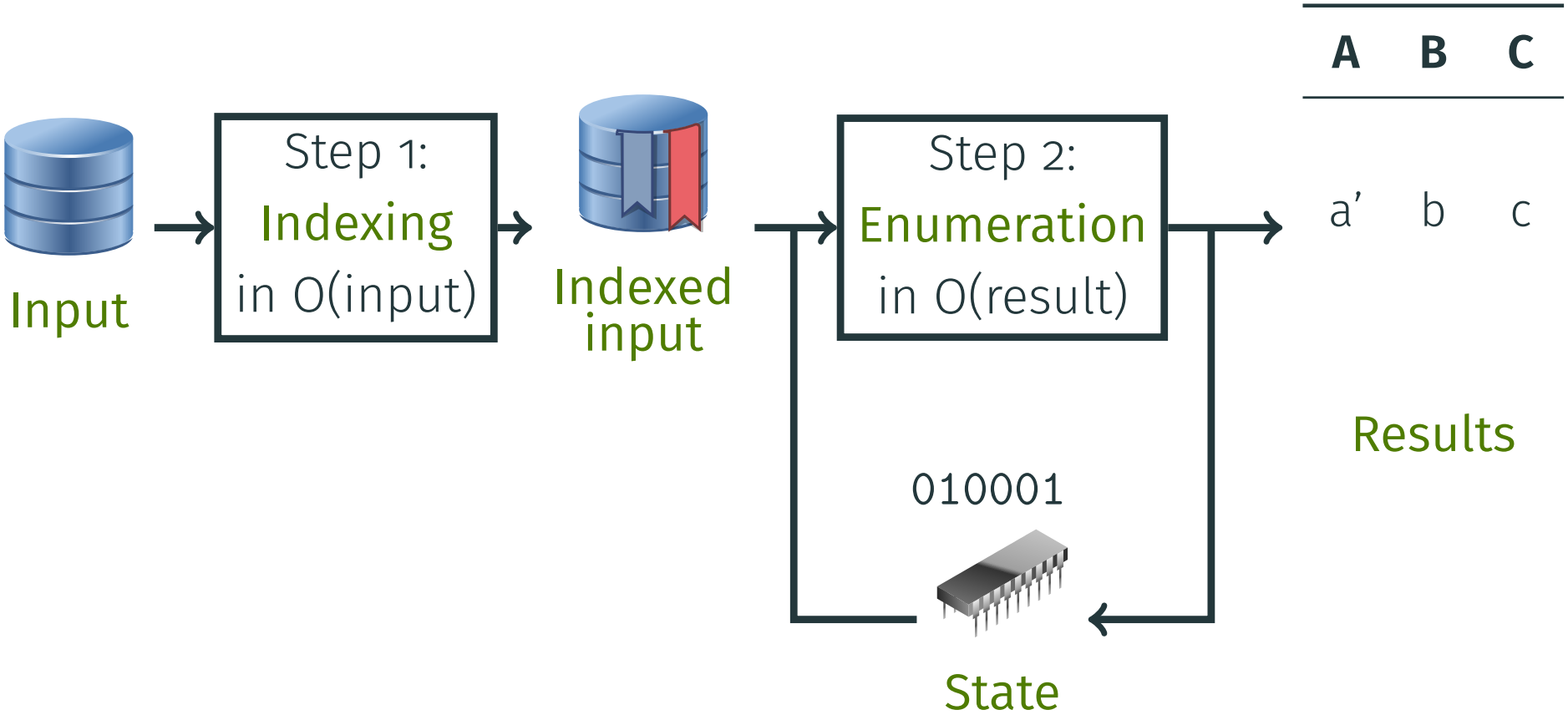
Enumeration algorithm



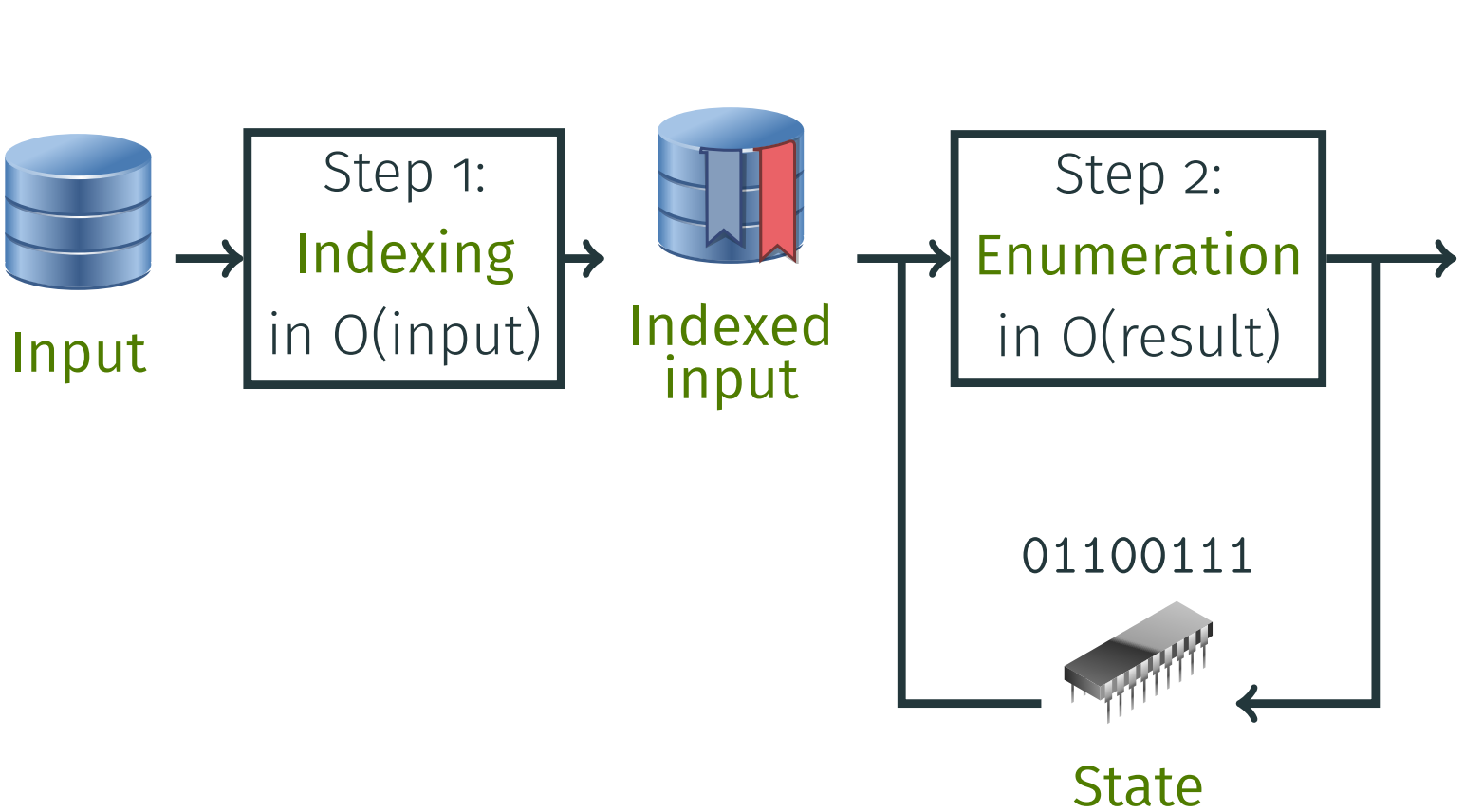
Enumeration algorithm



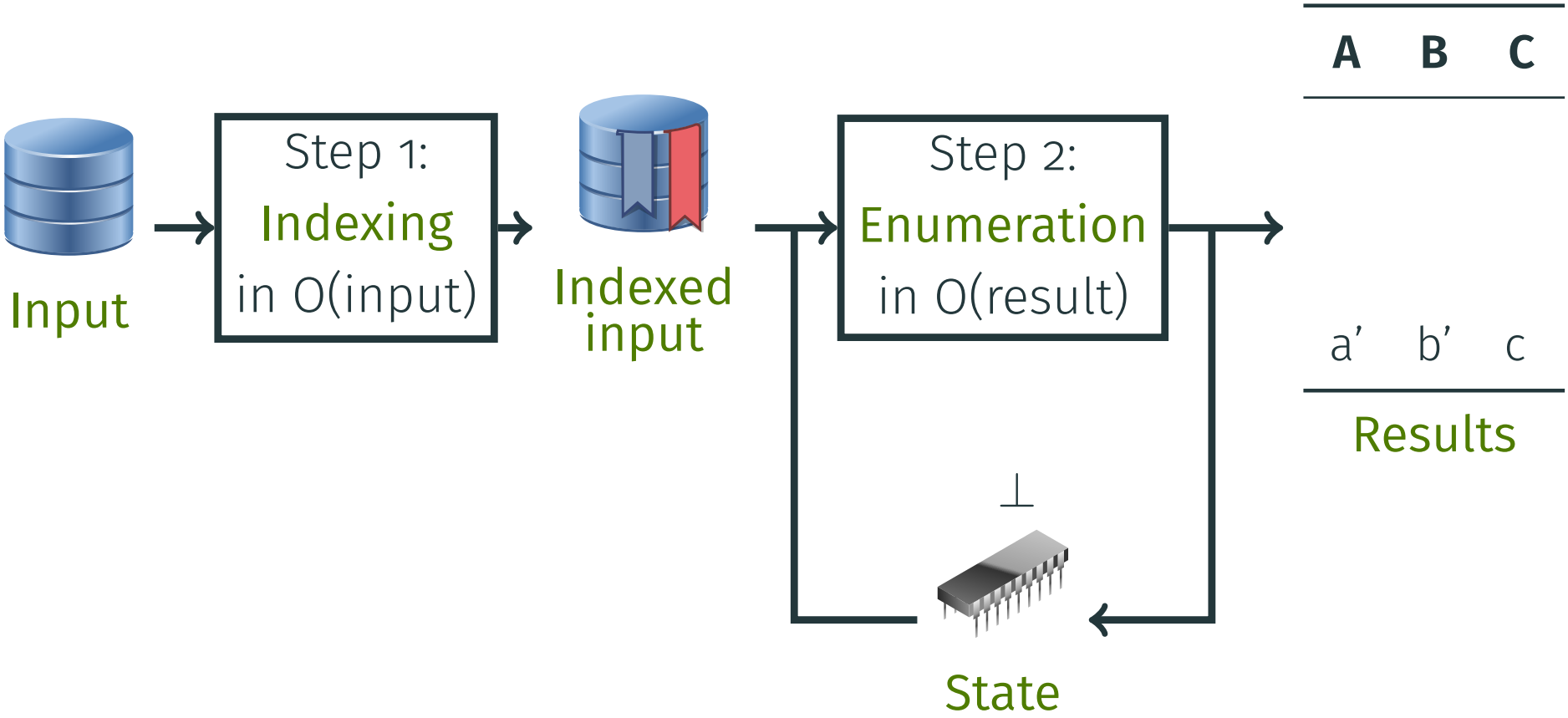
Enumeration algorithm



Enumeration algorithm

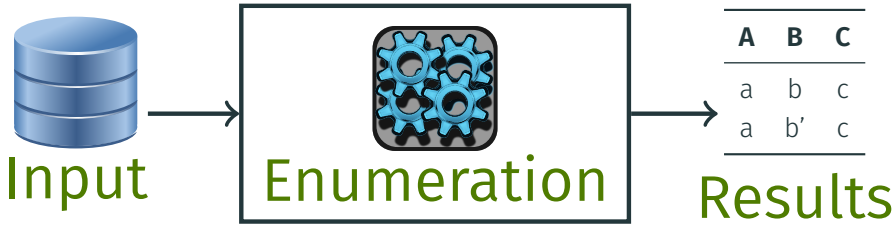


Enumeration algorithm



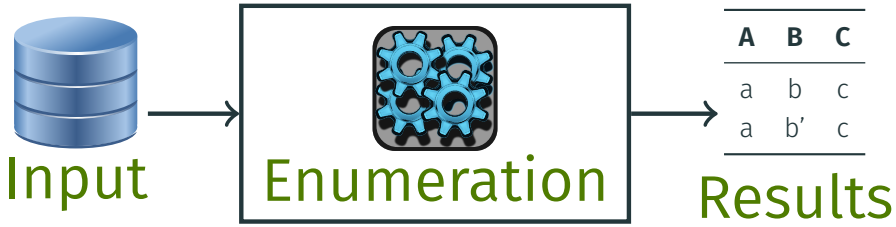
General idea for enumeration

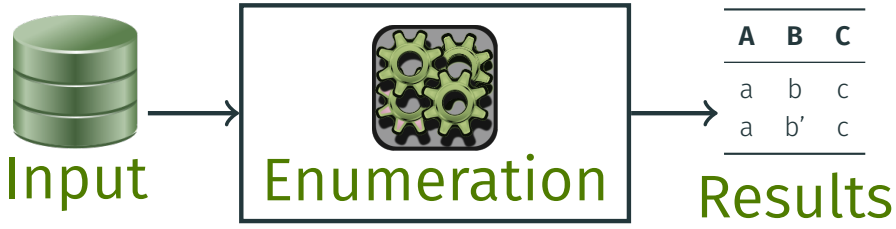
Currently:



General idea for enumeration

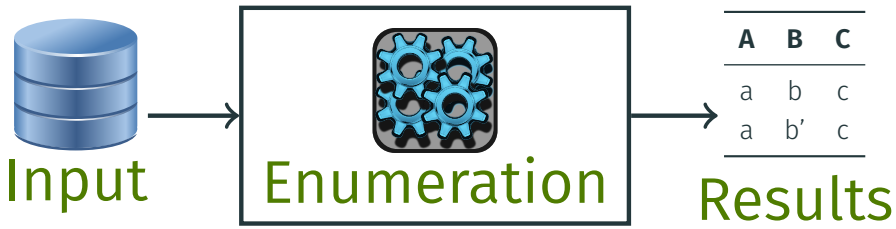
Currently:

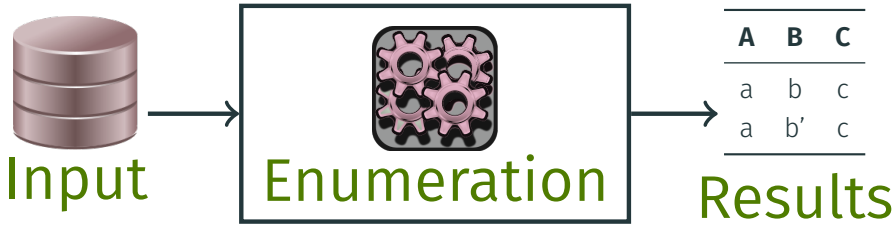




General idea for enumeration

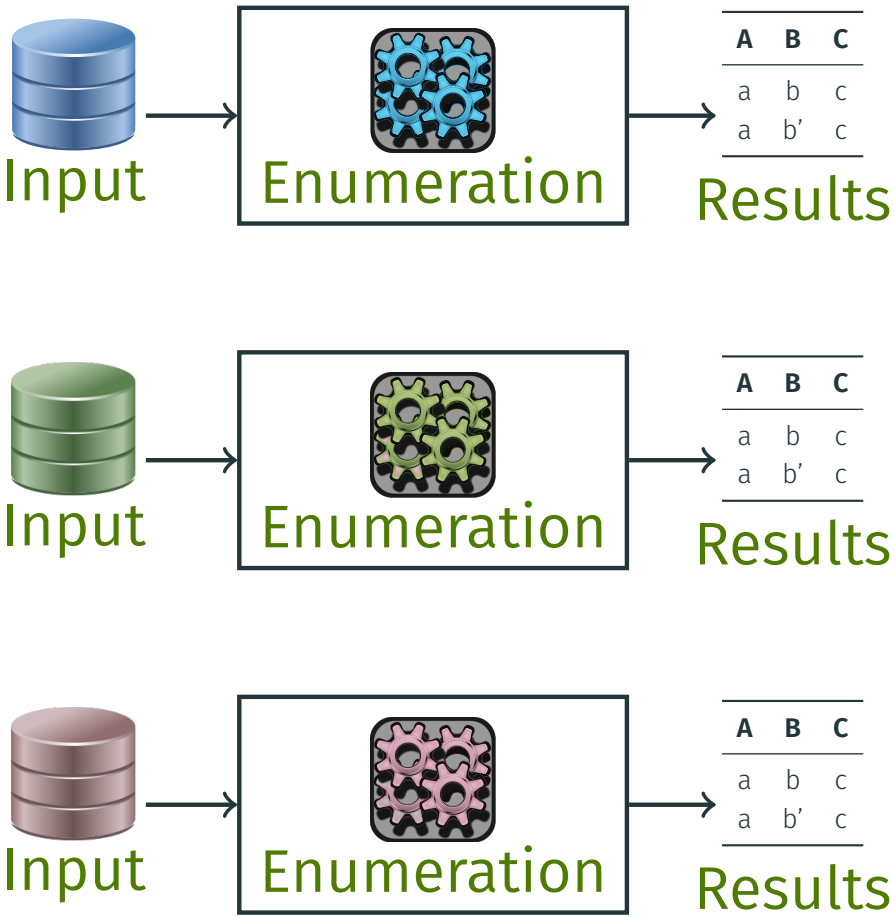
Currently:





General idea for enumeration

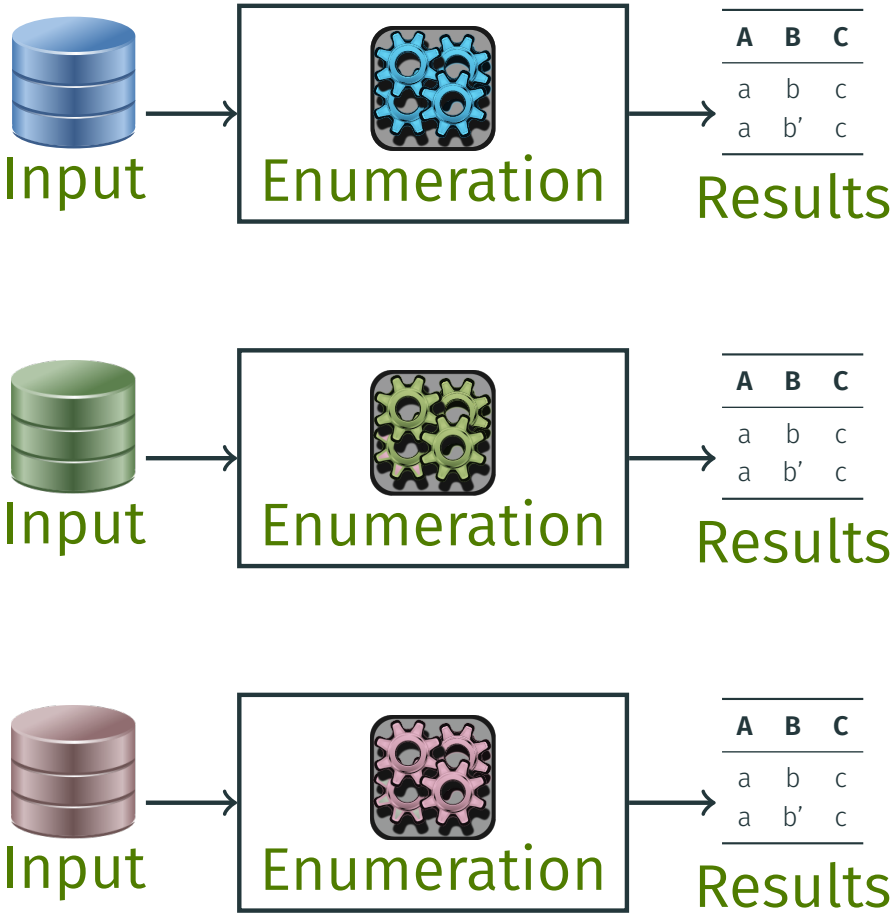
Currently:



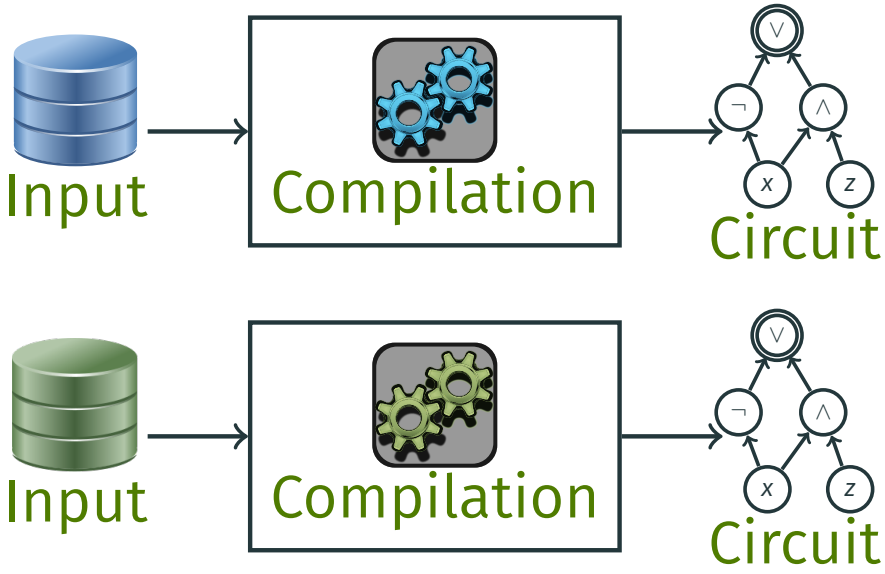
Our idea:

General idea for enumeration

Currently:

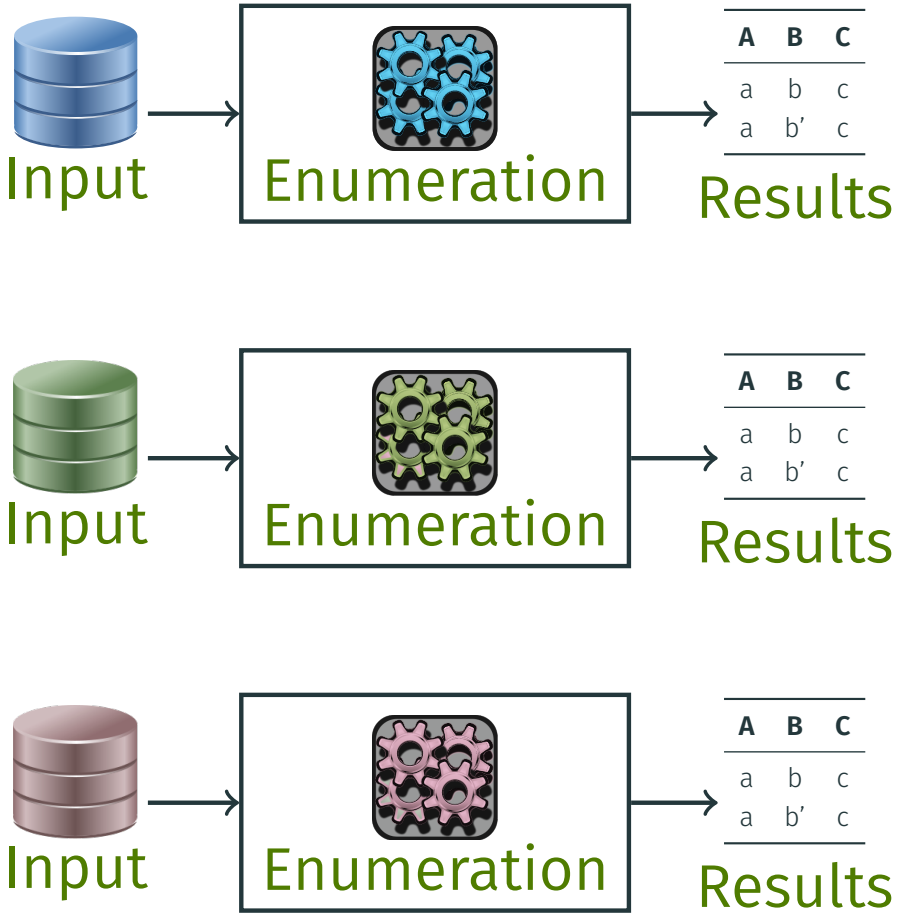


Our idea:

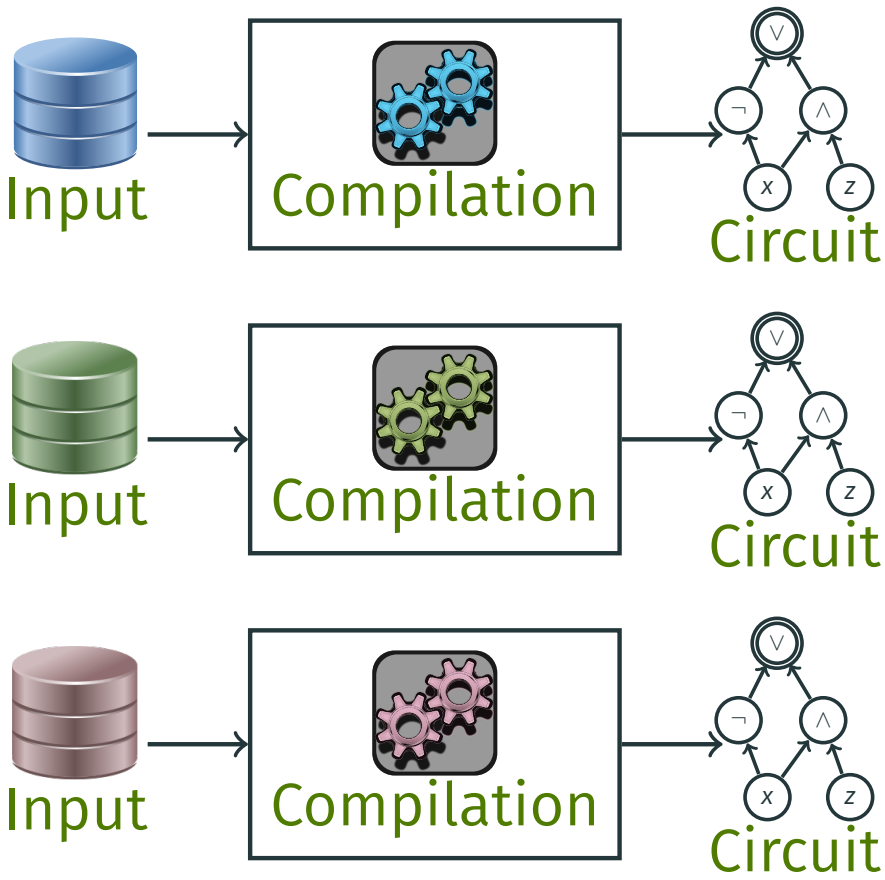


General idea for enumeration

Currently:

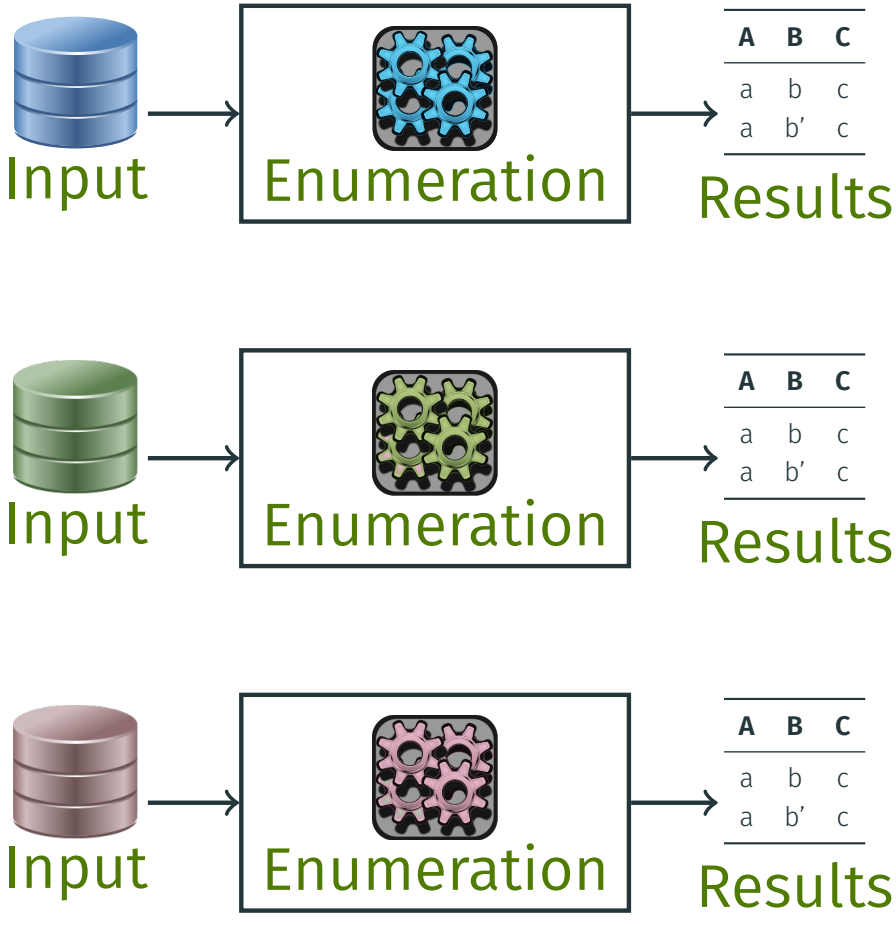


Our idea:

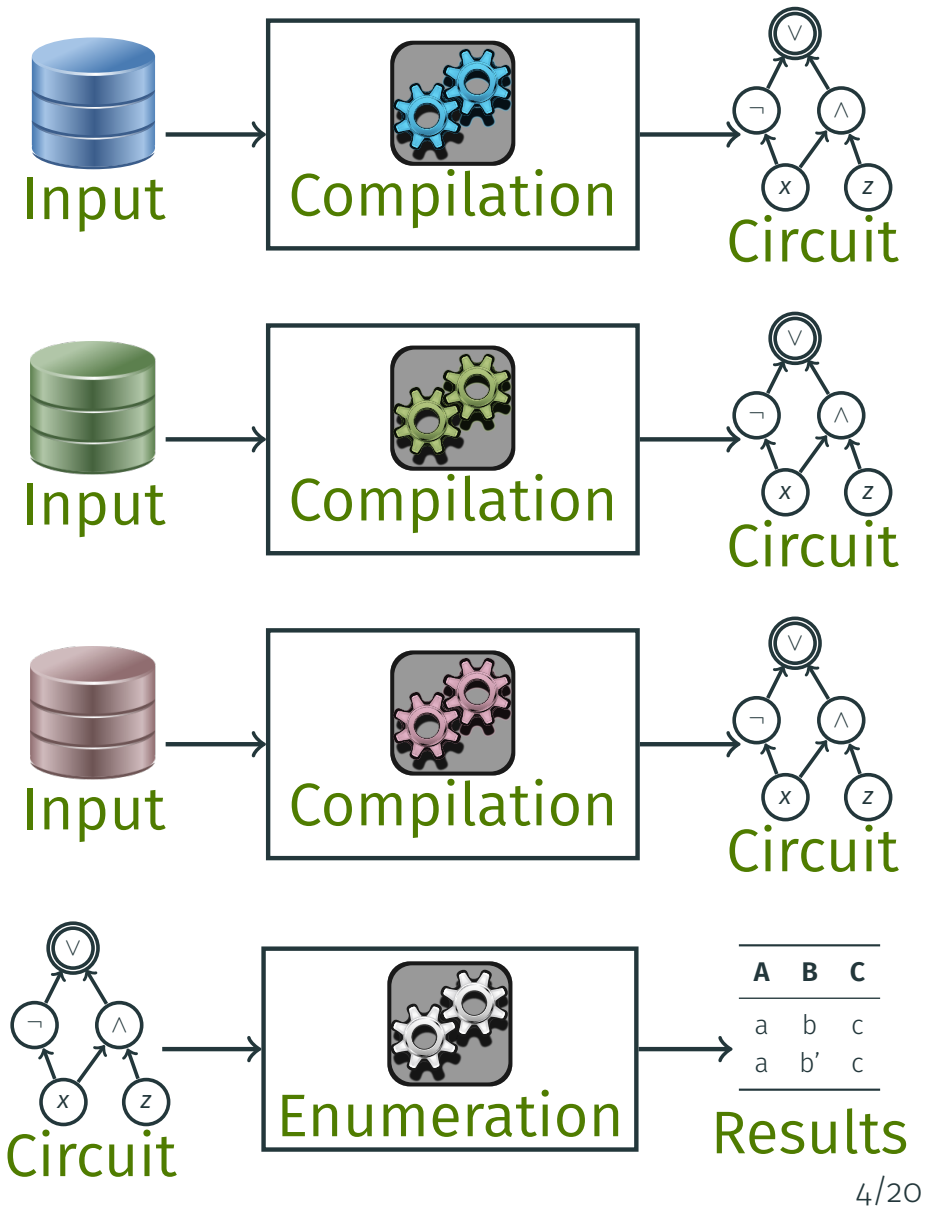


General idea for enumeration

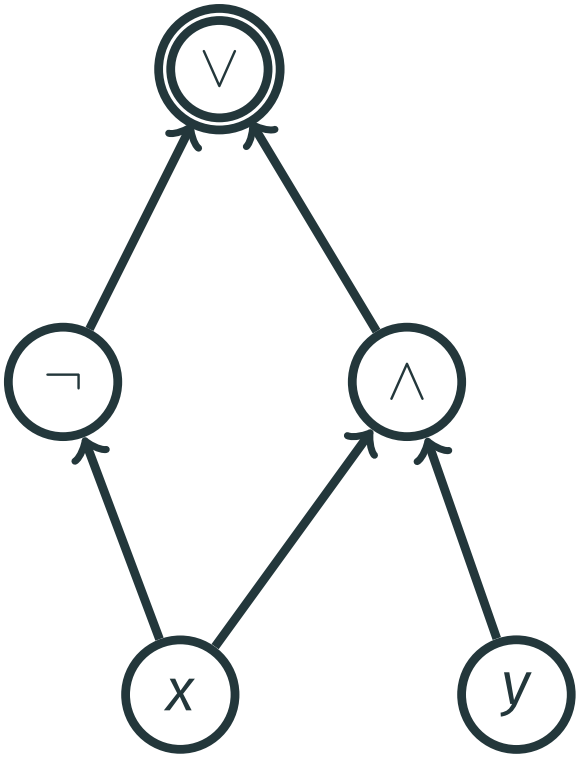
Currently:



Our idea:

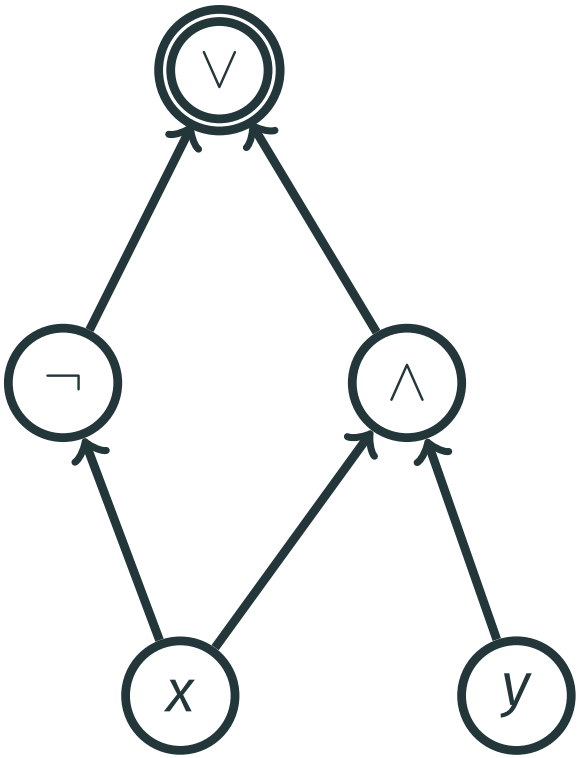


Boolean circuits



- Directed acyclic graph of gates
- Output gate:
- Variable gates: x
- Internal gates: $\vee$ $\wedge$ $\neg$

Boolean circuits

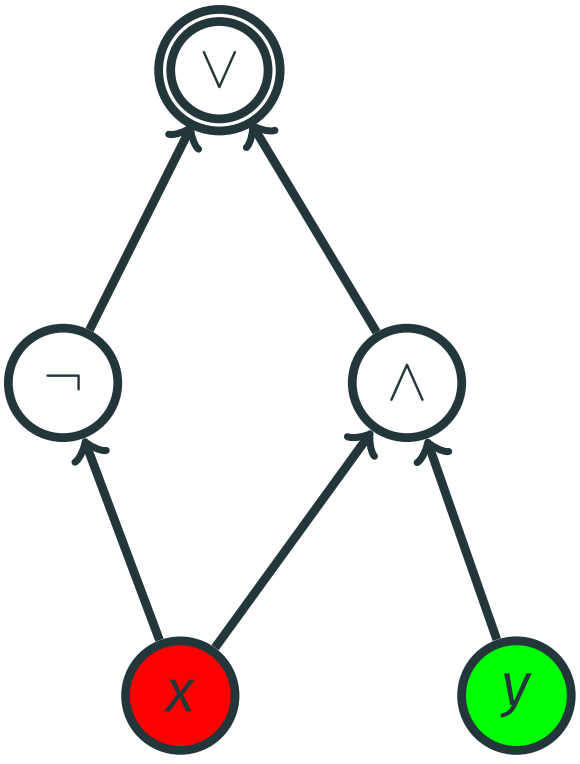


- Directed acyclic graph of gates
- Output gate: 
- Variable gates: 

- Internal gates: 

- Valuation: function from variables to $\{0, 1\}$
  Example: $\nu = \{x \mapsto 0,\ y \mapsto 1\}$...

Boolean circuits

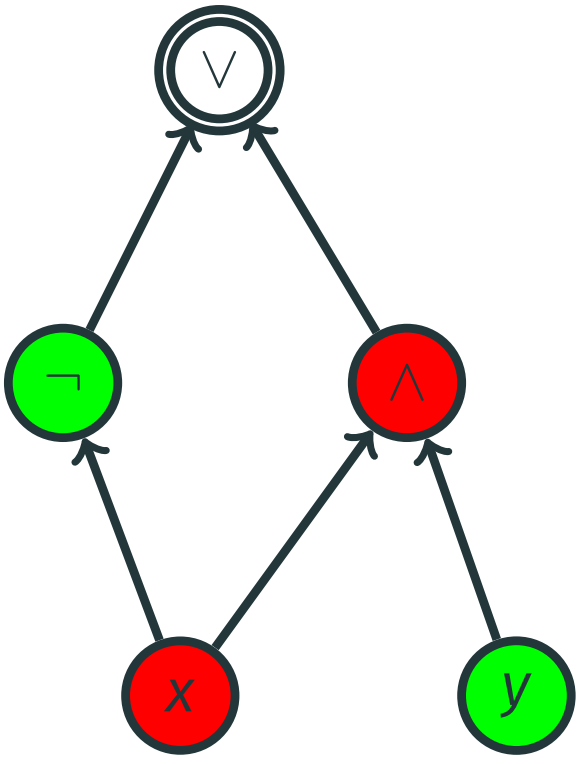


- Directed acyclic graph of gates
- Output gate: 
- Variable gates: 

- Internal gates: 

- Valuation: function from variables to $\{0, 1\}$
  Example: $\nu = \{x \mapsto 0,\ y \mapsto 1\}$...

