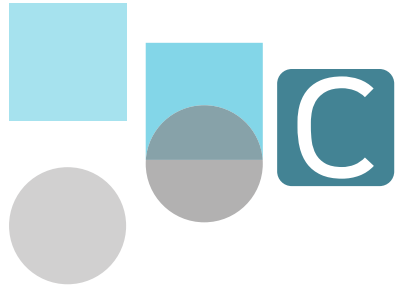


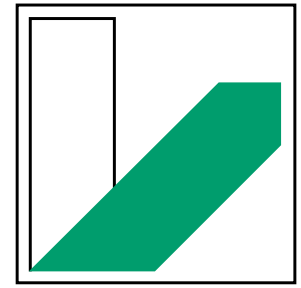

Enumeration for MSO Queries on Trees via Circuits

Antoine Amarilli[1] Pierre Bourhis[2], Louis Jachiet[2], Stefan Mengel[3], Matthias Niewerth[4]

July 14, 2019

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Circuits pour l'énumération de MSO sur des arbres

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Query evaluation

The main database problem is query evaluation

Data D: what we know

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Query Q: question asked about the data

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Result: all results of the query Q on the data D

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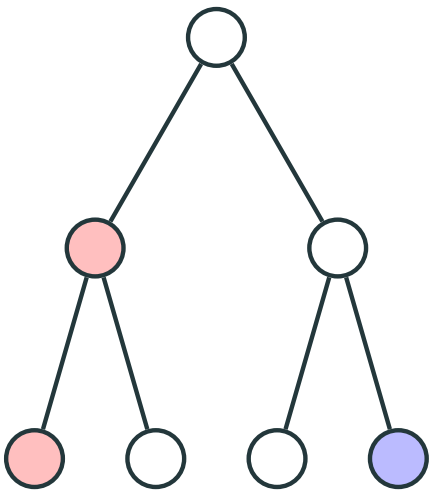
Measure of efficiency: computational complexity:

- Combined complexity: D and Q are **inputs**
- Data complexity: Q is **fixed**, D is the **input**

Structured data: trees



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  - Important special case: text

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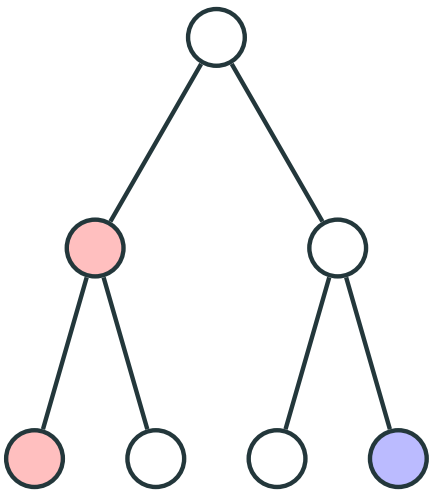

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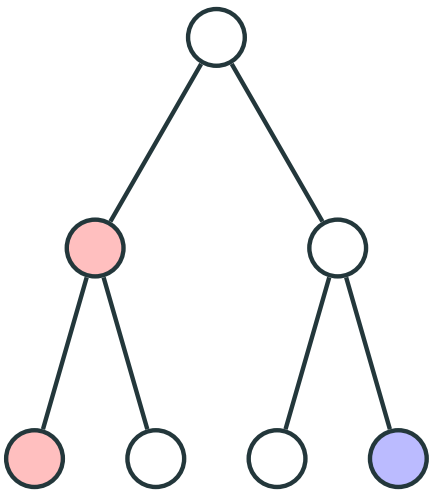
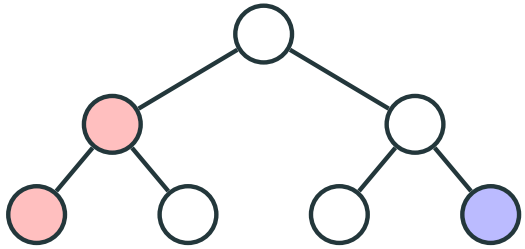


- Several kinds of data are structured as a **tree** (HTML pages, XML, folder hierarchies...)
  - Important special case: **text**
- On this kind of data, we can use **more efficient** query evaluation techniques
- Natural query language: **monadic second-order logic** (MSO)
  - Very expressive
  - Corresponds to **tree automata**
  - Data complexity is in **linear time**

Query evaluation on trees

Data: a tree T where nodes have a color from an alphabet ○ ● ●



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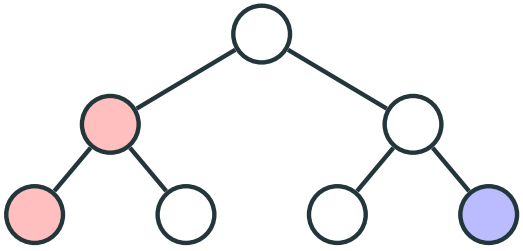

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- $P_{●}(x)$ means "x is blue"
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"Is there both a pink and a blue node?"

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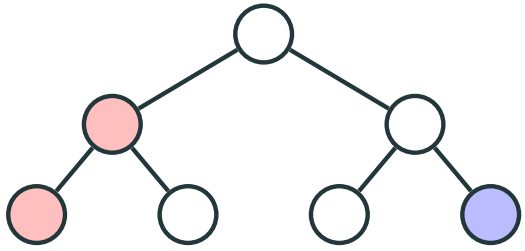

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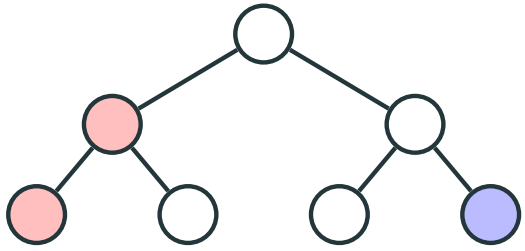

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- Data complexity: linear in T
- Combined complexity: cannot be elementary

Beyond Boolean queries

- Boolean queries only gives YES/NO answers
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  → We need a new definition of complexity

Idea: Enumeration algorithms

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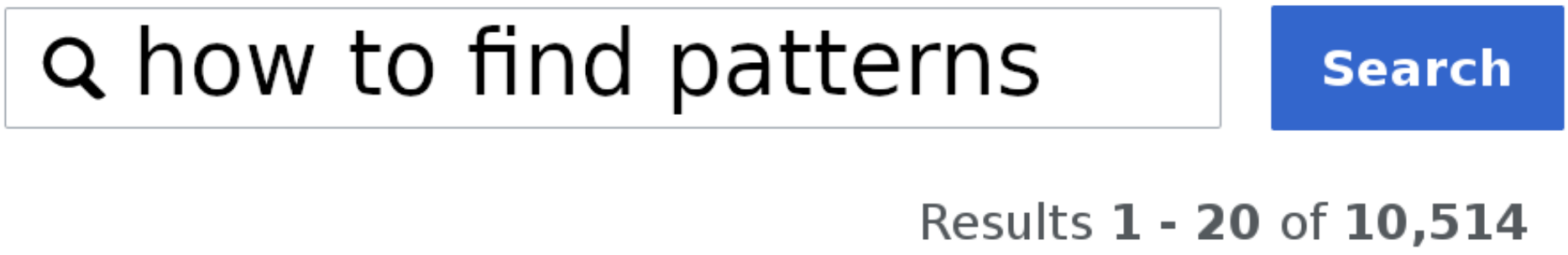

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Results **1 - 20** of **10,514**

...

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how to find patterns

Search

Results **1 - 20** of **10,514**

...

View (previous 20 | next 20) (20 | 50 | 100 | 250 | 500)

Idea: Enumeration algorithms

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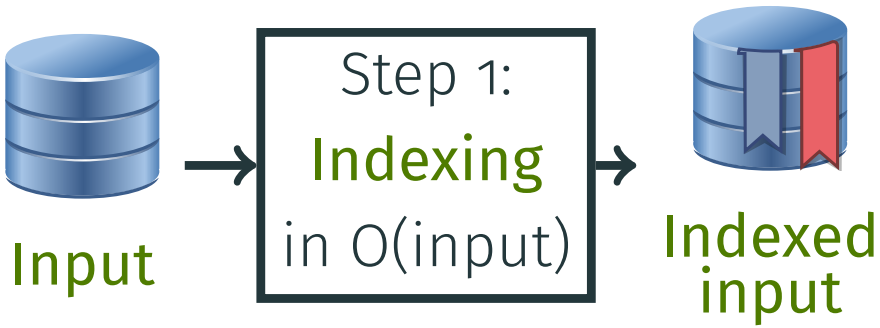
→ Formalization: enumeration algorithms