Boolean circuits

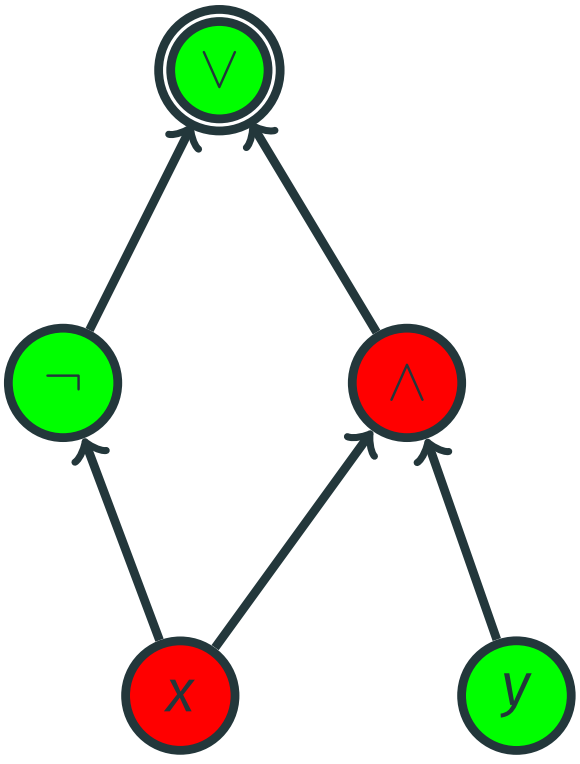


- Directed acyclic graph of gates
- Output gate: 
- Variable gates: 

- Internal gates: 

- Valuation: function from variables to $\{0, 1\}$
  Example: $\nu = \{x \mapsto 0,\ y \mapsto 1\}$...

Boolean circuits

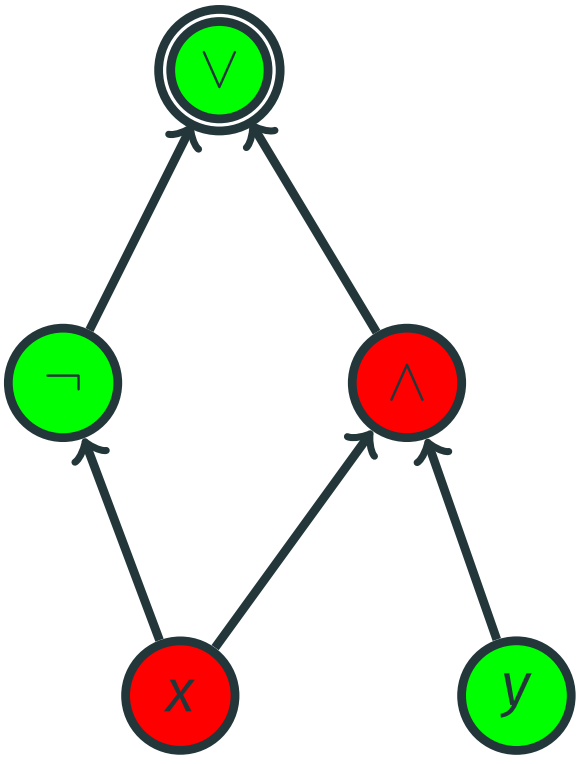


- Directed acyclic graph of gates
- Output gate: 
- Variable gates: 

- Internal gates: 

- Valuation: function from variables to $\{0, 1\}$
  Example: $\nu = \{x \mapsto 0,\ y \mapsto 1\}$... mapped to 1

Boolean circuits

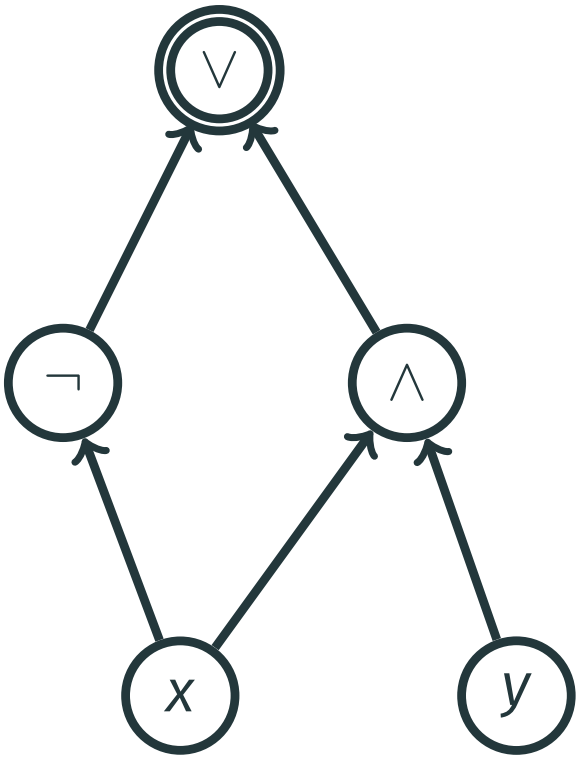


- Directed acyclic graph of gates
- Output gate: 
- Variable gates: 

- Internal gates: 

- Valuation: function from variables to $\{0, 1\}$
  Example: $\nu = \{x \mapsto 0,\ y \mapsto 1\}$... mapped to 1
- Assignment: set of variables mapped to 1
  Example: $S_\nu = \{y\}$; more concise than ν

Boolean circuits



- Directed acyclic graph of gates
- Output gate:
- Variable gates: x
- Internal gates: $\vee$ $\wedge$ $\neg$
- Valuation: function from variables to $\{0, 1\}$
  Example: $\nu = \{x \mapsto 0,\ y \mapsto 1\}$... mapped to 1
- Assignment: set of variables mapped to 1
  Example: $S_\nu = \{y\}$; more concise than ν

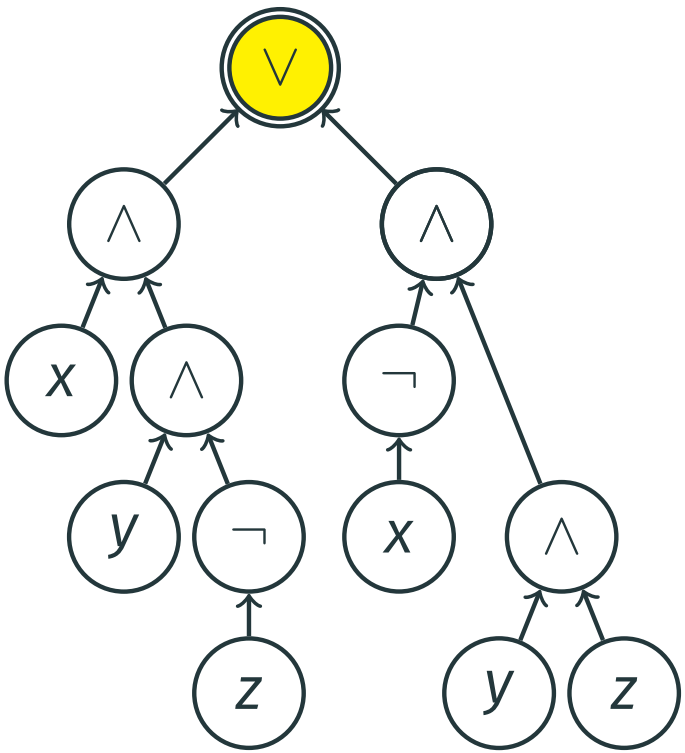
Our task: Enumerate all satisfying assignments of an input circuit

Circuit restrictions

d-DNNF:

- $\vee$ are all deterministic:

The inputs are mutually exclusive
(= no valuation ν makes two inputs simultaneously evaluate to 1)



Circuit restrictions

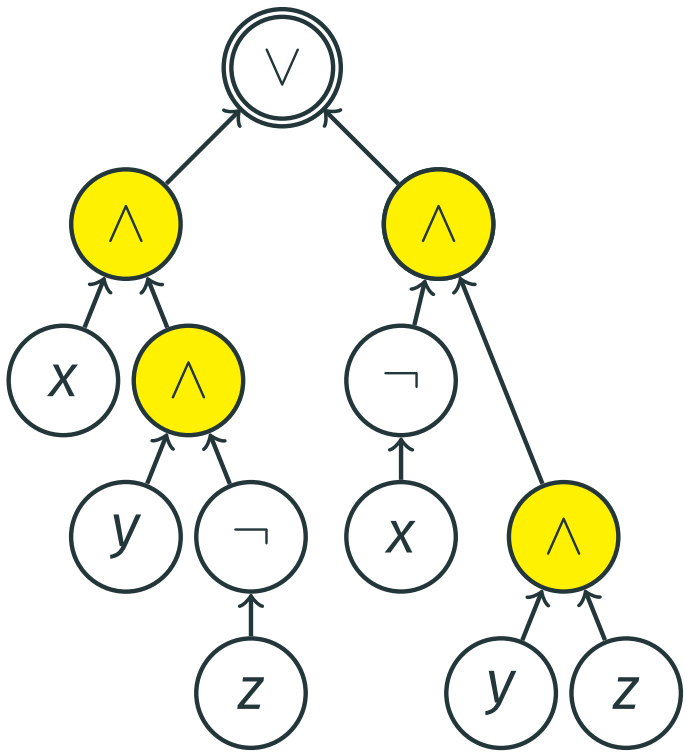
d-DNNF:

- $\vee$ are all deterministic:

The inputs are mutually exclusive
(= no valuation ν makes two inputs simultaneously evaluate to 1)

- $\wedge$ are all decomposable:

The inputs are independent
(= no variable x has a path to two different inputs)



Circuit restrictions

d-DNNF:

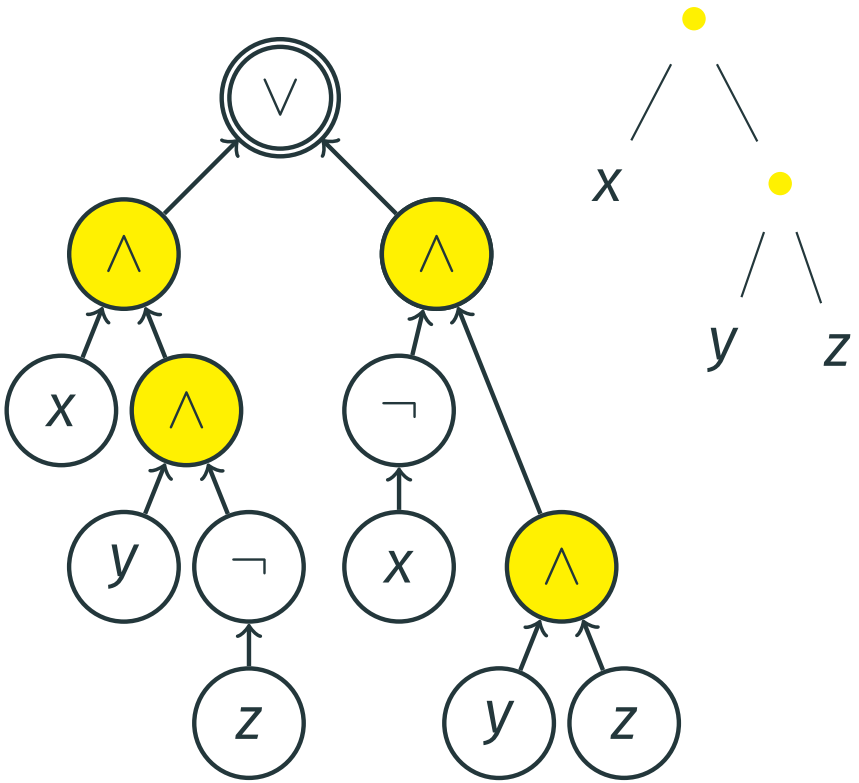
- $\vee$ are all deterministic:

The inputs are mutually exclusive
(= no valuation ν makes two inputs simultaneously evaluate to 1)

- $\wedge$ are all decomposable:

The inputs are independent
(= no variable x has a path to two different inputs)

v-tree: $\wedge$-gates follow a tree on the variables



Main results

Theorem

Given a d-DNNF circuit C with a v-tree T, we can enumerate its satisfying assignments with preprocessing linear in $|C| + |T|$ and delay linear in each assignment

Main results

Theorem

Given a d-DNNF circuit C with a v-tree T, we can enumerate its satisfying assignments with preprocessing linear in $|C| + |T|$ and delay linear in each assignment

Also: restrict to assignments of constant size $k \in \mathbb{N}$
(at most k variables are set to 1):

Theorem

Given a d-DNNF circuit C with a v-tree T, we can enumerate its satisfying assignments of size $\leq k$ with preprocessing linear in $|C| + |T|$ and constant delay

Application 1: Factorized databases

Orders (O for short)

customer	day	dish
Elise	Monday	burger
Elise	Friday	burger
Steve	Friday	hotdog
Joe	Friday	hotdog

Dish (D for short)

dish	item
burger	patty
burger	onion
burger	bun
hotdog	bun
hotdog	onion
hotdog	sausage

Items (I for short)

item	price
patty	6
onion	2
bun	2
sausage	4

Consider the join of the above relations:

O(customer, day, dish), D(dish, item), I(item, price)

customer	day	dish	item	price
Elise	Monday	burger	patty	6
Elise	Monday	burger	onion	2
Elise	Monday	burger	bun	2
Elise	Friday	burger	patty	6
Elise	Friday	burger	onion	2
Elise	Friday	burger	bun	2
...	...	...	...	...

(Slides courtesy of Dan Olteanu)

Application 1: Factorized databases

O(customer, day, dish), D(dish, item), I(item, price)

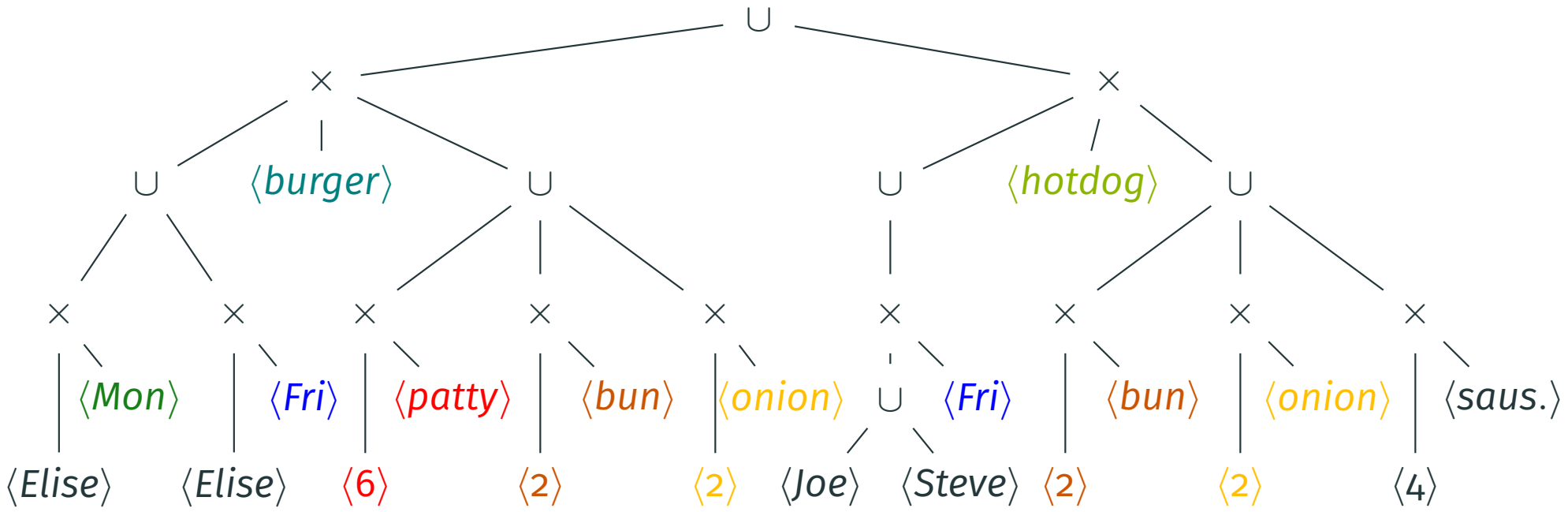
customer	day	dish	item	price
Elise	Monday	burger	patty	6
Elise	Monday	burger	onion	2
Elise	Monday	burger	bun	2
Elise	Friday	burger	patty	6
Elise	Friday	burger	onion	2
Elise	Friday	burger	bun	2
...	...	...	...	...

A relational algebra expression encoding the above query result is:

$$
\begin{array}{lllllllllll}
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Monday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \mathrm{patty} \rangle & \times & \langle 6 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Monday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \mathrm{onion} \rangle & \times & \langle 2 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Monday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \mathrm{bun} \rangle & \times & \langle 2 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Friday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \mathrm{patty} \rangle & \times & \langle 6 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Friday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \mathrm{onion} \rangle & \times & \langle 2 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Friday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \mathrm{bun} \rangle & \times & \langle 2 \rangle & \cup \ldots
\end{array}
$$

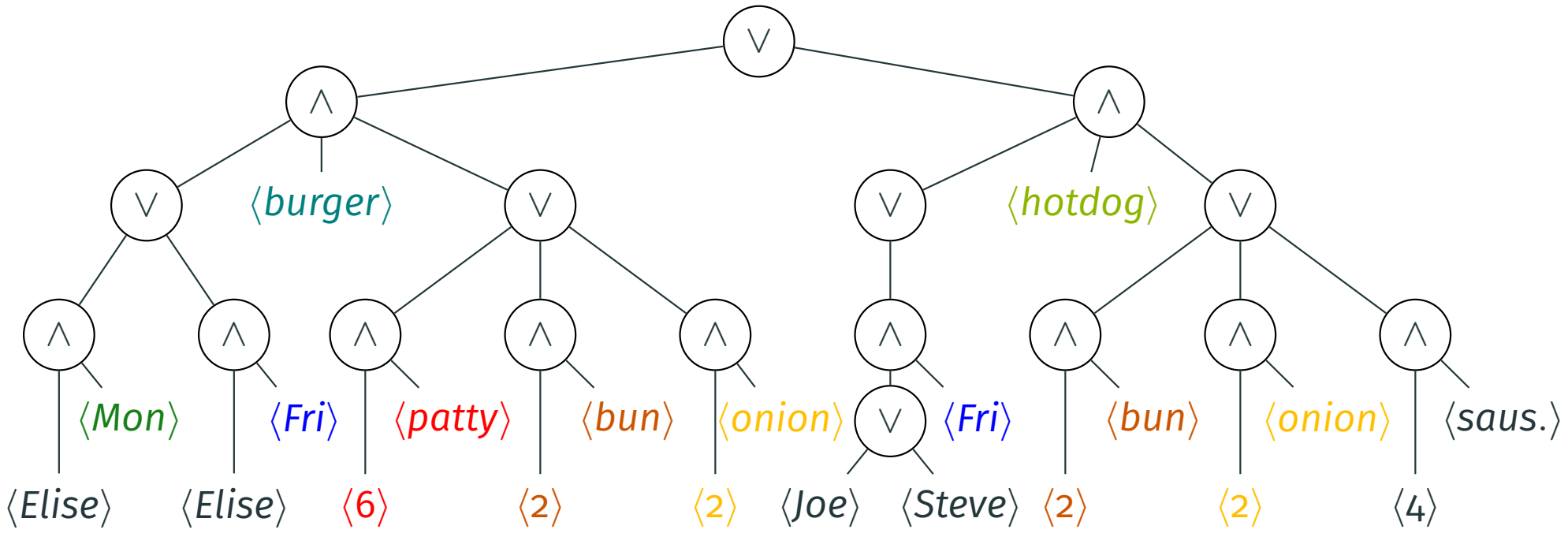
(Slides courtesy of Dan Olteanu)

Application 1: Factorized databases



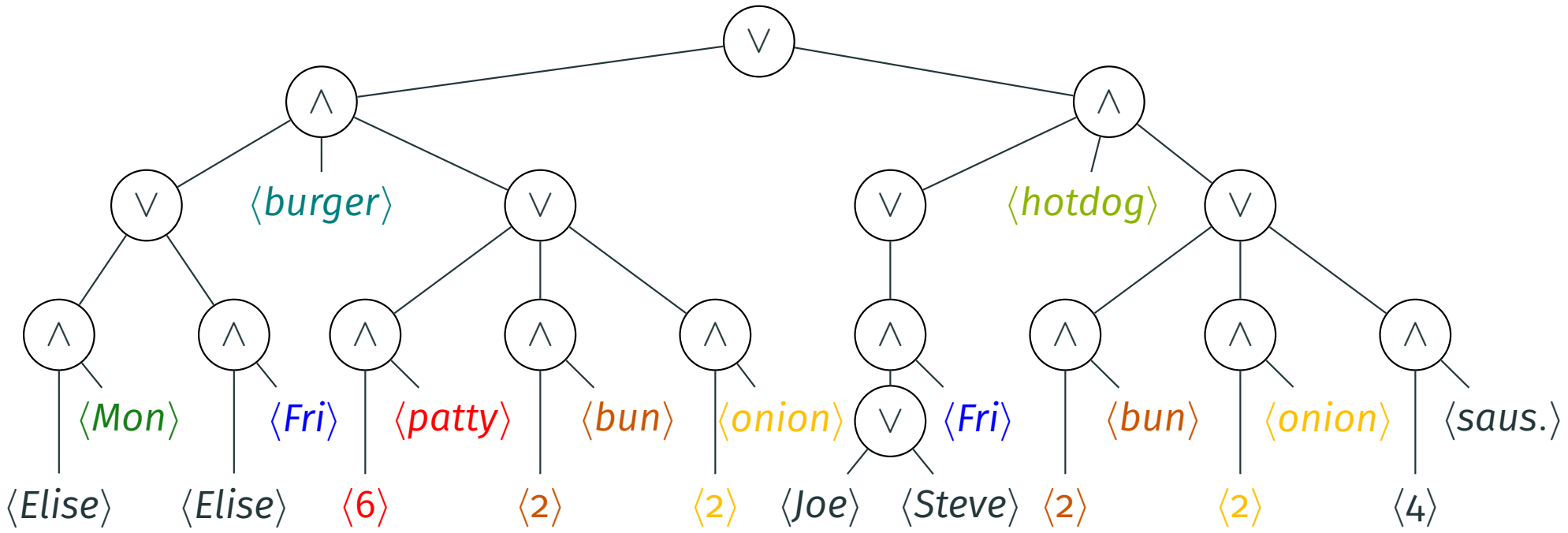
(Slides courtesy of Dan Olteanu)

Application 1: Factorized databases



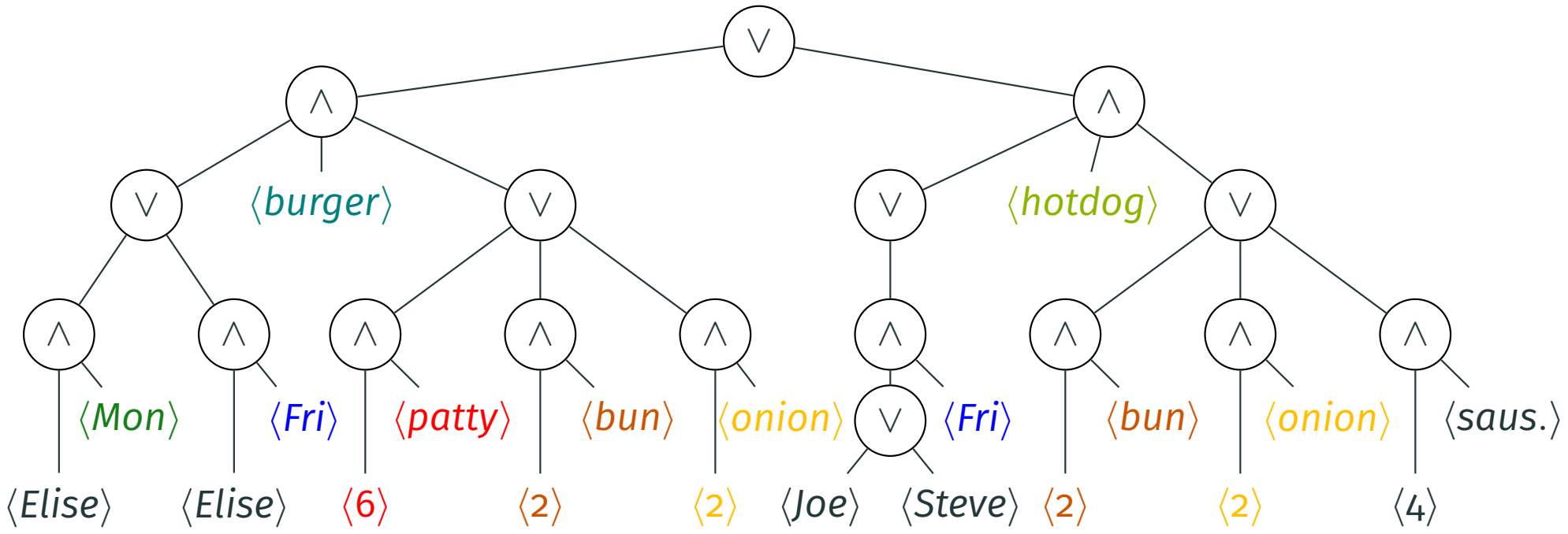
(Slides courtesy of Dan Olteanu)

Application 1: Factorized databases



- Decomposable: by definition (following the schema)
- Deterministic: we do not obtain the same tuple multiple times

Application 1: Factorized databases



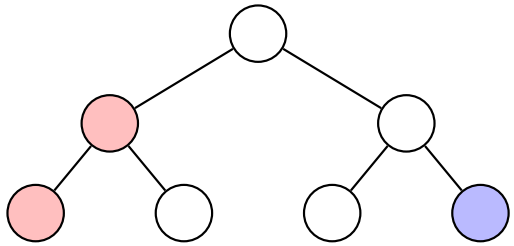
- Decomposable: by definition (following the schema)
- Deterministic: we do not obtain the same tuple multiple times

Theorem (Strenghtens result of [Olteanu and Závodnỳ, 2015])

Given a deterministic factorized representation, we can enumerate its tuples with linear preprocessing and constant delay

Query evaluation on trees

Database: a tree T where nodes have a color from an alphabet ○ ● ●



Query Q: a sentence in monadic second-order logic (MSO)

- $P_{●}(x)$ means "x is blue"
- $x \to y$ means "x is the parent of y"

"Is there both a pink and a blue node?"

$\exists x\, y\ P_{●}(x) \wedge P_{●}(y)$

Result: TRUE/FALSE indicating if T satisfies the query Q

Computational complexity as a function of the tree T
(the query Q is fixed)

(Slides courtesy of Pierre Bourhis)

Application 2: Query evaluation

- Compute the results (a, b, c) of a query $Q(x, y, z)$ on a tree T
  - $\rightarrow$ Generalizes to bounded-treewidth databases

Application 2: Query evaluation

- Compute the results (a, b, c) of a query $Q(x, y, z)$ on a tree T
  - $\rightarrow$ Generalizes to bounded-treewidth databases
- Query given as a deterministic tree automaton
  - $\rightarrow$ Captures monadic second-order (data-independent translation)
  - $\rightarrow$ Captures conjunctive queries, SQL, etc.

Application 2: Query evaluation

- Compute the results (a, b, c) of a query $Q(x, y, z)$ on a tree T
  - $\rightarrow$ Generalizes to bounded-treewidth databases
- Query given as a deterministic tree automaton
  - $\rightarrow$ Captures monadic second-order (data-independent translation)
  - $\rightarrow$ Captures conjunctive queries, SQL, etc.

$\rightarrow$ We can construct a d-DNNF that describes the query results

Application 2: Query evaluation

- Compute the results (a, b, c) of a query $Q(x, y, z)$ on a tree T
  - $\rightarrow$ Generalizes to bounded-treewidth databases
- Query given as a deterministic tree automaton
  - $\rightarrow$ Captures monadic second-order (data-independent translation)
  - $\rightarrow$ Captures conjunctive queries, SQL, etc.

$\rightarrow$ We can construct a d-DNNF that describes the query results

Theorem (Recaptures [Bagan, 2006], [Kazana and Segoufin, 2013])
For any constant $k \in \mathbb{N}$ and fixed MSO query Q,
given a database D of treewidth $\leq k$, the results of Q on D
can be enumerated with linear preprocessing in D and linear delay
in each answer ($\rightarrow$ constant delay for free first-order variables)

Application 2bis: Query evaluation under relabelings

- Compute the results of a query on data that can be updated
- Goal: avoid running the linear preprocessing at each update
- Update complexity: time required to perform an update and reset the enumeration

Application 2bis: Query evaluation under relabelings

- Compute the results of a query on data that can be updated
- Goal: avoid running the linear preprocessing at each update
- Update complexity: time required to perform an update and reset the enumeration

Type of updates:

- Relabel a tree node
  - $\rightarrow$ On a treelike instance, add/remove a unary fact
- Insert and delete a tree leaf

Application 2bis: Query evaluation under relabelings (results)

Work	Data	Delay	Updates

Application 2bis: Query evaluation under relabelings (results)

Work	Data	Delay	Updates
[Bagan, 2006], [Kazana and Segoufin, 2013]	trees	$O(1)$	N/A

Application 2bis: Query evaluation under relabelings (results)

Work	Data	Delay	Updates
[Bagan, 2006], [Kazana and Segoufin, 2013]	trees	$O(1)$	N/A
[Losemann and Martens, 2014]	words	$O(\log n)$	$O(\log n)$

Application 2bis: Query evaluation under relabelings (results)

Work	Data	Delay	Updates
[Bagan, 2006], [Kazana and Segoufin, 2013]	trees	$O(1)$	N/A
[Losemann and Martens, 2014]	words	$O(\log n)$	$O(\log n)$
[Losemann and Martens, 2014]	trees	$O(\log^2 n)$	$O(\log^2 n)$

Application 2bis: Query evaluation under relabelings (results)

Work	Data	Delay	Updates
[Bagan, 2006], [Kazana and Segoufin, 2013]	trees	$O(1)$	N/A
[Losemann and Martens, 2014]	words	$O(\log n)$	$O(\log n)$
[Losemann and Martens, 2014]	trees	$O(\log^2 n)$	$O(\log^2 n)$
[Niewerth and Segoufin, 2018]	words	$O(1)$	$O(\log n)$

Application 2bis: Query evaluation under relabelings (results)

Work	Data	Delay	Updates
[Bagan, 2006], [Kazana and Segoufin, 2013]	trees	$O(1)$	N/A
[Losemann and Martens, 2014]	words	$O(\log n)$	$O(\log n)$
[Losemann and Martens, 2014]	trees	$O(\log^2 n)$	$O(\log^2 n)$
[Niewerth and Segoufin, 2018]	**words**	$O(1)$	$O(\log n)$
[Amarilli, Bourhis, Mengel, 2018]	**trees**	$O(1)$	$O(\log n)$ for relabelings

Application 2bis: Query evaluation under relabelings (results)

Work	Data	Delay	Updates
[Bagan, 2006], [Kazana and Segoufin, 2013]	trees	$O(1)$	N/A
[Losemann and Martens, 2014]	words	$O(\log n)$	$O(\log n)$
[Losemann and Martens, 2014]	trees	$O(\log^2 n)$	$O(\log^2 n)$
[Niewerth and Segoufin, 2018]	words	$O(1)$	$O(\log n)$
[Amarilli, Bourhis, Mengel, 2018]	trees	$O(1)$	$O(\log n)$ for relabelings

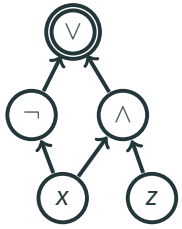
Theorem ([Amarilli, Bourhis, Mengel, 2018], to appear at ICDT)

For any constant $k \in \mathbb{N}$ and fixed MSO query Q,
given a database D of treewidth $\leq k$, the results of Q on D
can be enumerated with linear preprocessing in D and linear delay
in each answer ($\rightarrow$ constant delay for free first-order variables)
and logarithmic update time for relabelings

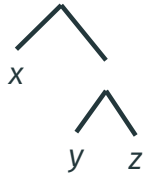
Proof techniques

Proof overview

Preprocessing phase:



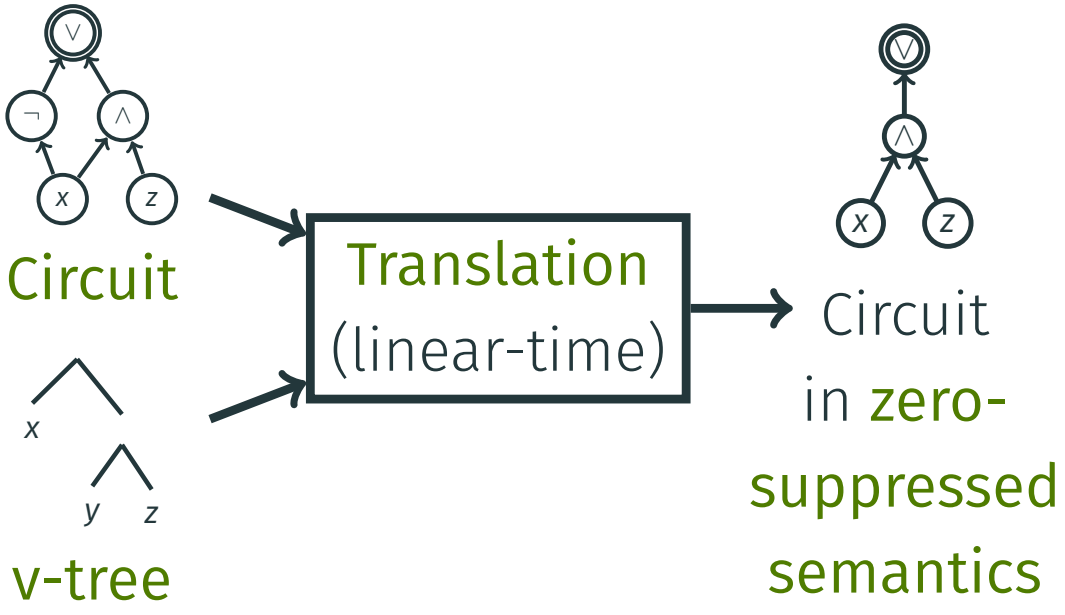
Circuit



v-tree

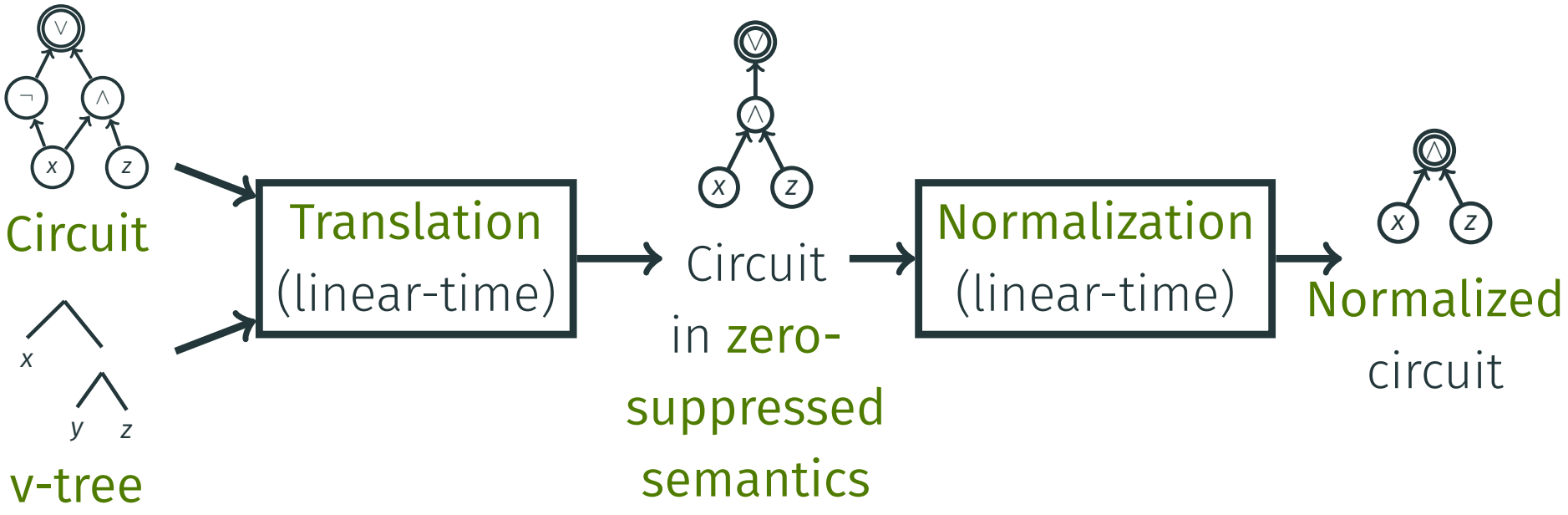
Proof overview

Preprocessing phase:



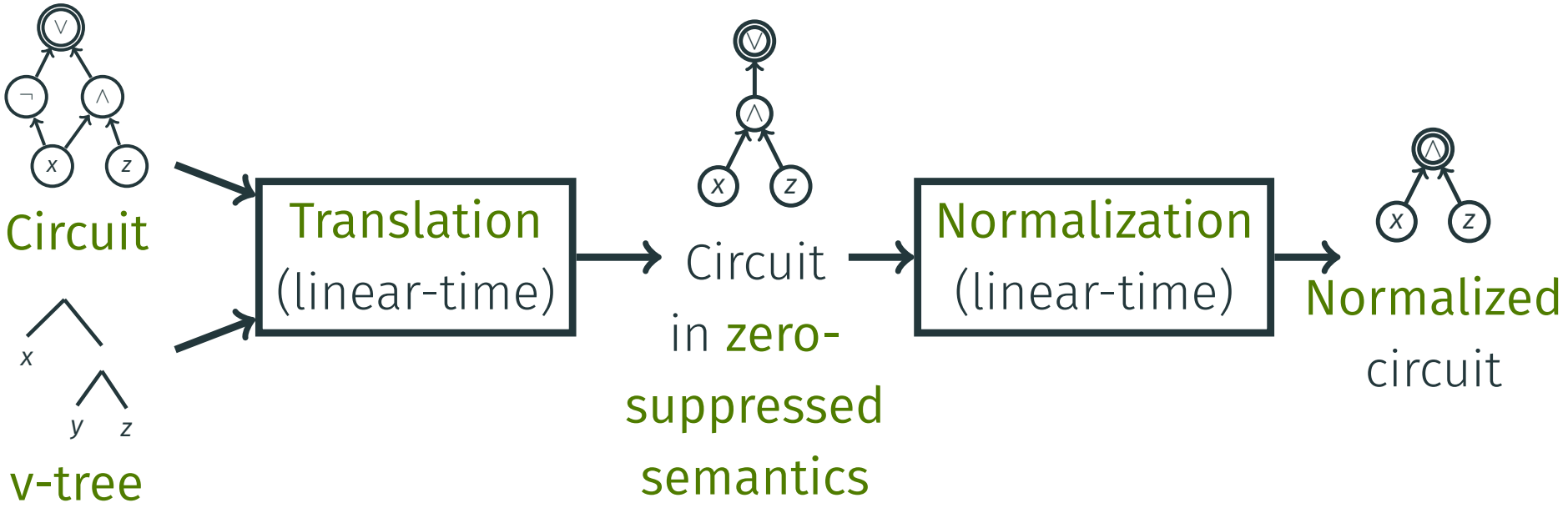
Proof overview

Preprocessing phase:

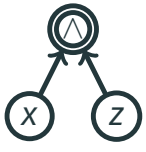


Proof overview

Preprocessing phase:



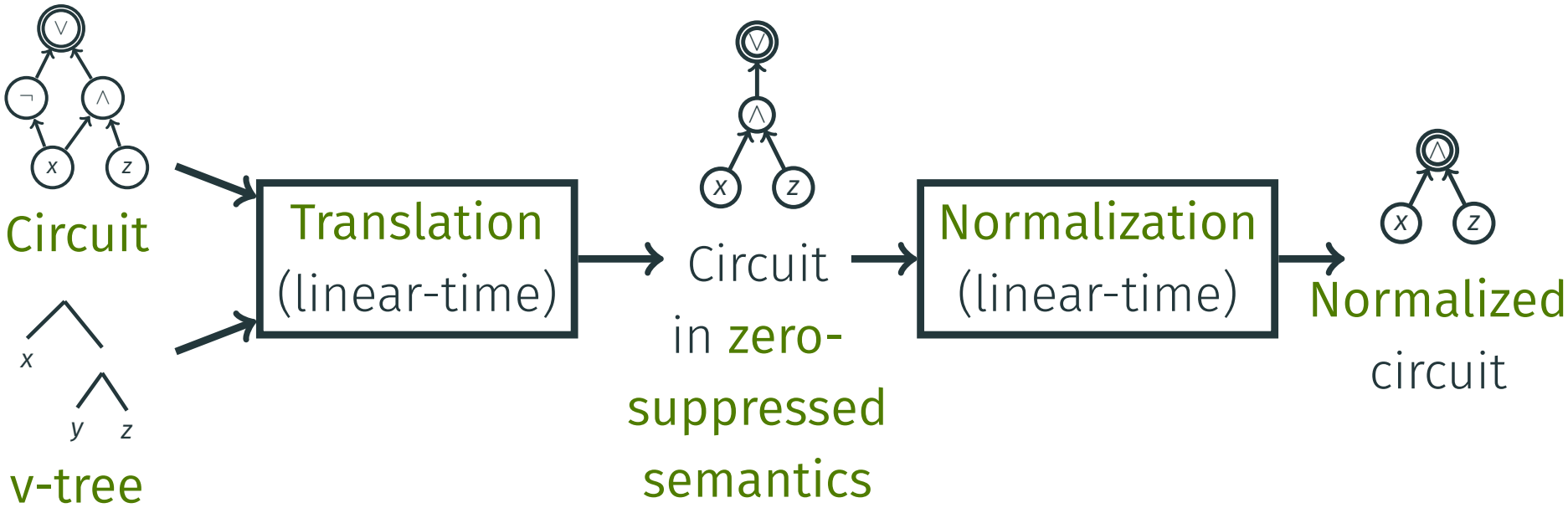
Enumeration phase:



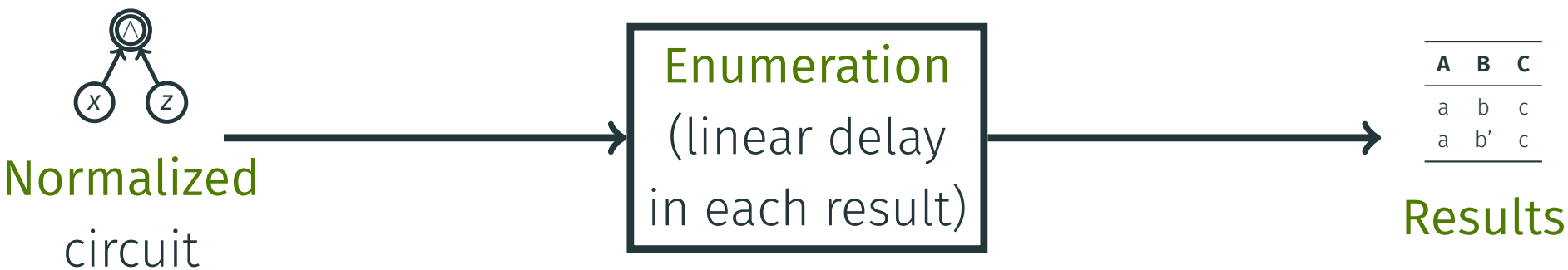
Normalized
circuit

Proof overview

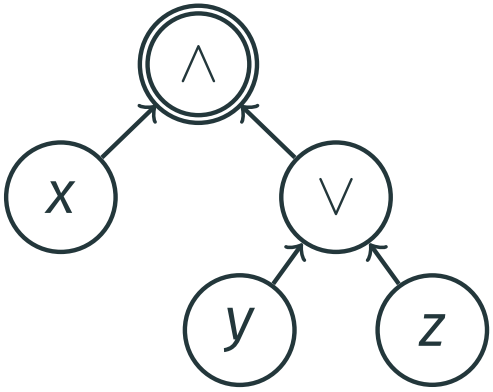
Preprocessing phase:



Enumeration phase:

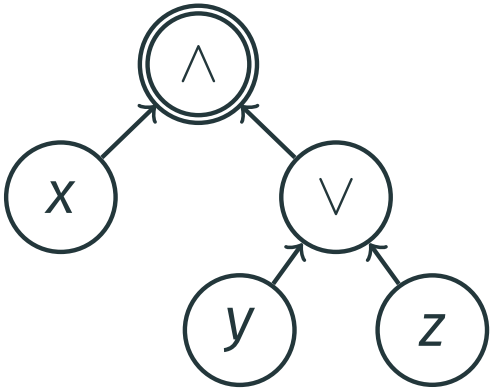


Zero-suppressed semantics



Special zero-suppressed semantics for circuits:

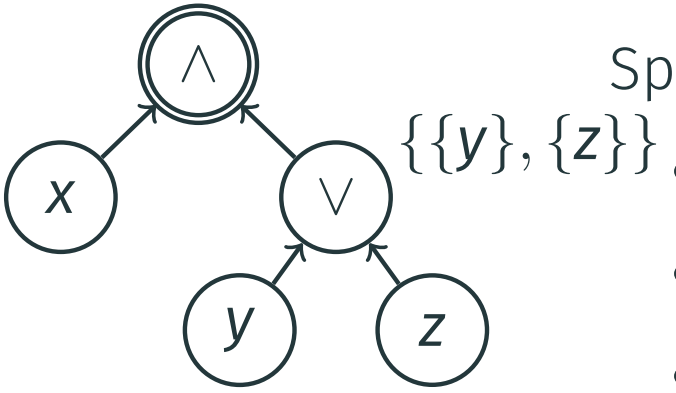
Zero-suppressed semantics



Special zero-suppressed semantics for circuits:

- No NOT-gate
- Each gate captures a set of assignments
- Bottom-up definition with $\times$ and $\cup$

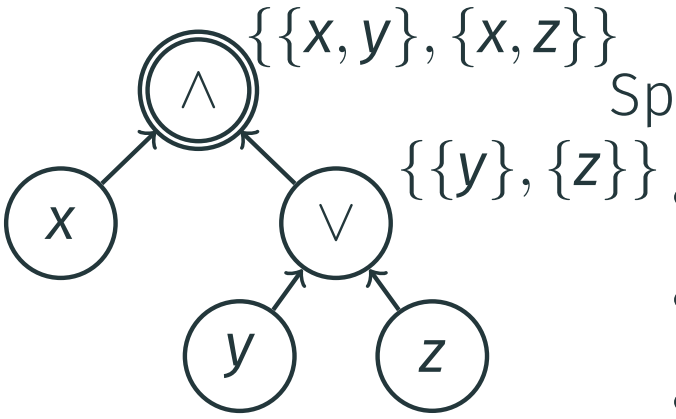
Zero-suppressed semantics



Special zero-suppressed semantics for circuits:

- No NOT-gate
- Each gate captures a set of assignments
- Bottom-up definition with $\times$ and $\cup$

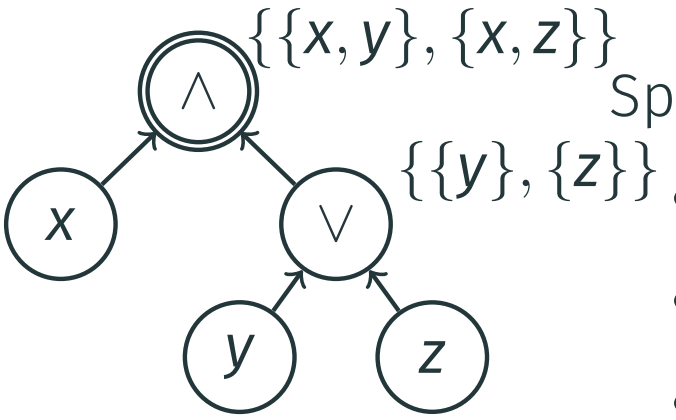
Zero-suppressed semantics



Special zero-suppressed semantics for circuits:

- No NOT-gate
- Each gate captures a set of assignments
- Bottom-up definition with $\times$ and $\cup$

Zero-suppressed semantics

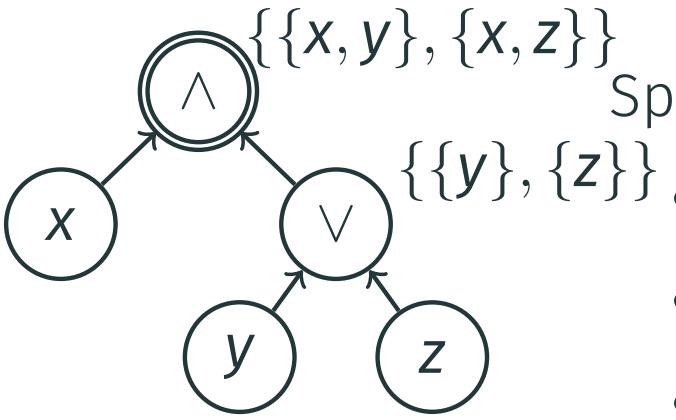


Special zero-suppressed semantics for circuits:

- No NOT-gate
- Each gate captures a set of assignments
- Bottom-up definition with $\times$ and $\cup$

- d-DNNF: $\cup$ are disjoint, $\times$ are on disjoint sets

Zero-suppressed semantics



Special zero-suppressed semantics for circuits:

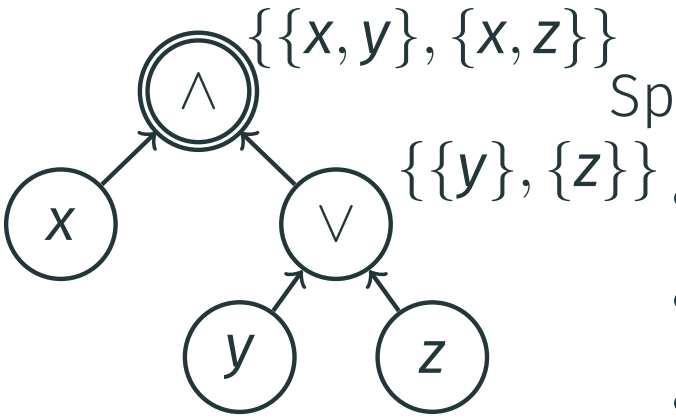
- No NOT-gate
- Each gate captures a set of assignments
- Bottom-up definition with $\times$ and $\cup$

- d-DNNF: $\cup$ are disjoint, $\times$ are on disjoint sets

Many equivalent ways to understand this:

- Generalization of factorized representations
- Analogue of zero-suppressed OBDDs (implicit negation)
- Arithmetic circuits: $\times$ and $+$ on polynomials

Zero-suppressed semantics



Special zero-suppressed semantics for circuits:

- No NOT-gate
- Each gate captures a set of assignments
- Bottom-up definition with $\times$ and $\cup$

- d-DNNF: $\cup$ are disjoint, $\times$ are on disjoint sets

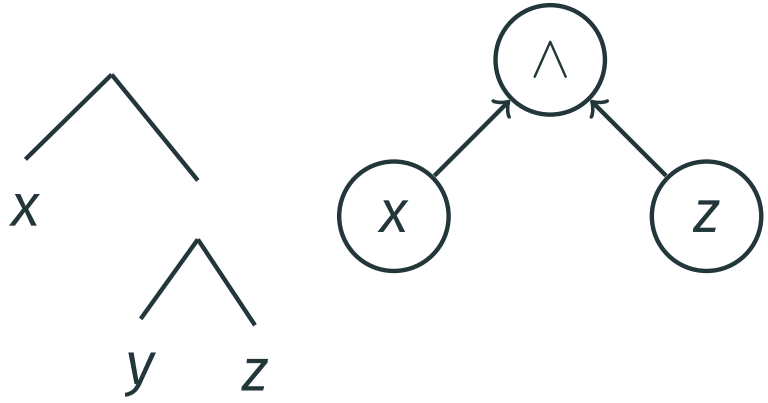
Many equivalent ways to understand this:

- Generalization of factorized representations
- Analogue of zero-suppressed OBDDs (implicit negation)
- Arithmetic circuits: $\times$ and $+$ on polynomials

Simplification: rewrite circuits to arity-two (fan-in ≤ 2)

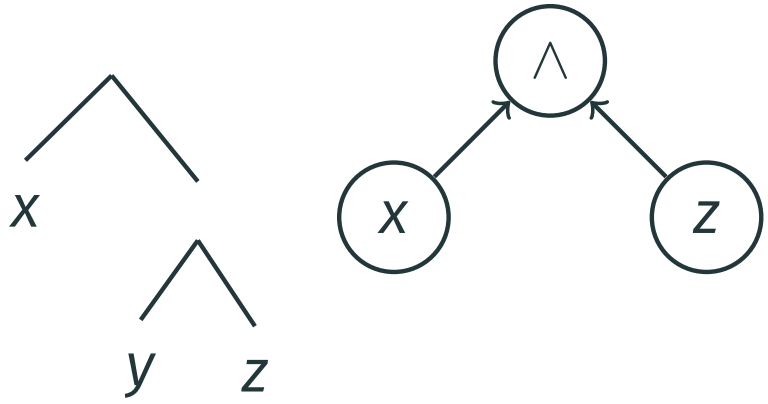
Translating to zero-suppressed semantics

- This is where we use the v-tree



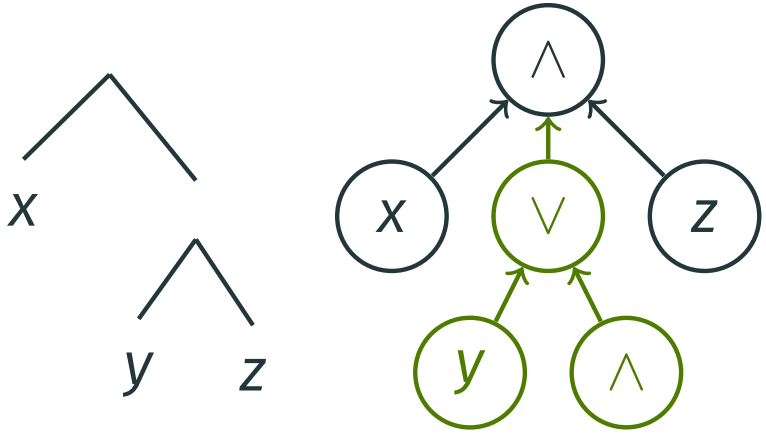
Translating to zero-suppressed semantics

- This is where we use the v-tree
- Add explicitly untested variables (smoothing)



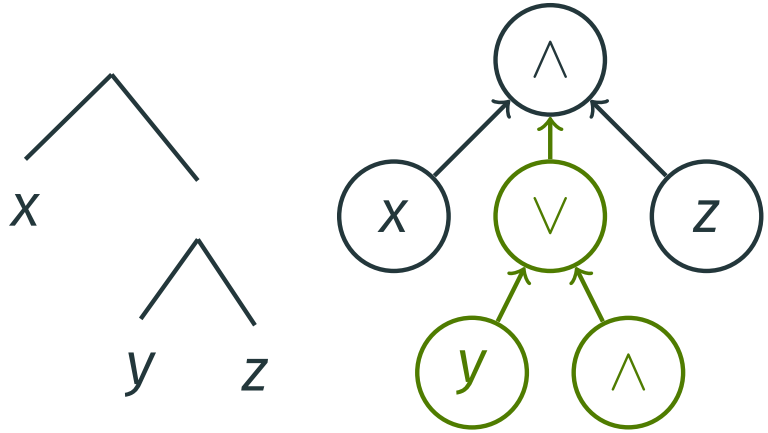
Translating to zero-suppressed semantics

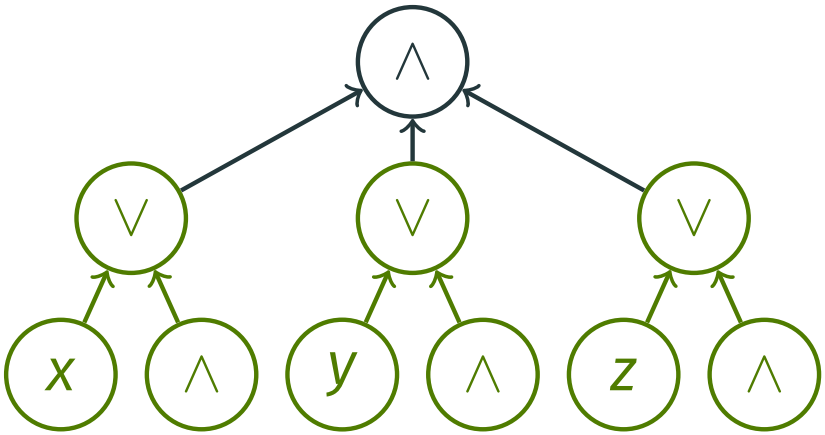
- This is where we use the v-tree
- Add explicitly untested variables (smoothing)



Translating to zero-suppressed semantics

- This is where we use the v-tree
- Add explicitly untested variables (smoothing)