Formalizing an enumeration algorithm

Input

Formalizing an enumeration algorithm

Formalizing an enumeration algorithm



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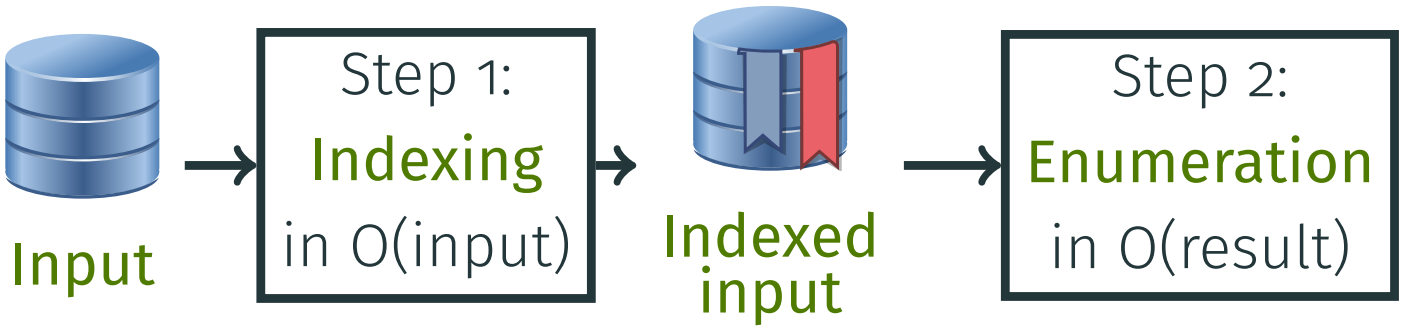

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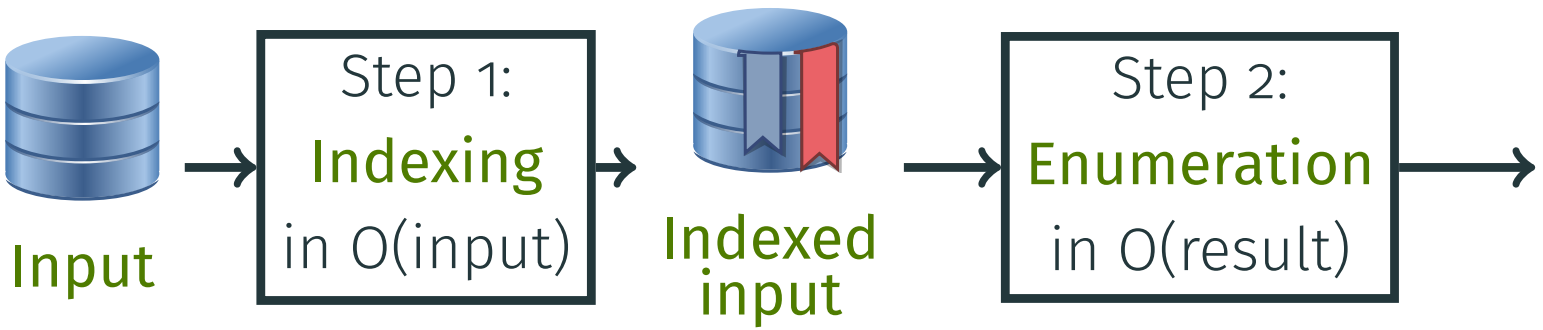

A	B	C
a	b	c

Results

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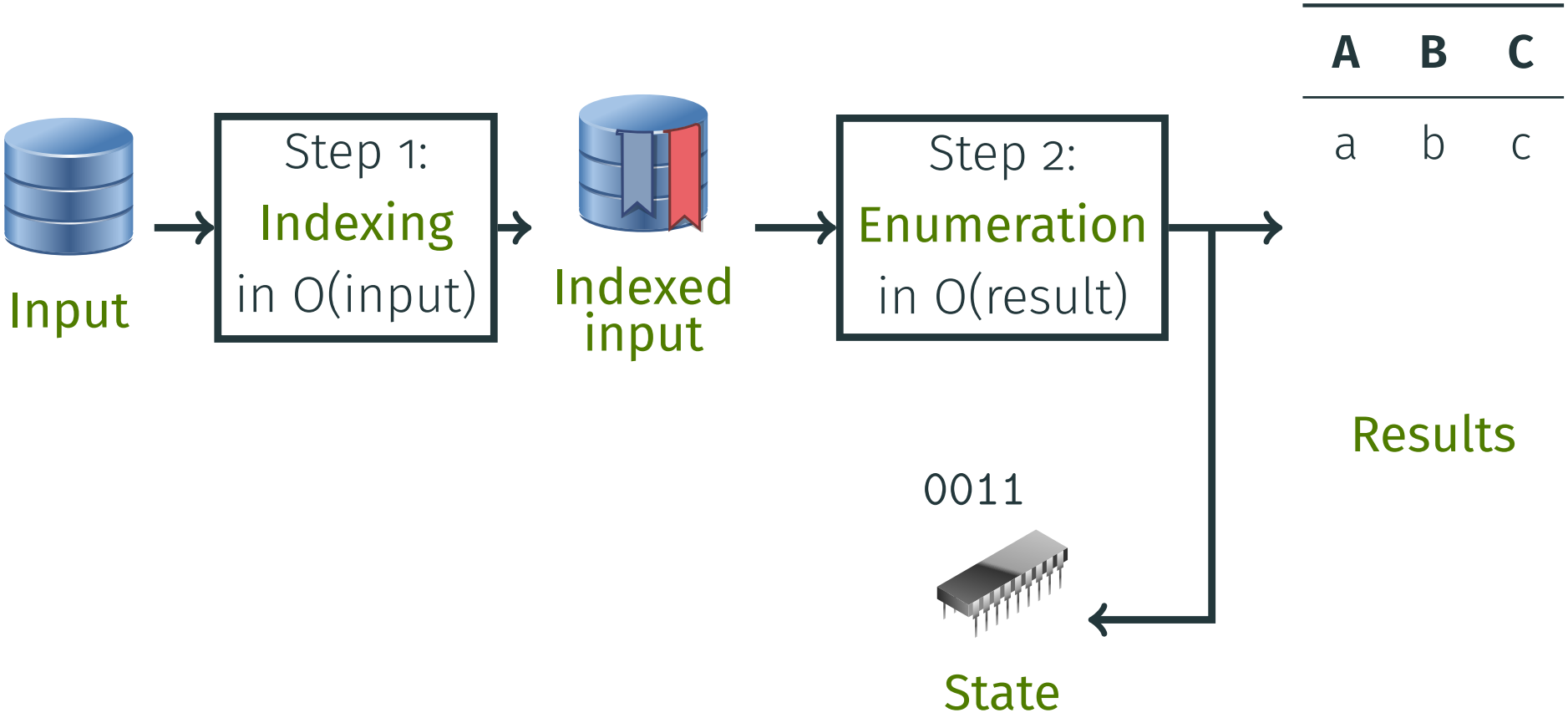

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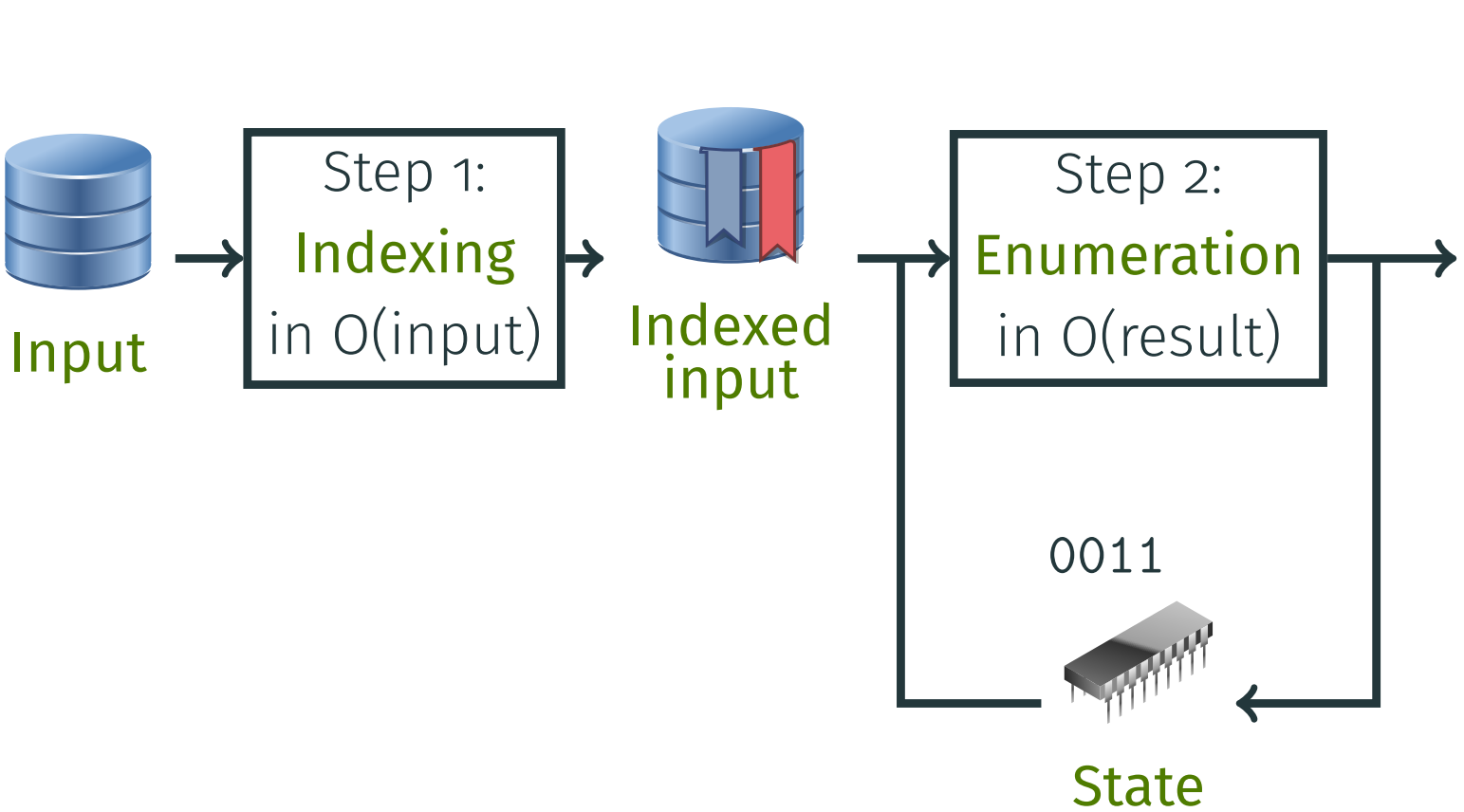