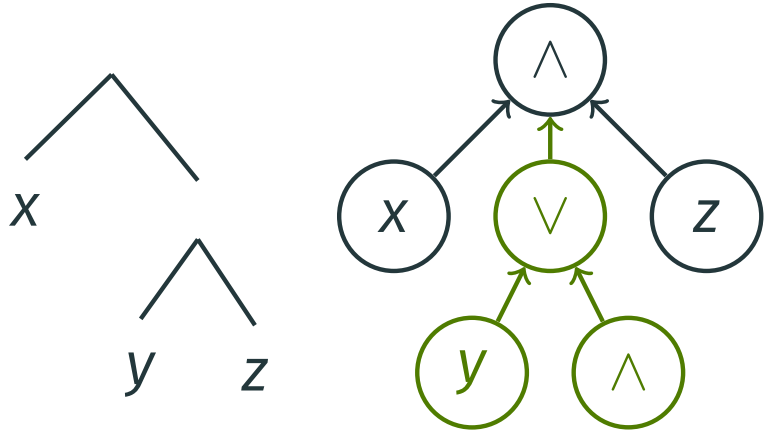


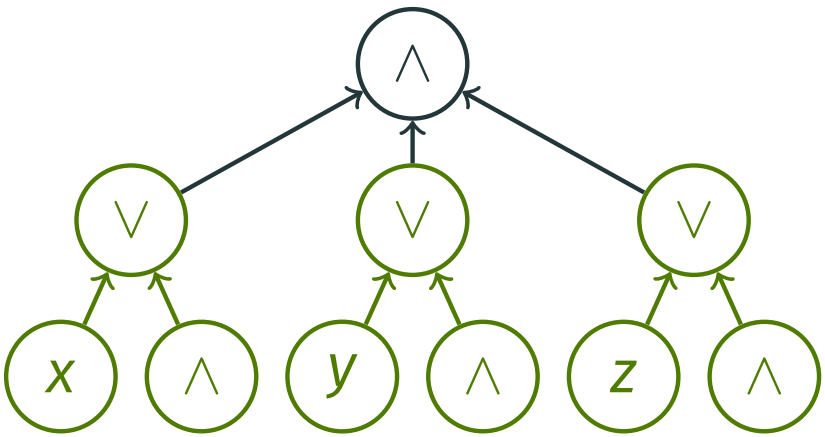


- Problem: quadratic blowup

Translating to zero-suppressed semantics

- This is where we use the v-tree
- Add explicitly untested variables (smoothing)

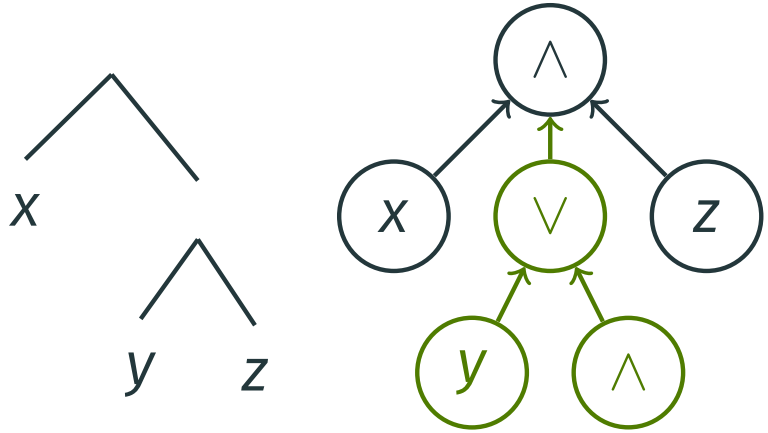


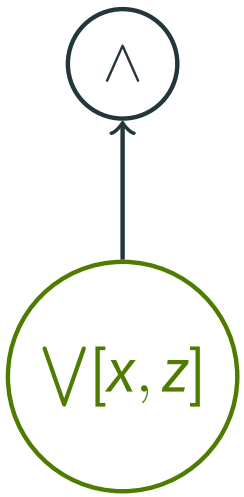


- Problem: quadratic blowup
- Solution:
  - Order $<$ on variables in the v-tree ($x < y < z$)
  - Interval $[x, z]$
  - Range gates to denote $\bigvee[x, z]$ in constant space

Translating to zero-suppressed semantics

- This is where we use the v-tree
- Add explicitly untested variables (smoothing)

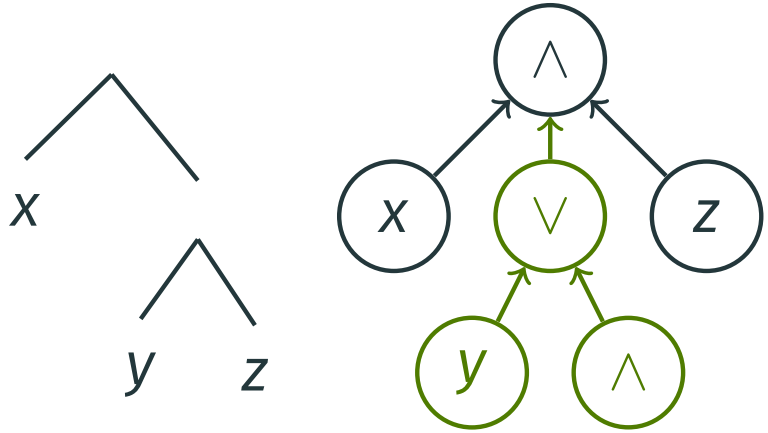


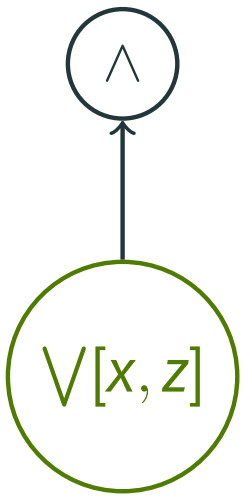


- Problem: quadratic blowup
- Solution:
  - Order $<$ on variables in the v-tree ($x < y < z$)
  - Interval $[x, z]$
  - Range gates to denote $\bigvee[x, z]$ in constant space

Translating to zero-suppressed semantics

- This is where we use the v-tree
- Add explicitly untested variables (smoothing)





- Problem: quadratic blowup
- Solution:
  - Order $<$ on variables in the v-tree ($x < y < z$)
  - Interval $[x, z]$
  - Range gates to denote $\bigvee[x, z]$ in constant space

$\rightarrow$ For MSO query evaluation: we can directly compute a circuit that captures the answers in zero-suppressed semantics

Enumerating assignments in the zero-suppressed semantics

Task: Enumerate the elements of the set $S(g)$ captured by a gate g

$\rightarrow$ E.g., for $S(g) = \{\{x, y\}, \{x, z\}\}$, enumerate $\{x, y\}$ and then $\{x, z\}$

Enumerating assignments in the zero-suppressed semantics

Task: Enumerate the elements of the set $S(g)$ captured by a gate g

$\rightarrow$ E.g., for $S(g) = \{\{x, y\}, \{x, z\}\}$, enumerate $\{x, y\}$ and then $\{x, z\}$

Base case: variable 

:

Enumerating assignments in the zero-suppressed semantics

Task: Enumerate the elements of the set $S(g)$ captured by a gate g

$\rightarrow$ E.g., for $S(g) = \{\{x, y\}, \{x, z\}\}$, enumerate $\{x, y\}$ and then $\{x, z\}$

Base case: variable 

: enumerate $\{x\}$ and stop

Enumerating assignments in the zero-suppressed semantics

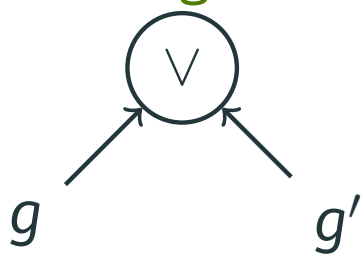
Task: Enumerate the elements of the set $S(g)$ captured by a gate g

$\rightarrow$ E.g., for $S(g) = \{\{x, y\}, \{x, z\}\}$, enumerate $\{x, y\}$ and then $\{x, z\}$

Base case: variable 

: enumerate $\{x\}$ and stop

OR-gate



Concatenation: enumerate $S(g)$
and then enumerate $S(g')$

Enumerating assignments in the zero-suppressed semantics

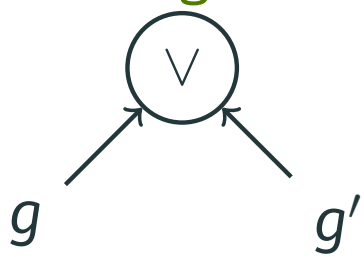
Task: Enumerate the elements of the set $S(g)$ captured by a gate g

$\rightarrow$ E.g., for $S(g) = \{\{x, y\}, \{x, z\}\}$, enumerate $\{x, y\}$ and then $\{x, z\}$

Base case: variable 

: enumerate $\{x\}$ and stop

OR-gate



Concatenation: enumerate $S(g)$
and then enumerate $S(g')$

Determinism: no duplicates

Enumerating assignments in the zero-suppressed semantics

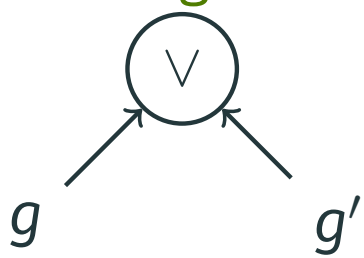
Task: Enumerate the elements of the set $S(g)$ captured by a gate g

$\rightarrow$ E.g., for $S(g) = \{\{x, y\}, \{x, z\}\}$, enumerate $\{x, y\}$ and then $\{x, z\}$

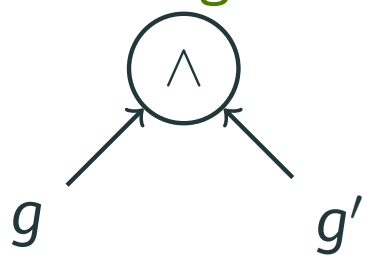
Base case: variable 

: enumerate $\{x\}$ and stop

OR-gate



AND-gate



Concatenation: enumerate $S(g)$ and then enumerate $S(g')$

Determinism: no duplicates

Lexicographic product: enumerate $S(g)$ and for each result t enumerate $S(g')$ and concatenate t with each result

Enumerating assignments in the zero-suppressed semantics

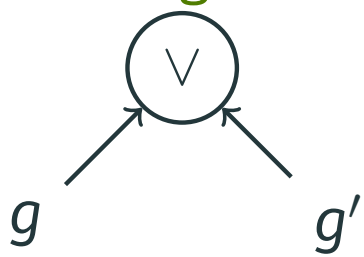
Task: Enumerate the elements of the set $S(g)$ captured by a gate g

$\rightarrow$ E.g., for $S(g) = \{\{x, y\}, \{x, z\}\}$, enumerate $\{x, y\}$ and then $\{x, z\}$

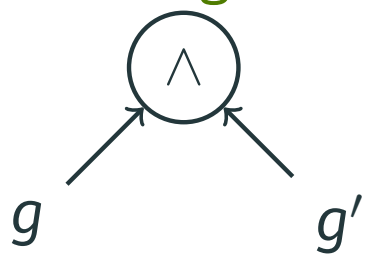
Base case: variable 

: enumerate $\{x\}$ and stop

OR-gate



AND-gate



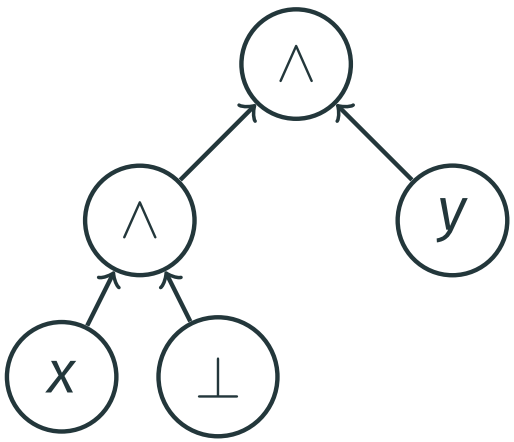
Concatenation: enumerate $S(g)$ and then enumerate $S(g')$

Determinism: no duplicates

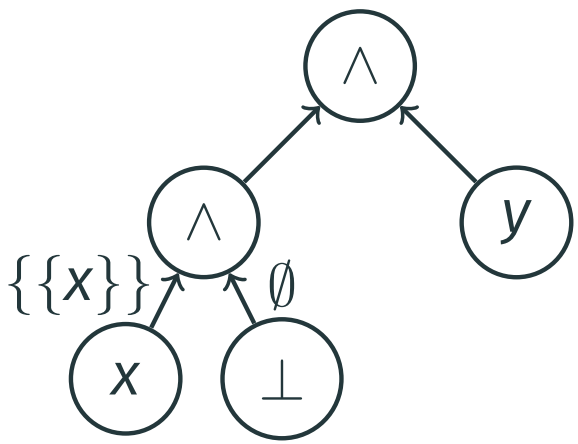
Lexicographic product: enumerate $S(g)$ and for each result t enumerate $S(g')$ and concatenate t with each result

Decomposability: no duplicates

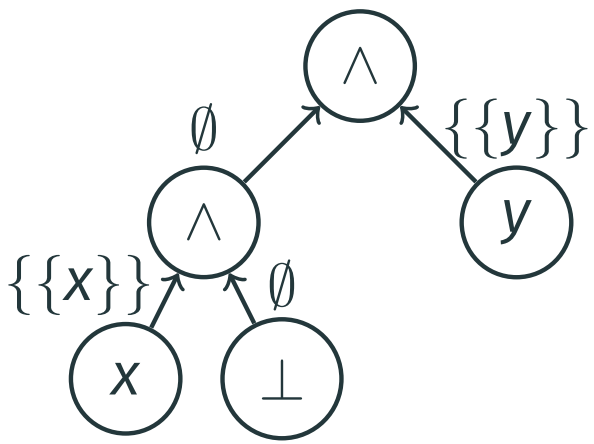
Normalization: handling $\emptyset$



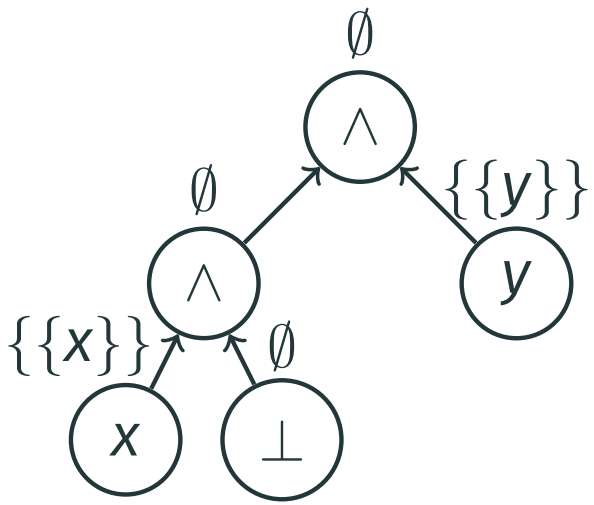
Normalization: handling $\emptyset$



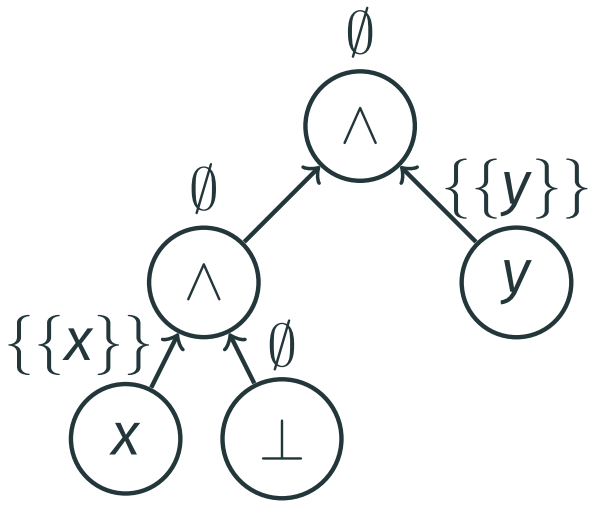
Normalization: handling $\emptyset$



Normalization: handling $\emptyset$

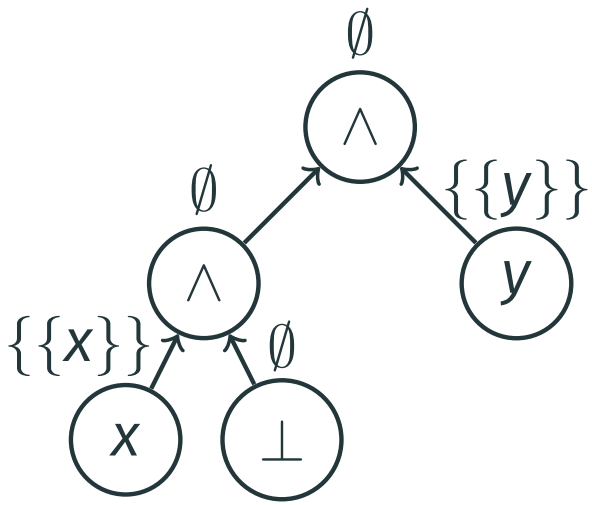


Normalization: handling $\emptyset$



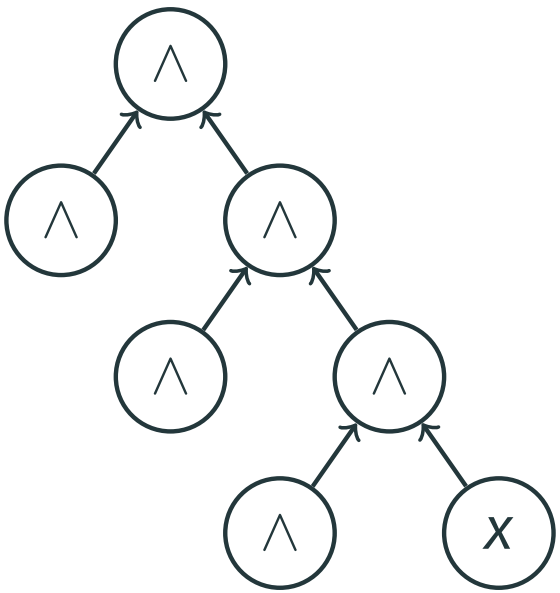
- Problem: if $S(g) = \emptyset$ we waste time

Normalization: handling $\emptyset$

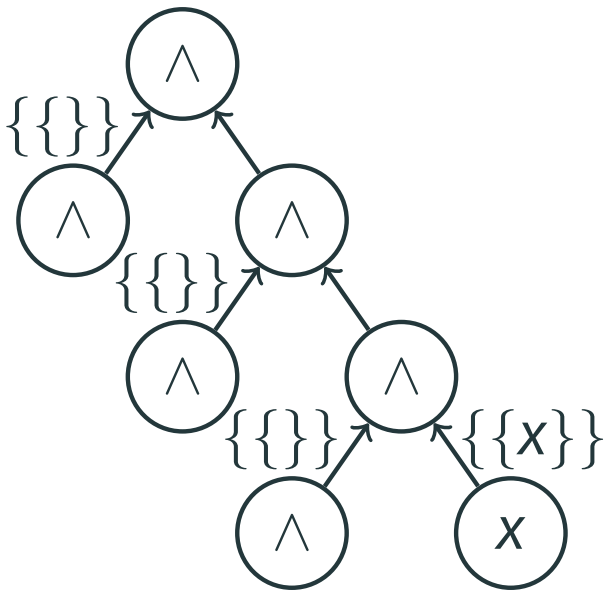


- Problem: if $S(g) = \emptyset$ we waste time
- Solution: compute bottom-up if $S(g) = \emptyset$

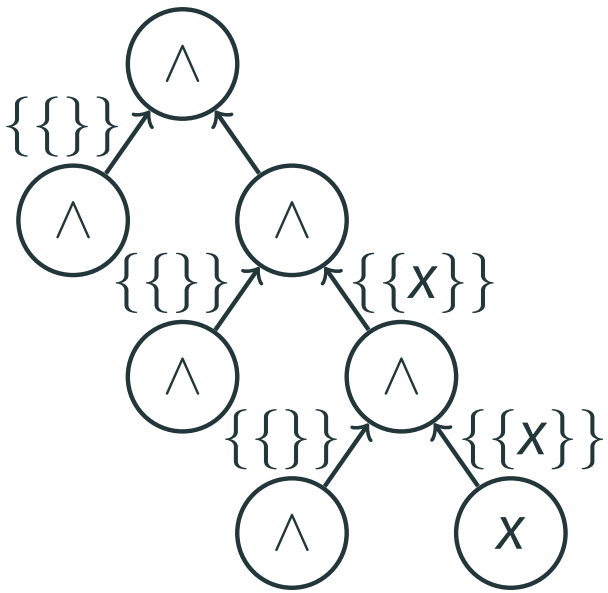
Normalization: handling empty assignments