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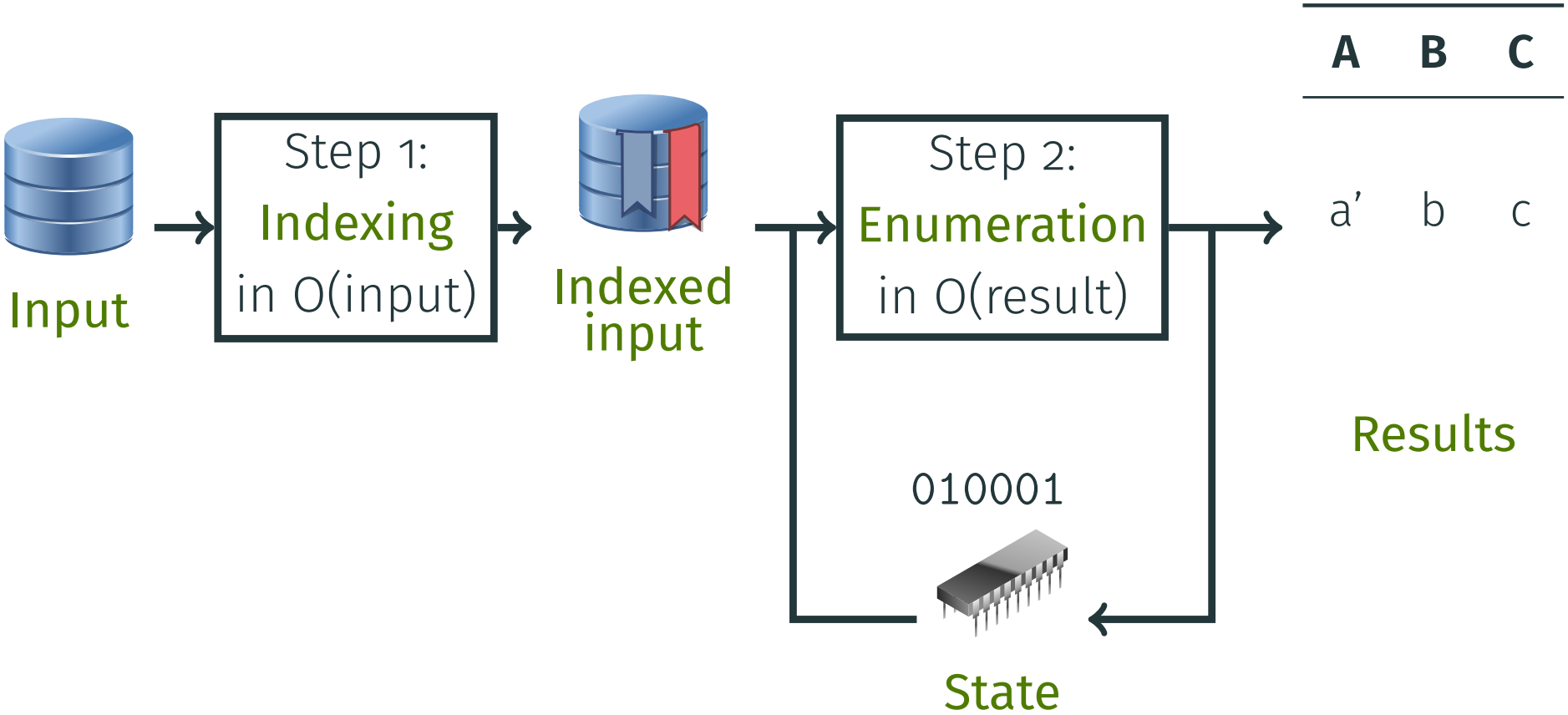

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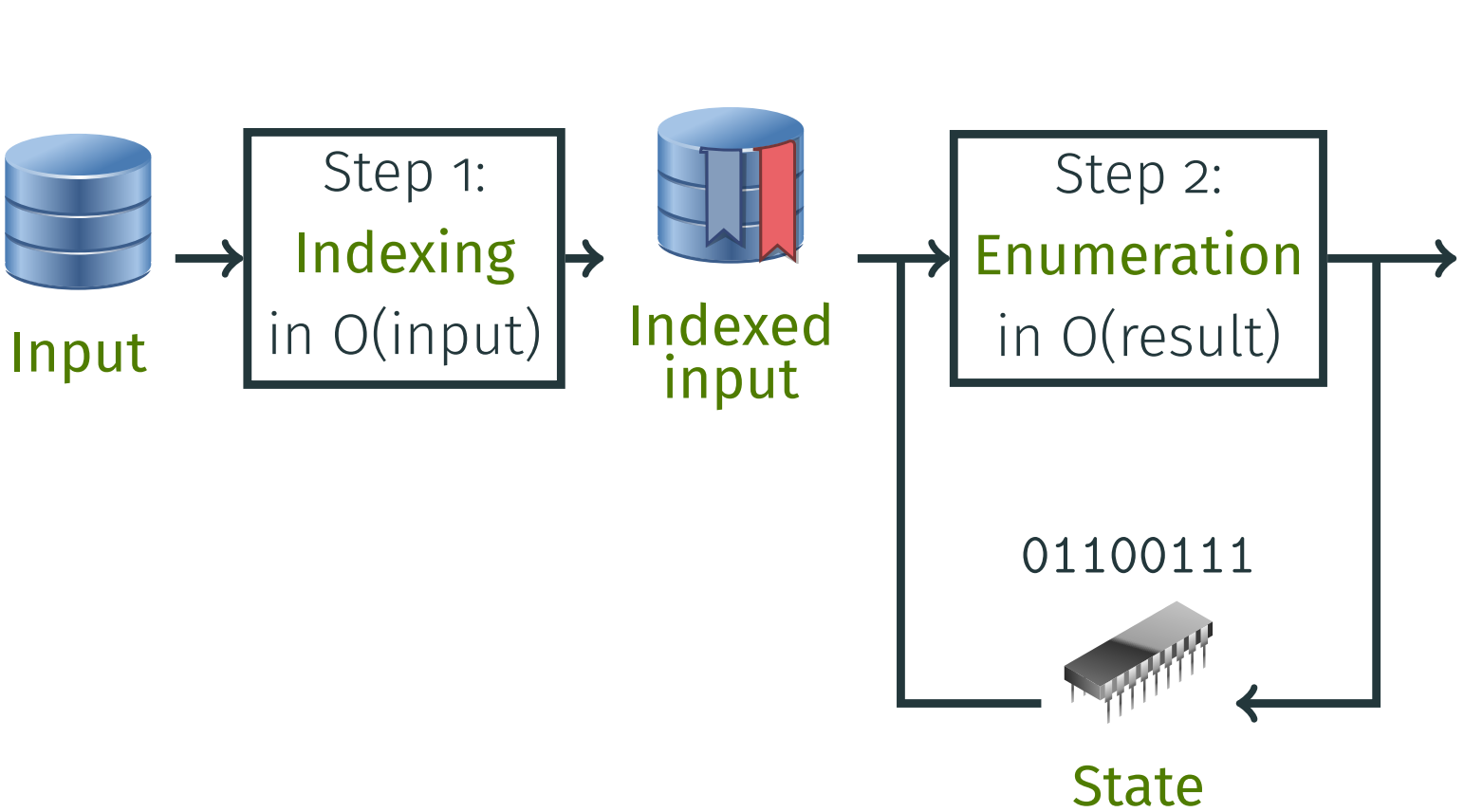

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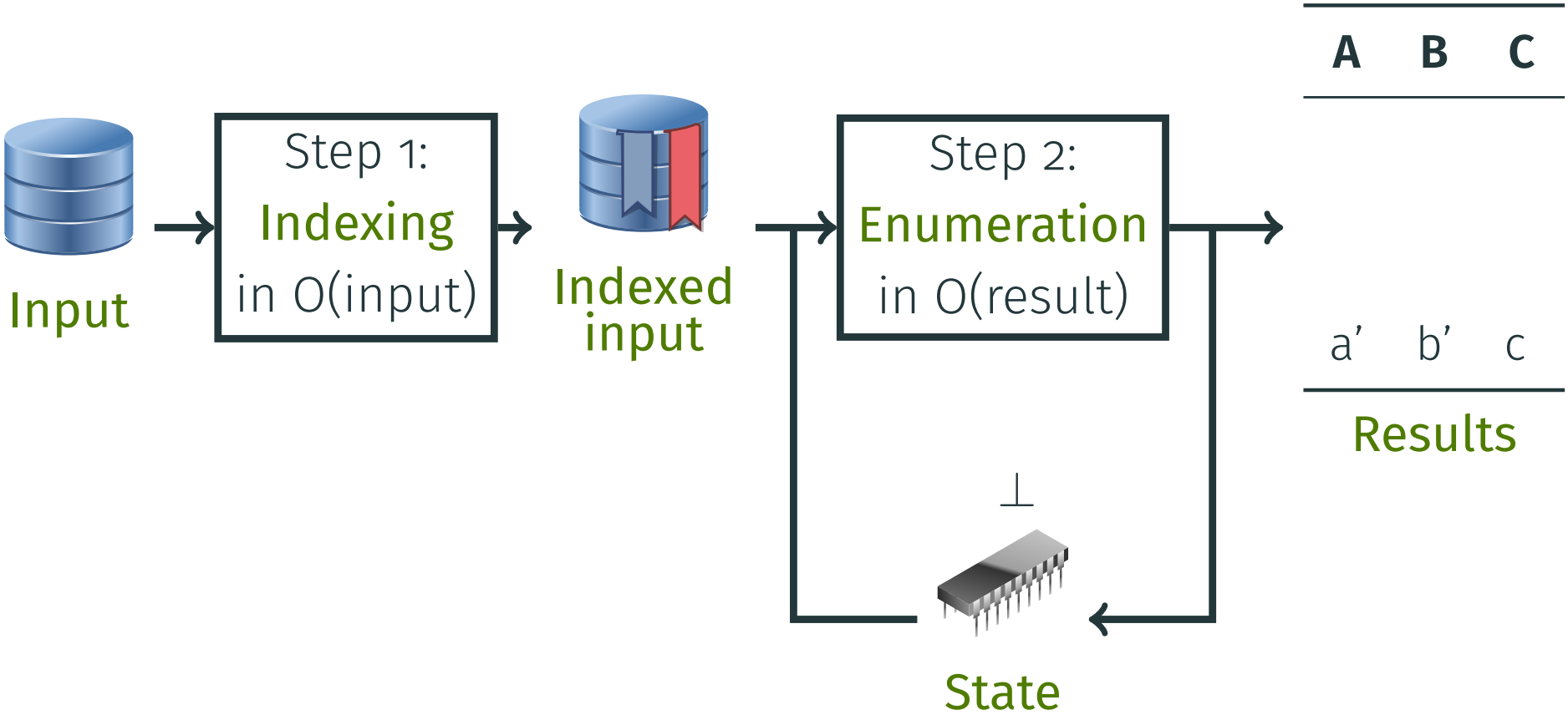

Main results

- We are given: the tree T, a non-Boolean query Q in MSO
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*For any **fixed** MSO query Q, given a tree T,*
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This was already known, so what's new?

- Modular proof based on a notion of set circuits
- Tractable in combined complexity for Q given as an automaton
- Efficiently update the preprocessing when the tree changes

Structure of the talk

- Building a set circuit: given a tree T and automaton A, we can build a set circuit C that represents the results

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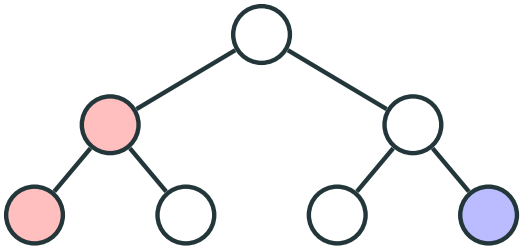
Structure of the talk

- Building a set circuit: given a tree T and automaton A, we can build a set circuit C that represents the results
- Enumeration on set circuits: given a set circuit C, we can enumerate efficiently the results that it captures (under some assumptions on C)
- New stuff:
  - Tractability in the automaton and application to text
  - Efficient updates of the index

Building a set circuit

Monadic second-order logic (MSO)



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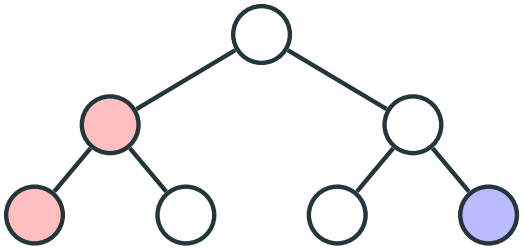

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- Propositional logic: formulas with AND $\wedge$, OR $\vee$, NOT $\neg$
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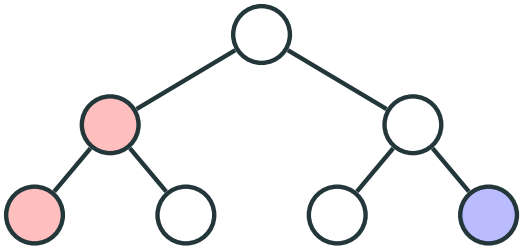

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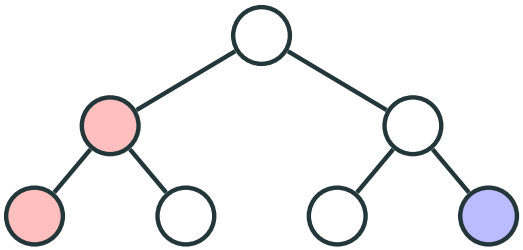

- $P_{\color{blue}\bigcirc}(x)$ means "x is blue"; also $P_{\color{red}\bigcirc}(x)$, $P_{\bigcirc}(x)$
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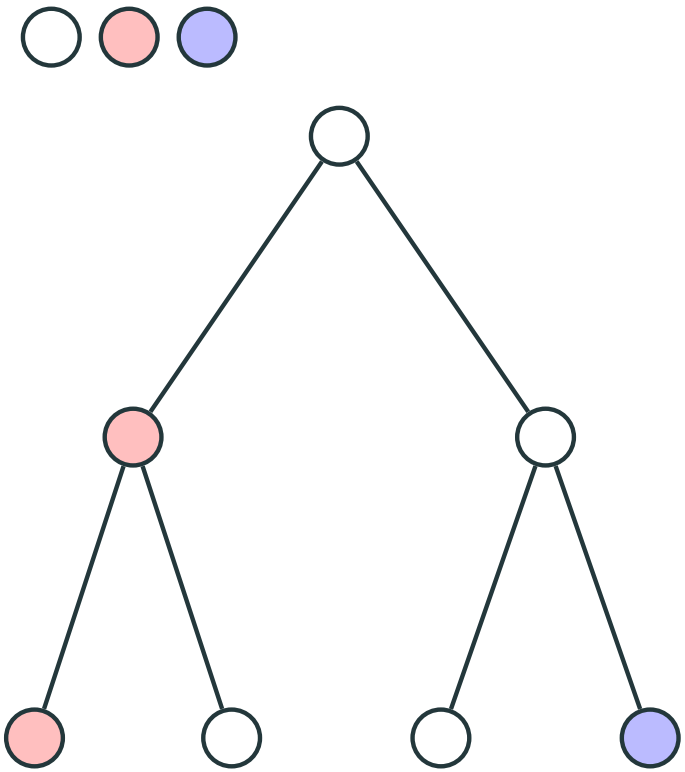
- Monadic second-order logic (MSO): adds quantifiers over sets
  - $\exists S\; \forall x\; S(x)$ means *"there is a set S containing every element x"*
  - Can express transitive closure $x \to^* y$, i.e., *"x is an ancestor of y"*
  - $\forall x\, P_{\color{red}\bigcirc}(x) \Rightarrow \exists y\, P_{\color{blue}\bigcirc}(y) \wedge x \to^* y$
    means *"There is a blue node below every pink node"*

Tree automata

Tree alphabet:



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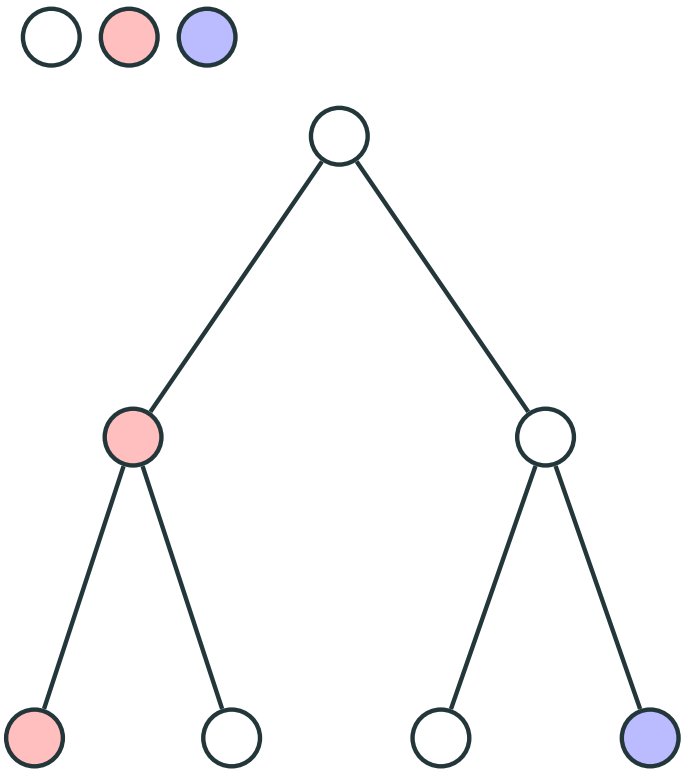

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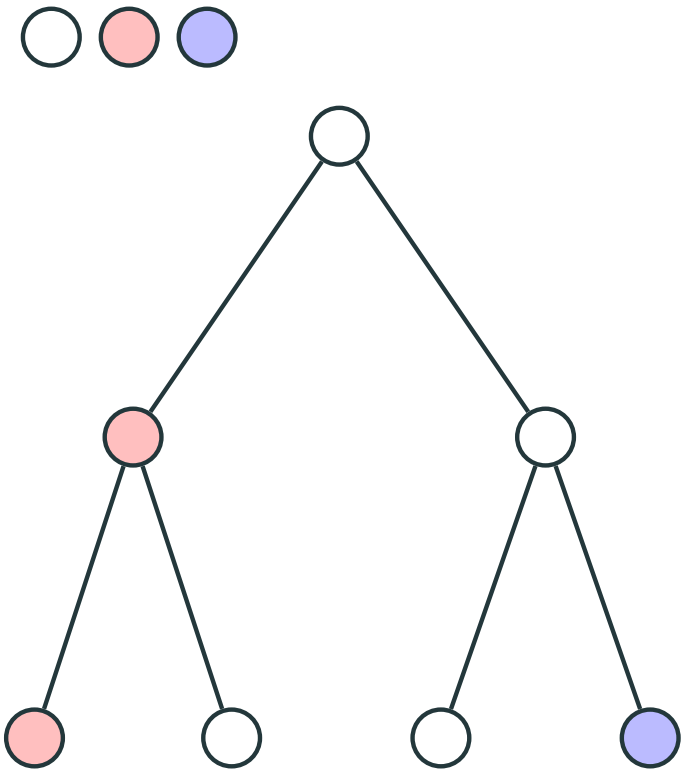

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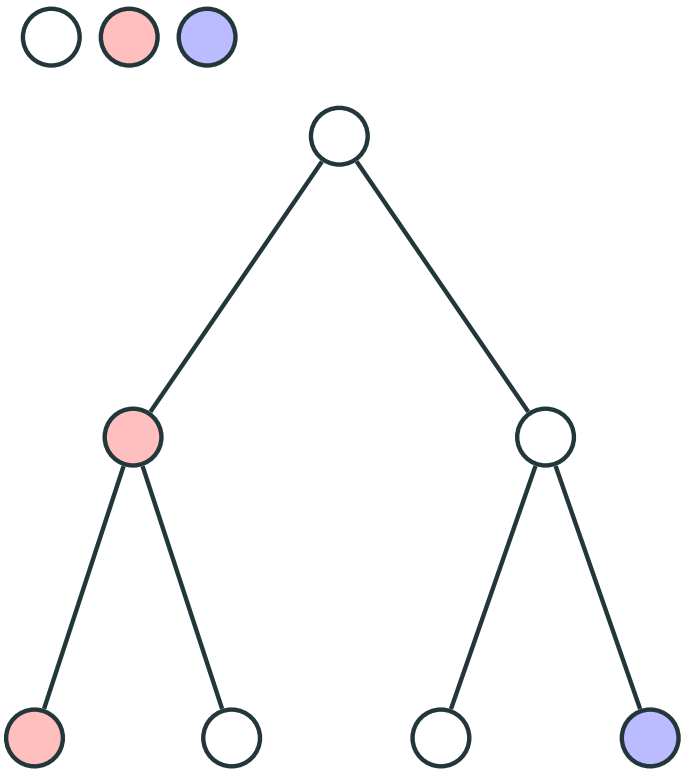

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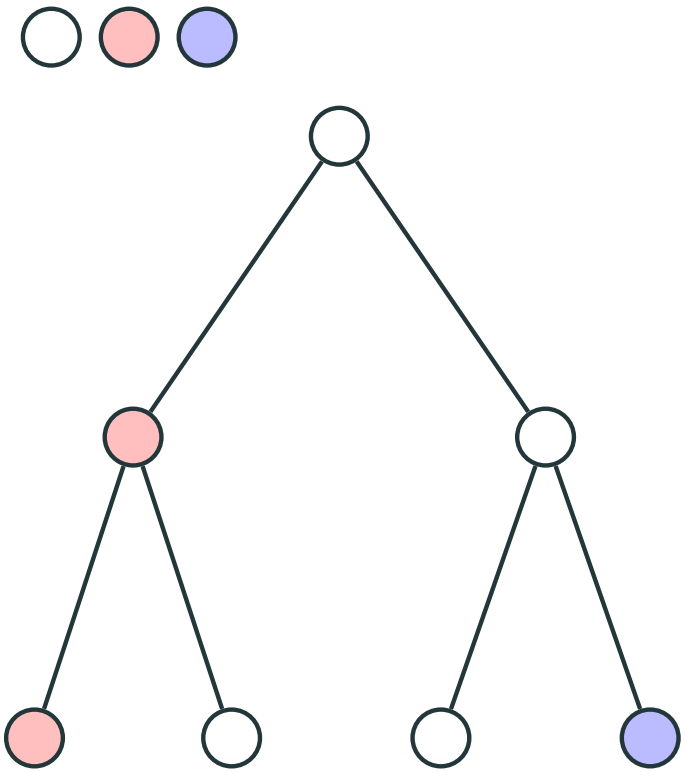

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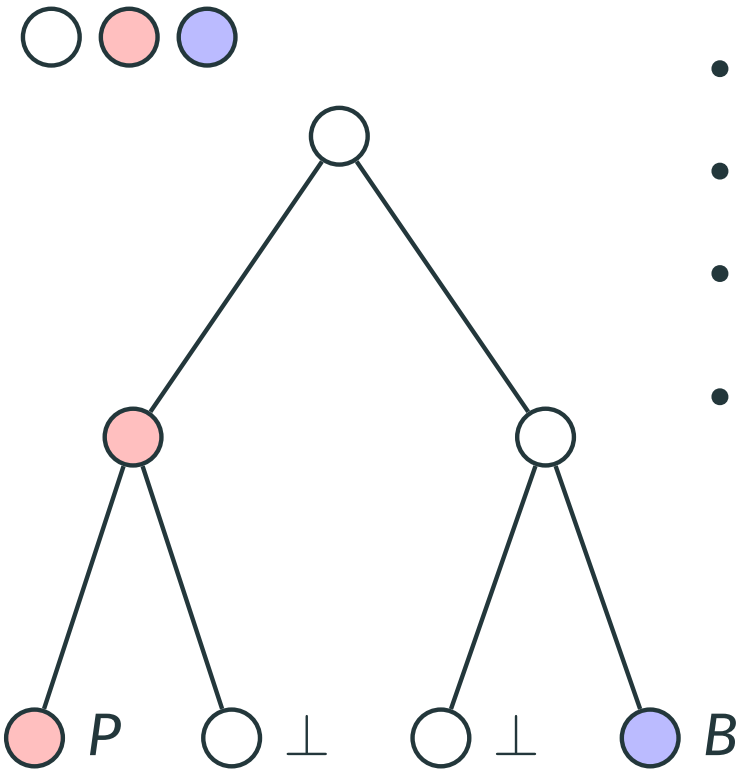