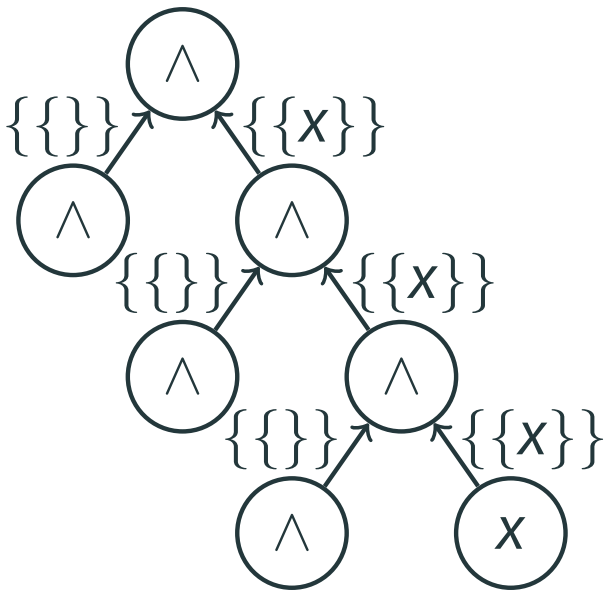
Normalization: handling empty assignments



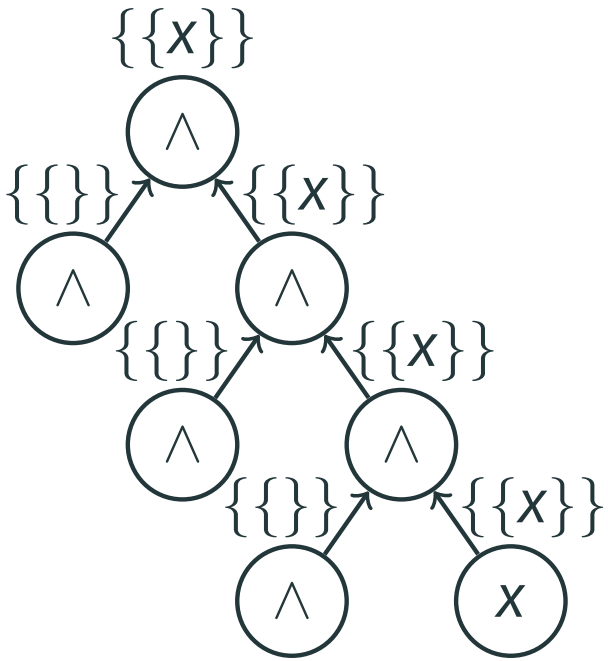
Normalization: handling empty assignments



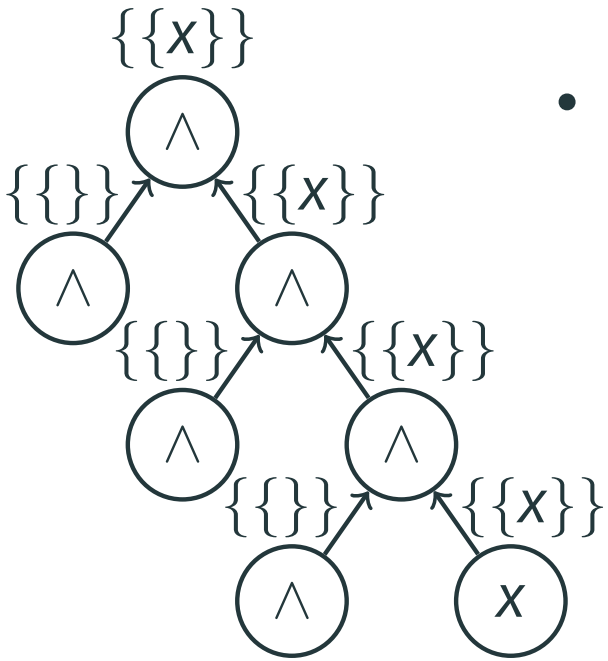
Normalization: handling empty assignments



Normalization: handling empty assignments

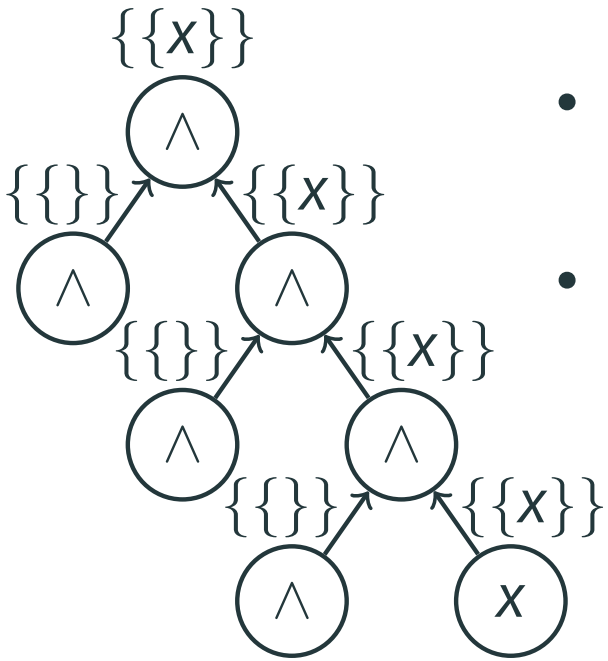


Normalization: handling empty assignments



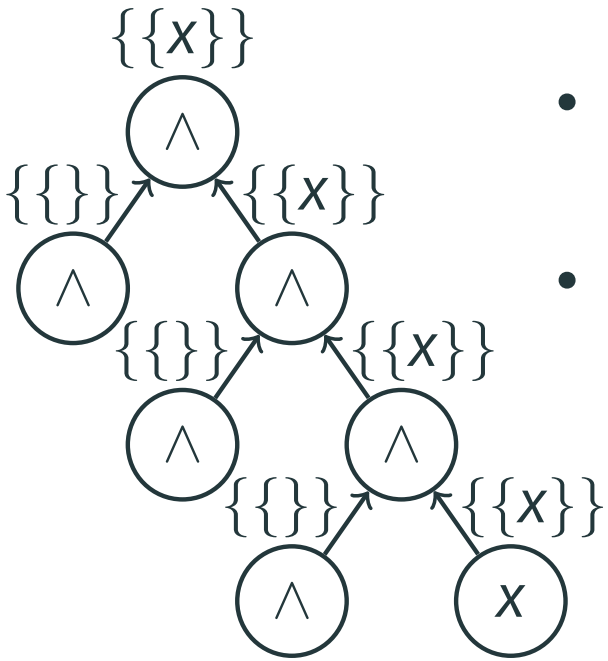
- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of AND-gates

Normalization: handling empty assignments



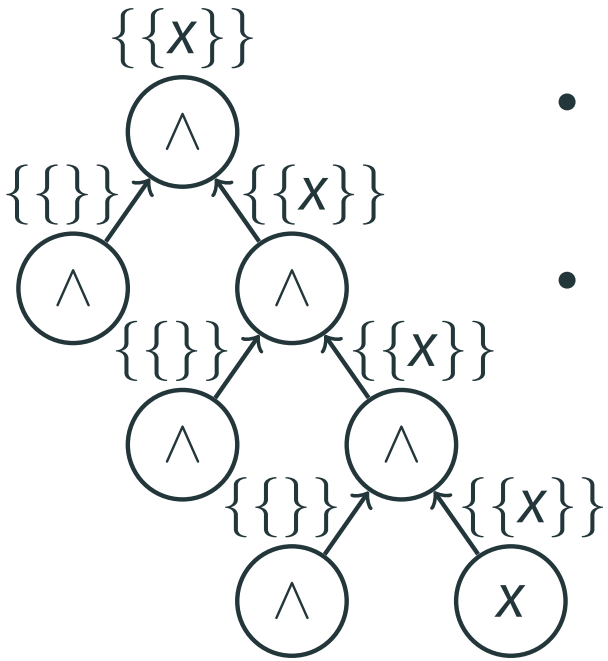
- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of AND-gates
- Solution:

Normalization: handling empty assignments



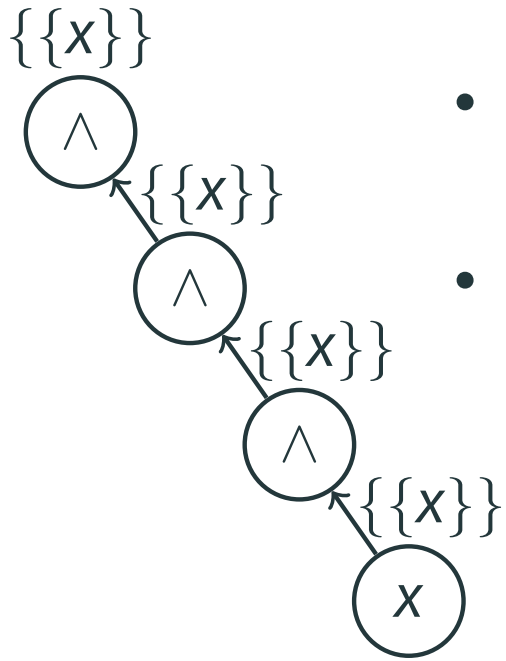
- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of AND-gates
- Solution:
  - split g between $S(g) \cap \{\}$ and $S(g) \setminus \{\}$ (homogenization)

Normalization: handling empty assignments



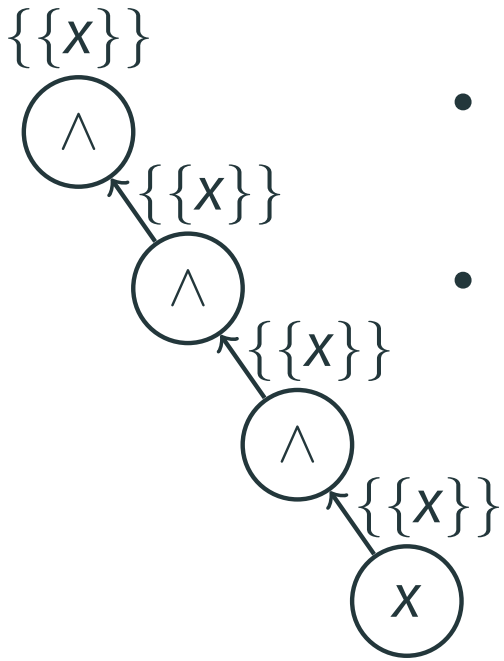
- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of AND-gates
- Solution:
  - split g between $S(g) \cap \{\}$ and $S(g) \setminus \{\}$ (homogenization)
  - remove inputs with $S(g) = \{\}$ for AND-gates

Normalization: handling empty assignments



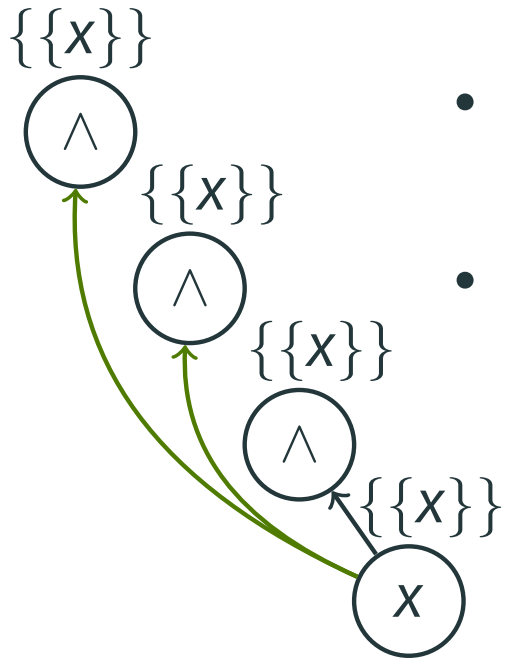
- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of AND-gates
- Solution:
  - split g between $S(g) \cap \{\}$ and $S(g) \setminus \{\}$ (homogenization)
  - remove inputs with $S(g) = \{\}$ for AND-gates

Normalization: handling empty assignments



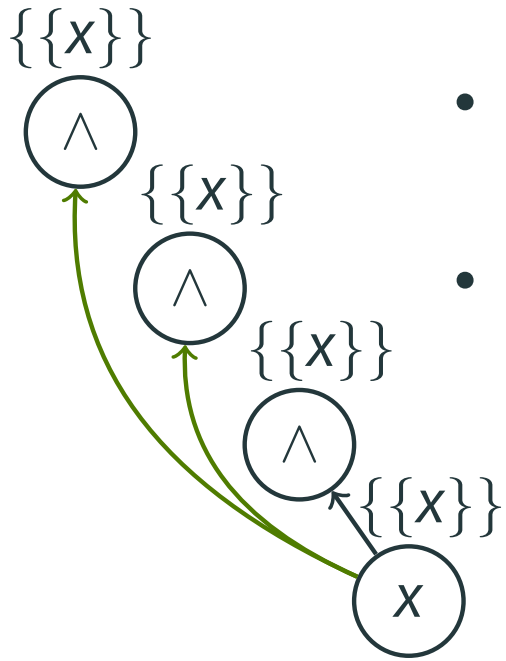
- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of AND-gates
- Solution:
  - split g between $S(g) \cap \{\}$ and $S(g) \setminus \{\}$ (homogenization)
  - remove inputs with $S(g) = \{\}$ for AND-gates
  - collapse AND-chains with fan-in 1

Normalization: handling empty assignments



- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of AND-gates
- Solution:
  - split g between $S(g) \cap \{\}$ and $S(g) \setminus \{\}$ (homogenization)
  - remove inputs with $S(g) = \{\}$ for AND-gates
  - collapse AND-chains with fan-in 1

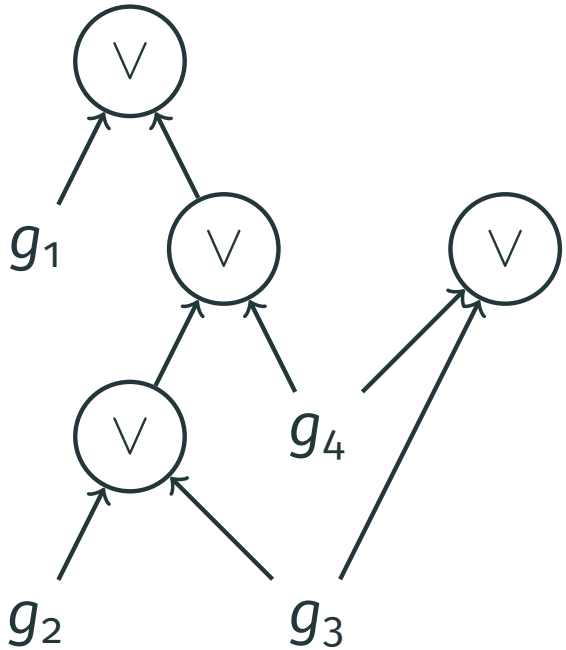
Normalization: handling empty assignments



- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of AND-gates
- Solution:
  - split g between $S(g) \cap \{\}$ and $S(g) \setminus \{\}$ (homogenization)
  - remove inputs with $S(g) = \{\}$ for AND-gates
  - collapse AND-chains with fan-in 1

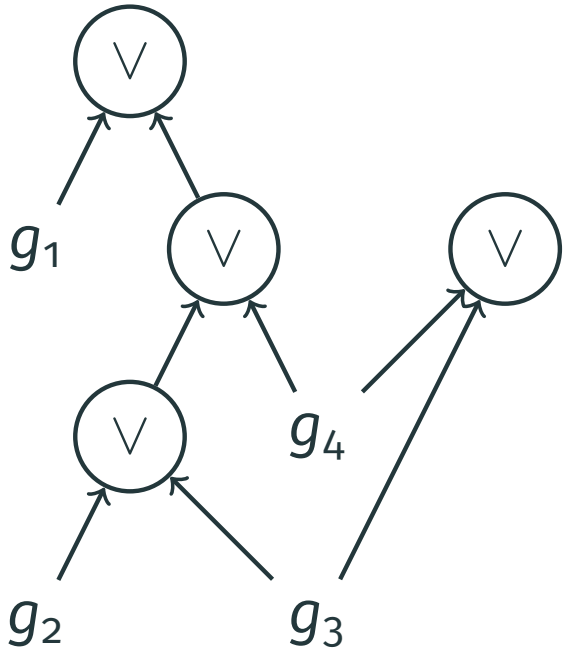
→ Now, traversing an AND-gate ensures that we make progress: it splits the assignments non-trivially

Normalization: handling OR-hierarchies



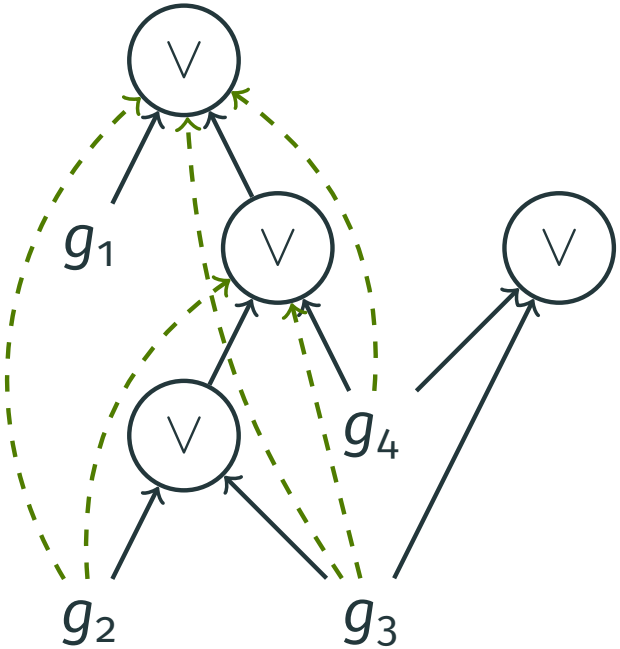
- Problem: we waste time in OR-hierarchies to find a reachable exit (non-OR gate)

Normalization: handling OR-hierarchies



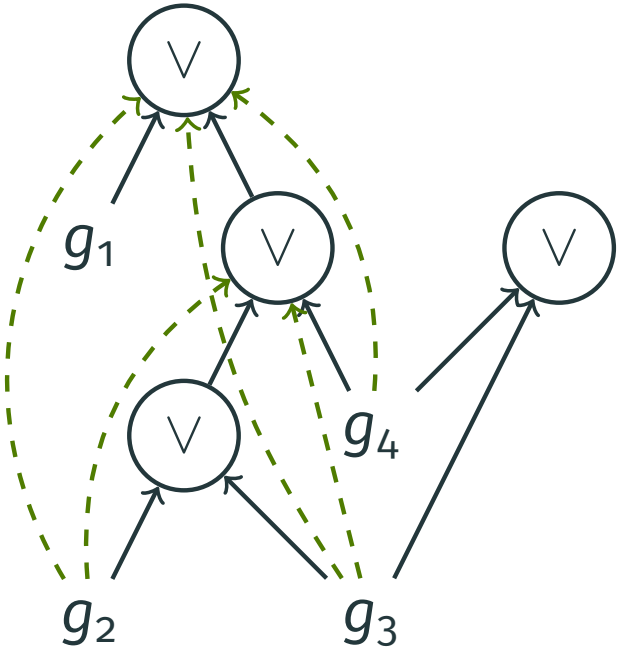
- Problem: we waste time in OR-hierarchies to find a reachable exit (non-OR gate)
- Solution: compute reachability index