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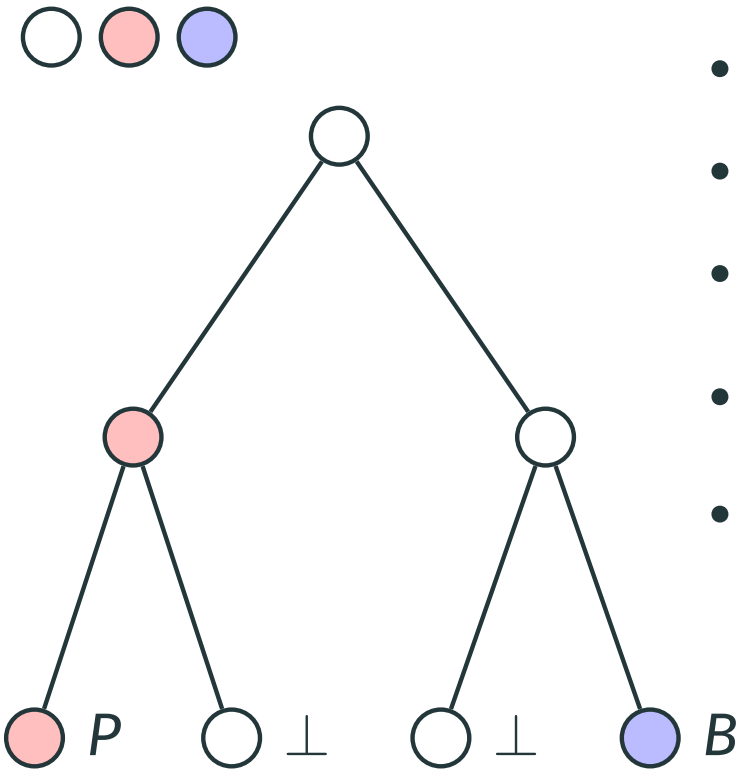

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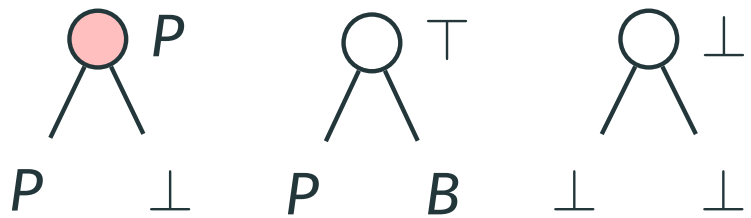

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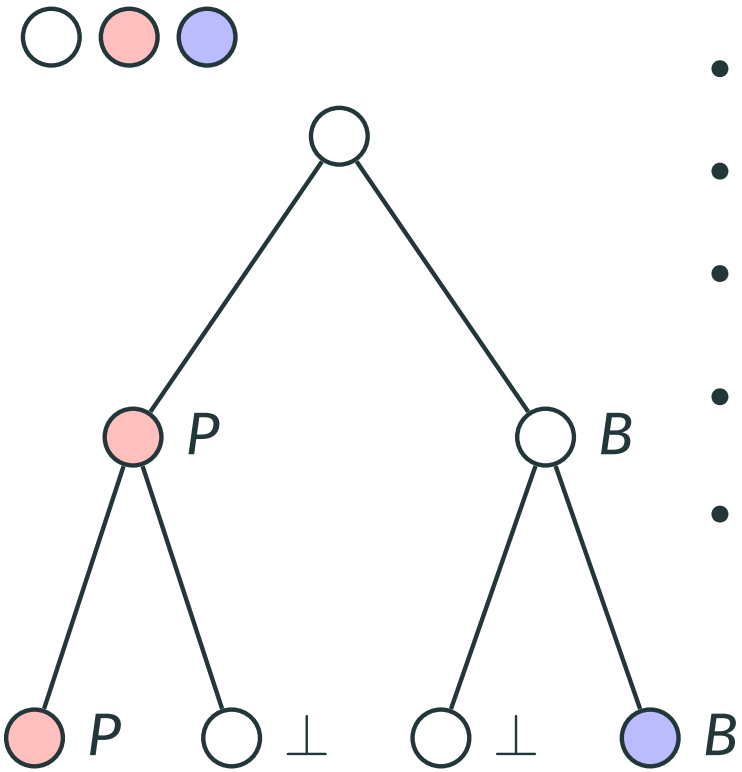

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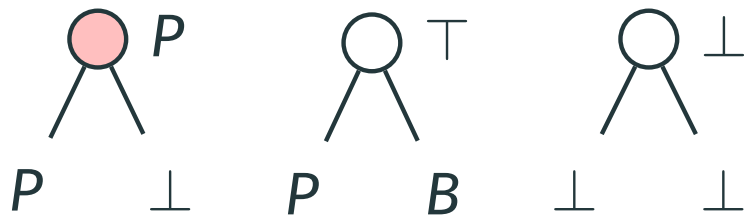
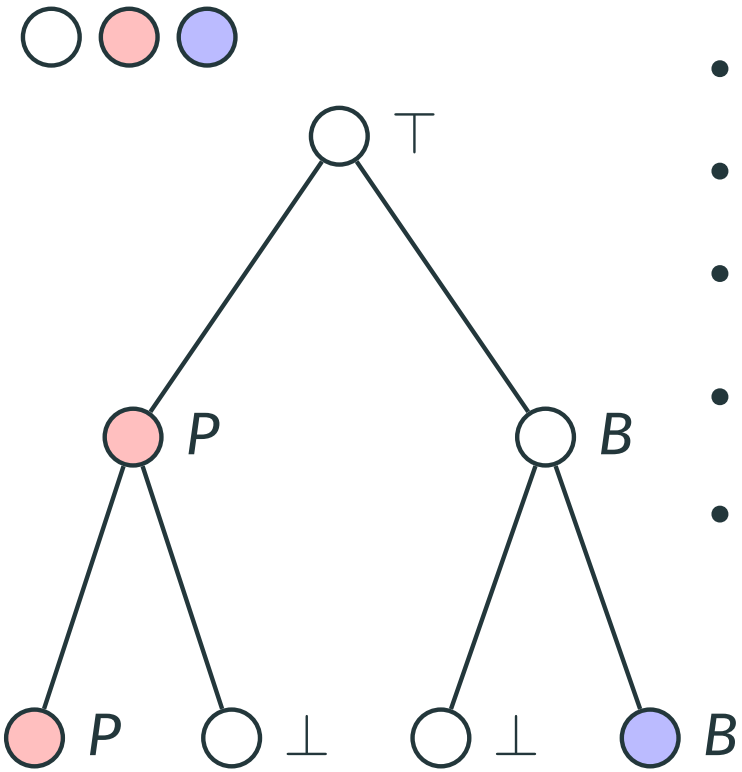


Tree automata

Tree alphabet:



- Bottom-up deterministic **tree automaton**
- *"Is there both a pink and a blue node?"*
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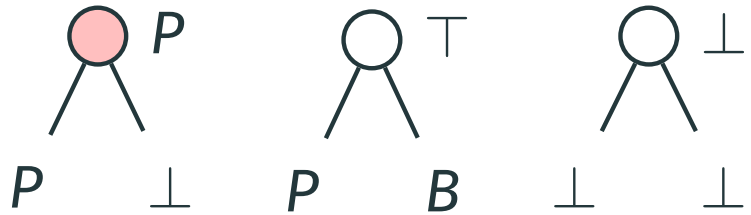

Boolean MSO query evaluation via automata

Theorem [Thatcher and Wright, 1968]

MSO and tree automata have the same expressive power on trees

→ *Given a Boolean MSO query, we can compute a tree automaton that accepts precisely the trees on which the query holds*

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Corollary

Evaluating a Boolean MSO query on a tree is in linear time in the tree

A small hack for non-Boolean queries

Remember our problem on non-Boolean queries:

Data: A tree T

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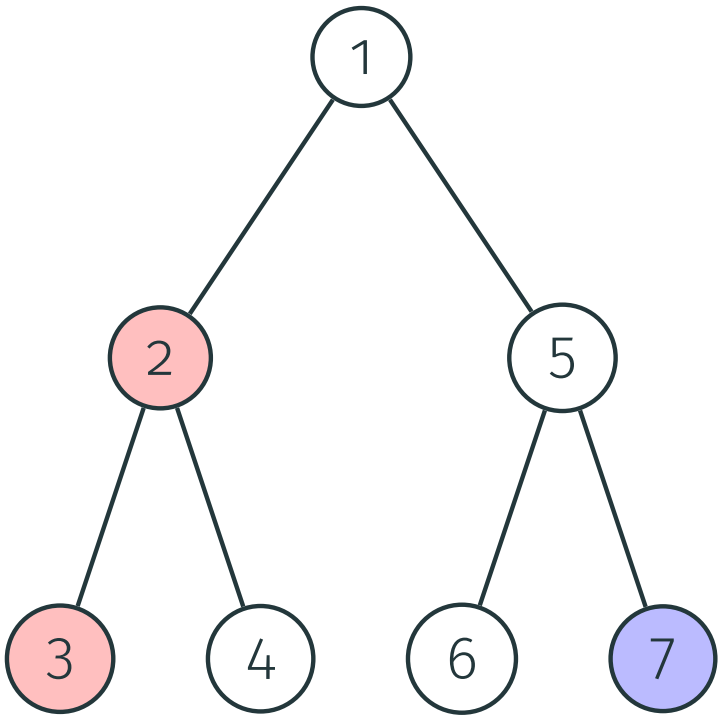

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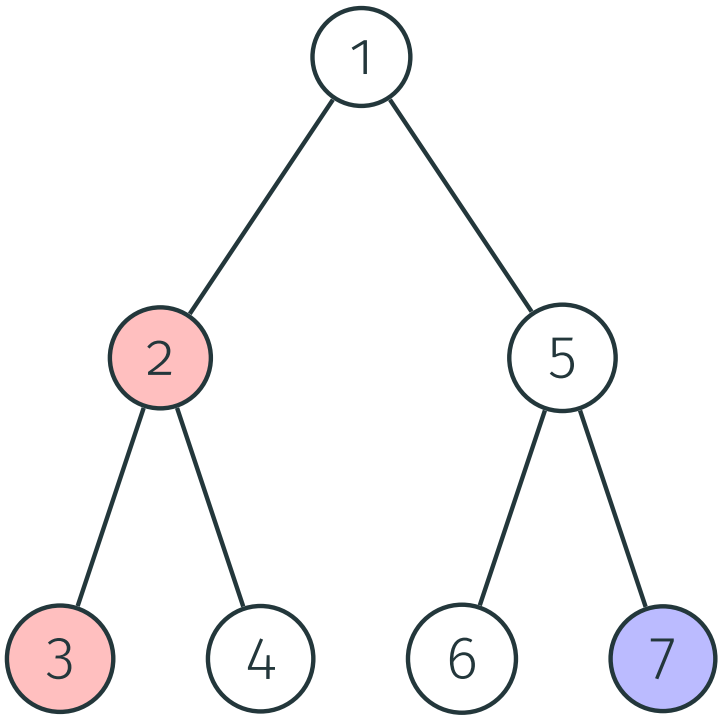

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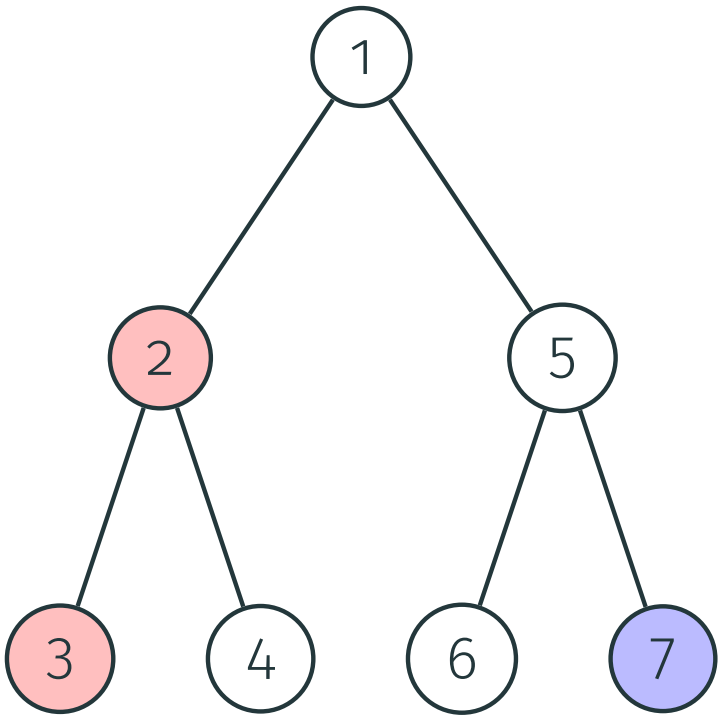

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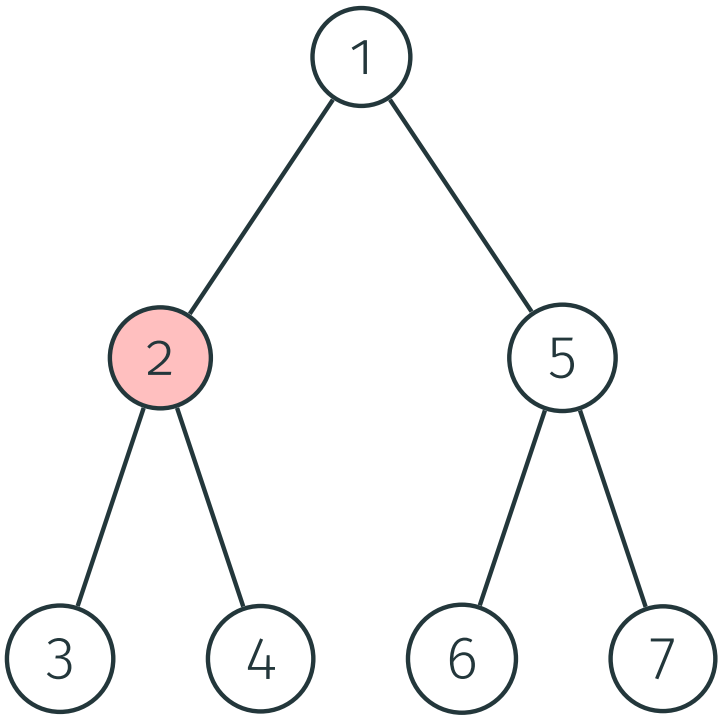

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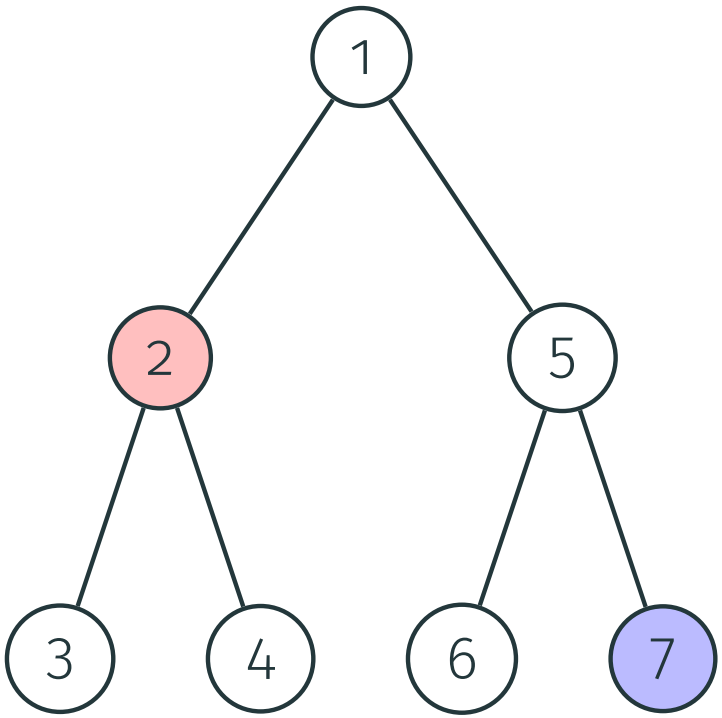

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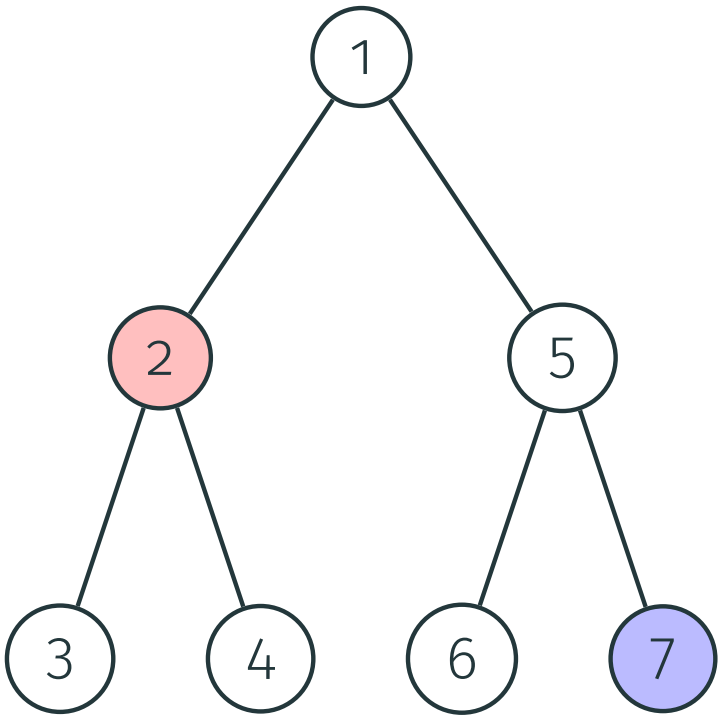