Normalization: handling OR-hierarchies



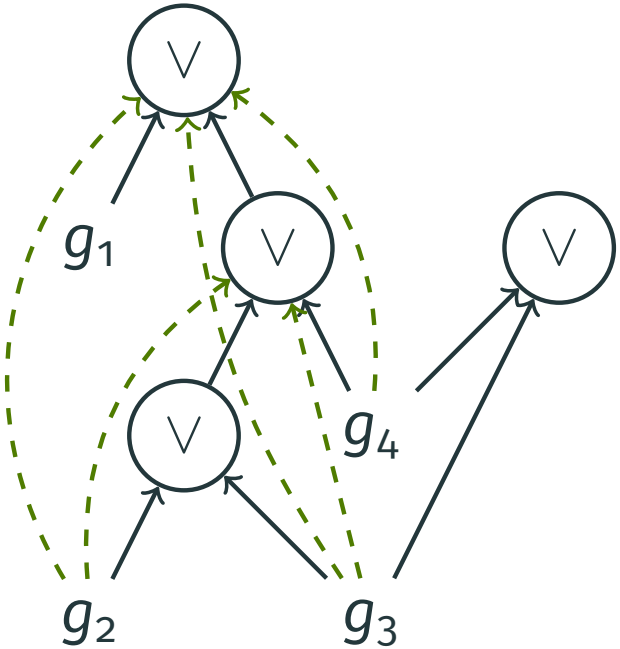
- Problem: we waste time in OR-hierarchies to find a reachable exit (non-OR gate)
- Solution: compute reachability index

Normalization: handling OR-hierarchies



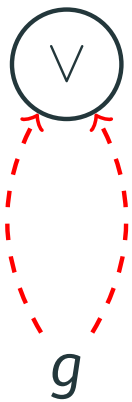
- Problem: we waste time in OR-hierarchies to find a reachable exit (non-OR gate)
- Solution: compute reachability index
- Problem: must be done in linear time

Normalization: handling OR-hierarchies

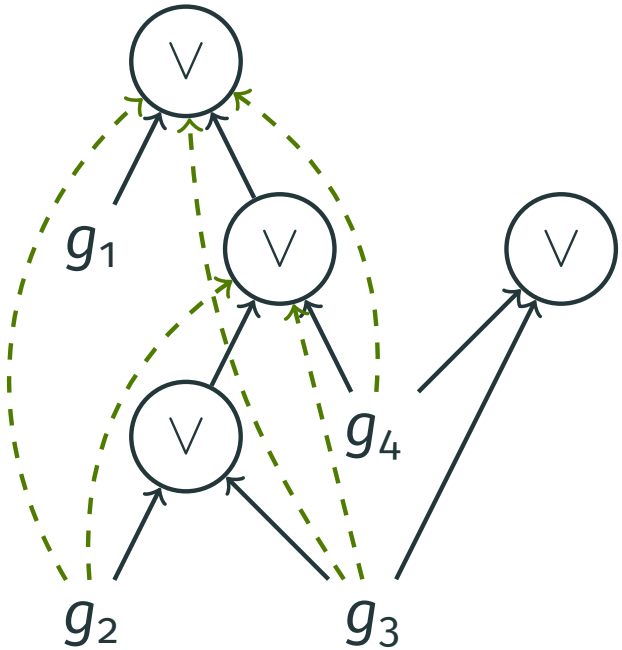


- Problem: we waste time in OR-hierarchies to find a reachable exit (non-OR gate)
- Solution: compute reachability index
- Problem: must be done in linear time

- Solution: Determinism ensures we have a multitree (we cannot have the pattern at the right)

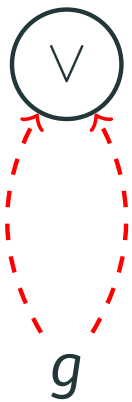


Normalization: handling OR-hierarchies

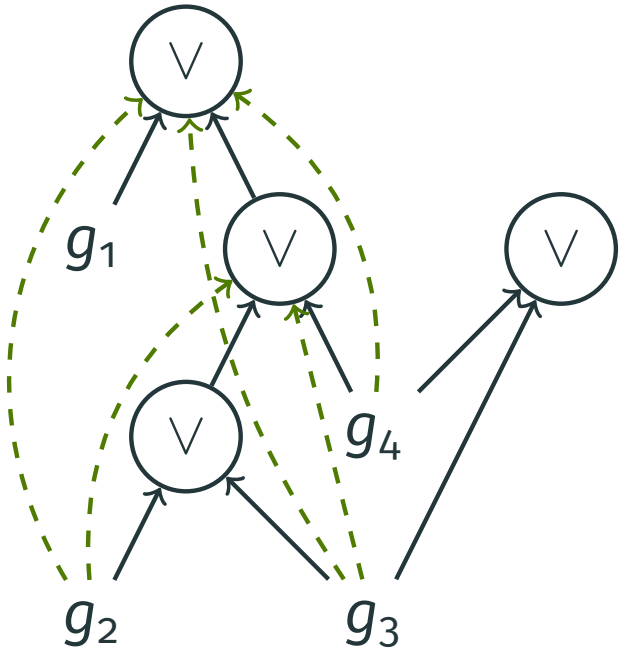


- Problem: we waste time in OR-hierarchies to find a reachable exit (non-OR gate)
- Solution: compute reachability index
- Problem: must be done in linear time

- Solution: Determinism ensures we have a multitree (we cannot have the pattern at the right)
- Custom constant-delay reachability index for multitrees

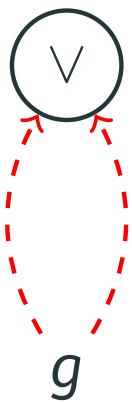


Normalization: handling OR-hierarchies

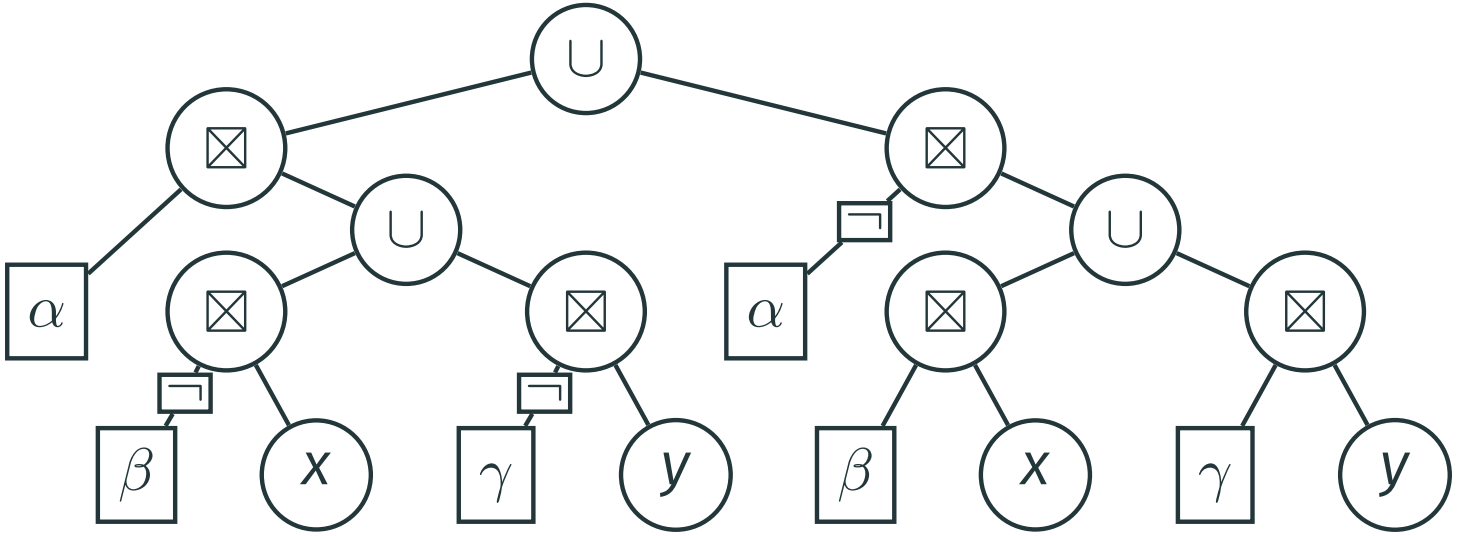


- Problem: we waste time in OR-hierarchies to find a reachable exit (non-OR gate)
- Solution: compute reachability index
- Problem: must be done in linear time

- Solution: Determinism ensures we have a multitree (we cannot have the pattern at the right)
- Custom constant-delay reachability index for multitrees
- For MSO query evaluation: upwards-deterministic circuit so we have a tree: simpler constant-memory index

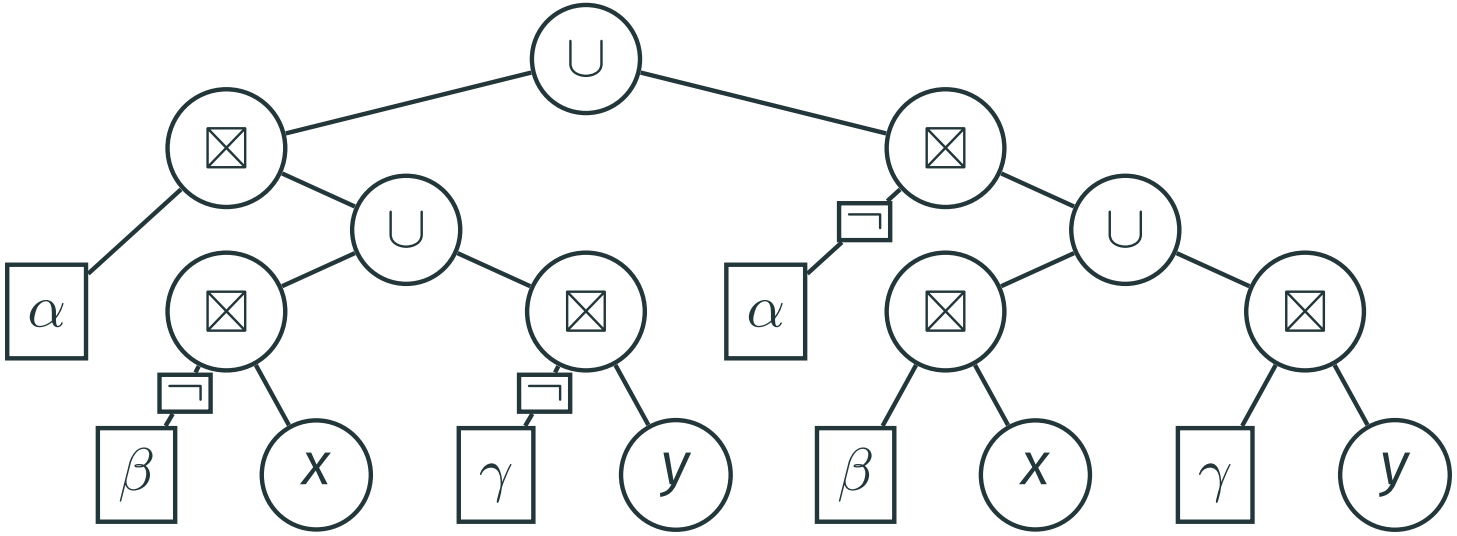


What's new for updates?



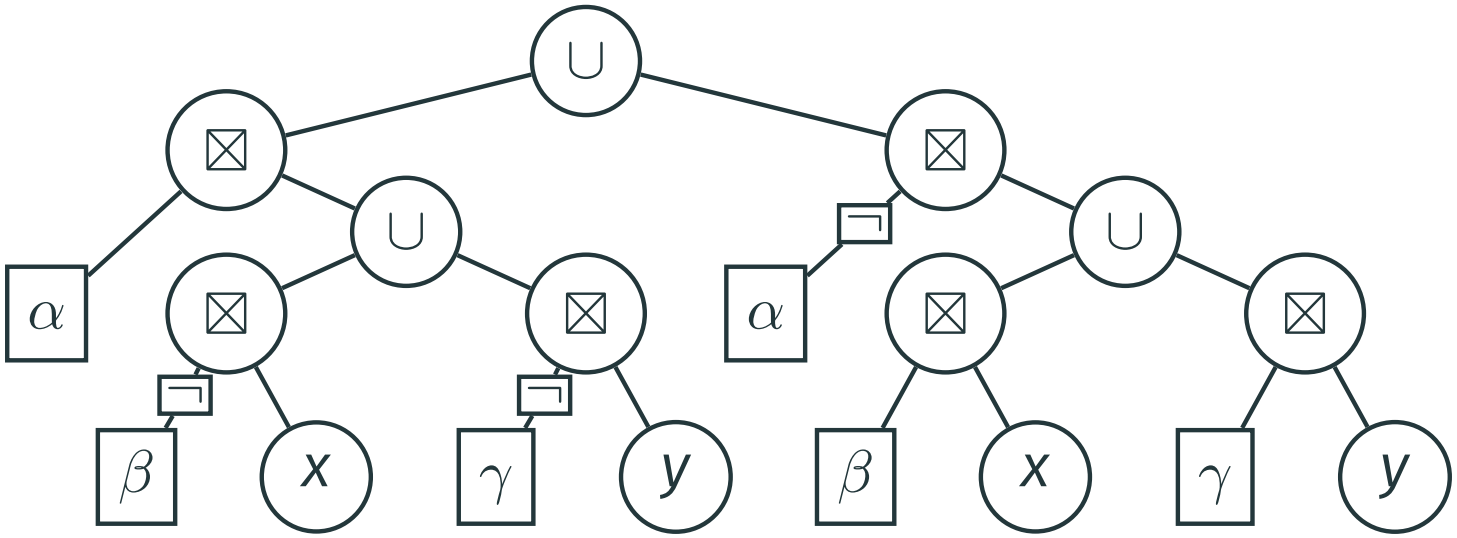
- Hybrid circuits:
  - (x) Set gates (zero-suppressed semantics)
  - [α] Boolean gates (usual semantics)
  - (⊠) Product between the two (→ togglable wire)

What's new for updates?



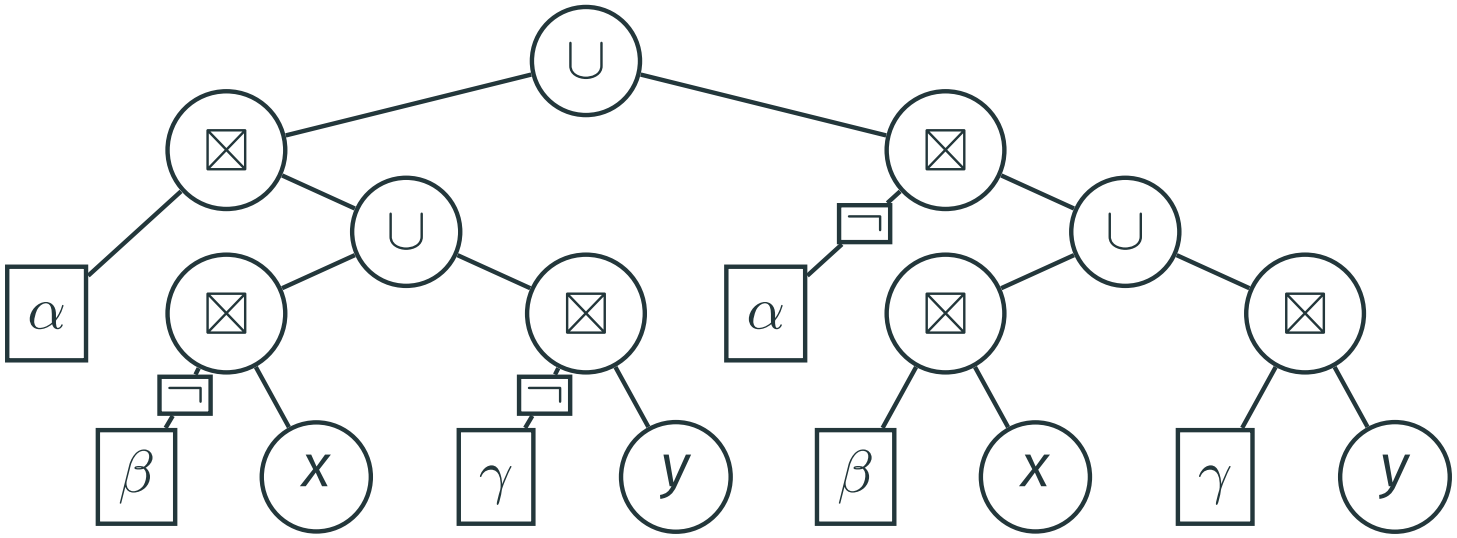
- Hybrid circuits:
  - (x) Set gates (zero-suppressed semantics)
  - [α] Boolean gates (usual semantics)
  - (⊠) Product between the two (→ togglable wire)
- Homogenization: transforms set gates into Boolean gates

What's new for updates?



- Hybrid circuits:
  - x Set gates (zero-suppressed semantics)
  - α Boolean gates (usual semantics)
  - $\boxtimes$ Product between the two ($\rightarrow$ togglable wire)
- Homogenization: transforms set gates into Boolean gates
- Reachability index for OR-hierarchies: trees with updates

What's new for updates?



- Hybrid circuits:
  - (x) Set gates (zero-suppressed semantics)
  - [α] Boolean gates (usual semantics)
  - (⊠) Product between the two (→ togglable wire)
- Homogenization: transforms set gates into Boolean gates
- Reachability index for OR-hierarchies: trees with updates
- Use balancing lemma to make the input tree balanced

Conclusion

Summary and conclusion

- Enumerate the satisfying assignments of structured d-DNNFs
  - → in delay linear in each assignment
  - → in constant delay for constant Hamming weight

→ Can recapture existing enumeration results

→ Useful general-purpose result for applications

Summary and conclusion

- Enumerate the satisfying assignments of structured d-DNNFs
  - → in delay linear in each assignment
  - → in constant delay for constant Hamming weight

→ Can recapture existing enumeration results

→ Useful general-purpose result for applications

Future work:

- Practice: implement the technique with automata
- Improvements: enumerate in order? (e.g., of increasing weight?)
- Updates: support insertions/deletions?

Summary and conclusion

- Enumerate the satisfying assignments of structured d-DNNFs
  - → in delay linear in each assignment
  - → in constant delay for constant Hamming weight

→ Can recapture existing enumeration results

→ Useful general-purpose result for applications

Future work:

- Practice: implement the technique with automata
- Improvements: enumerate in order? (e.g., of increasing weight?)
- Updates: support insertions/deletions?

Thanks for your attention!

References i


Amarilli, A., Bourhis, P., and Mengel, S. (2018).
Enumeration on Trees under Relabelings.
In *ICDT*.
To appear.


Bagan, G. (2006).
MSO queries on tree decomposable structures are computable with linear delay.
In *CSL*.


Kazana, W. and Segoufin, L. (2013).
Enumeration of monadic second-order queries on trees.
TOCL, 14(4).

References ii


Losemann, K. and Martens, W. (2014).
MSO queries on trees: enumerating answers under updates.
In *CSL-LICS*.


Niewerth, M. and Segoufin, L. (2018).
Enumeration of MSO queries on strings with constant delay and logarithmic updates.
In *PODS*.
To appear.


Olteanu, D. and Závodnỳ, J. (2015).
Size bounds for factorised representations of query results.
TODS, 40(1).