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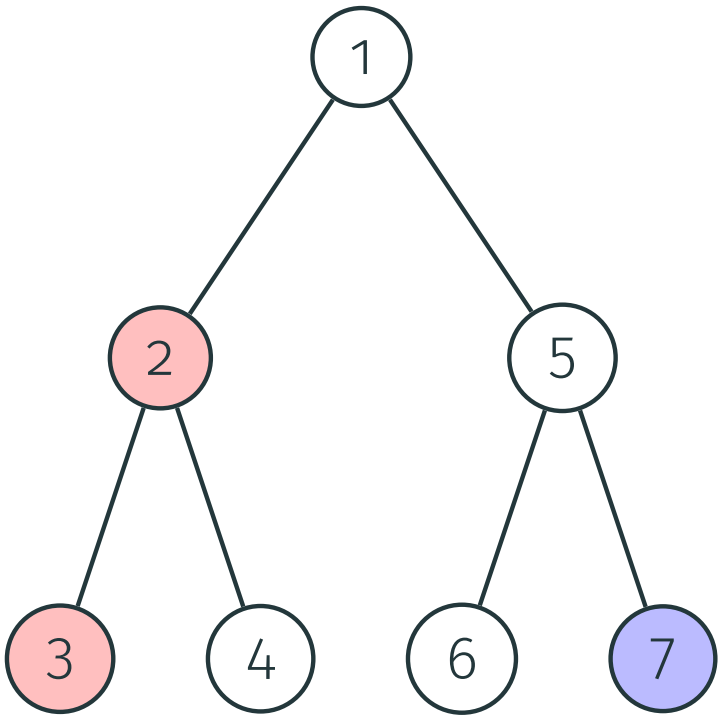

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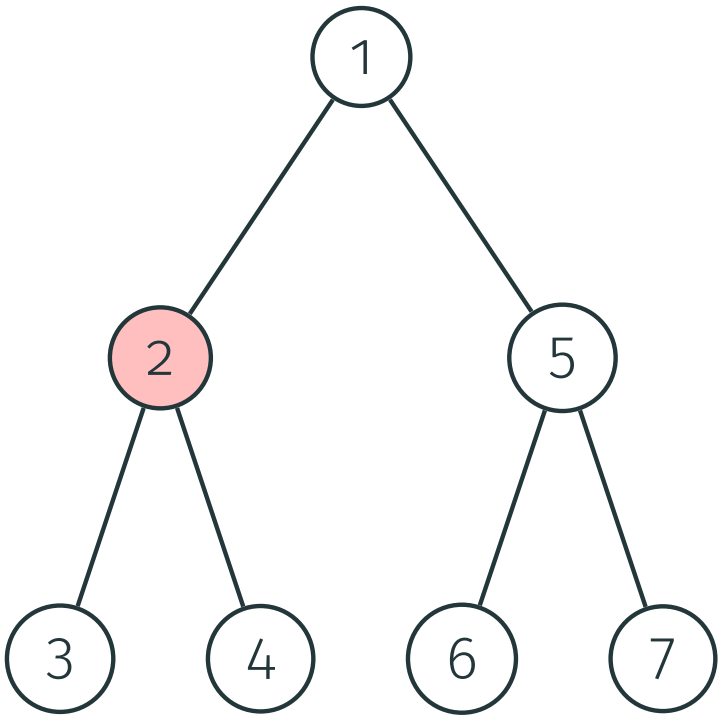

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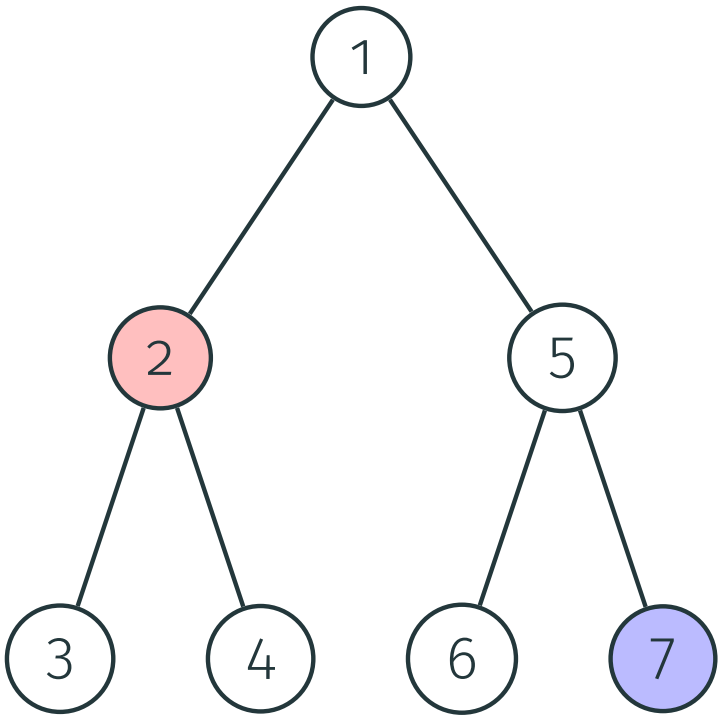

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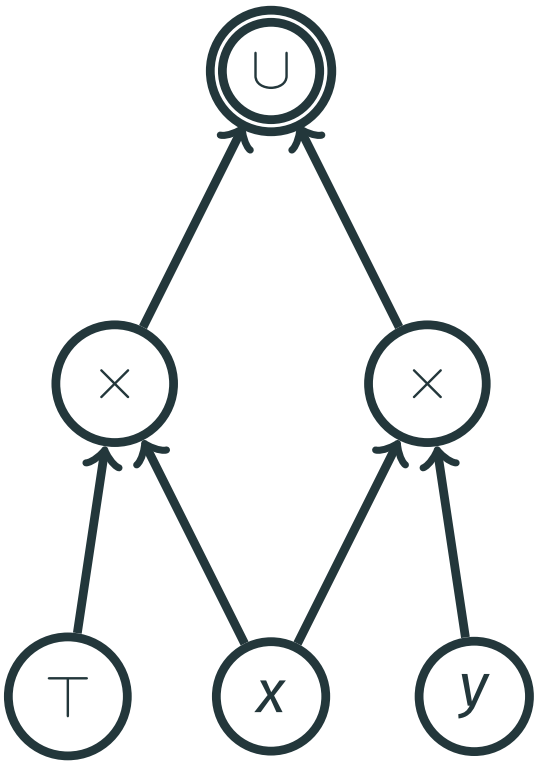
$\rightarrow$ The **results** that we want to enumerate are all **valuations** of T that make A accept

Set circuits

We want to represent these results as a set circuit:



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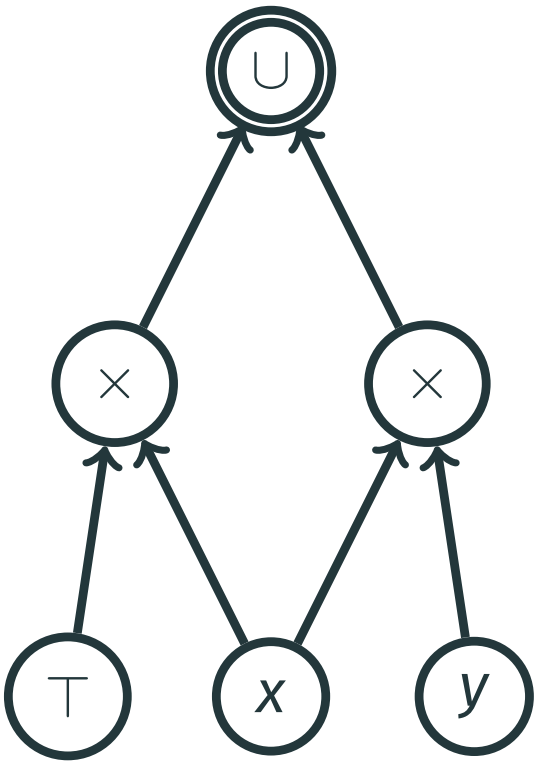

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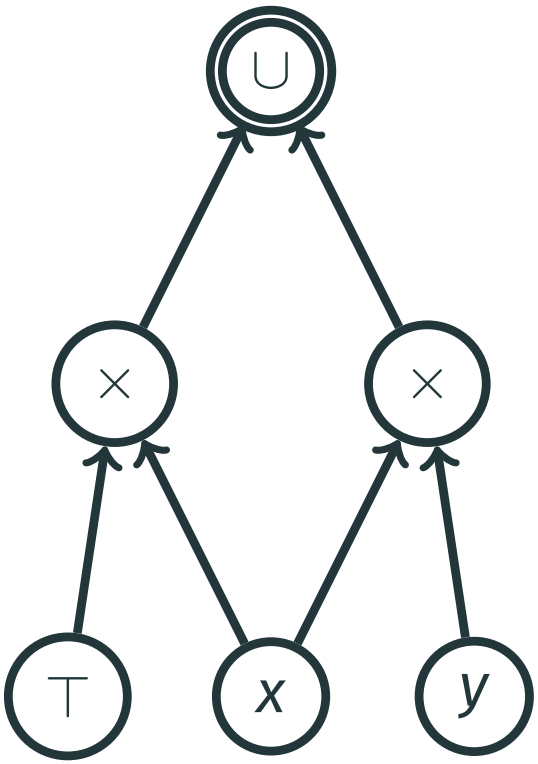

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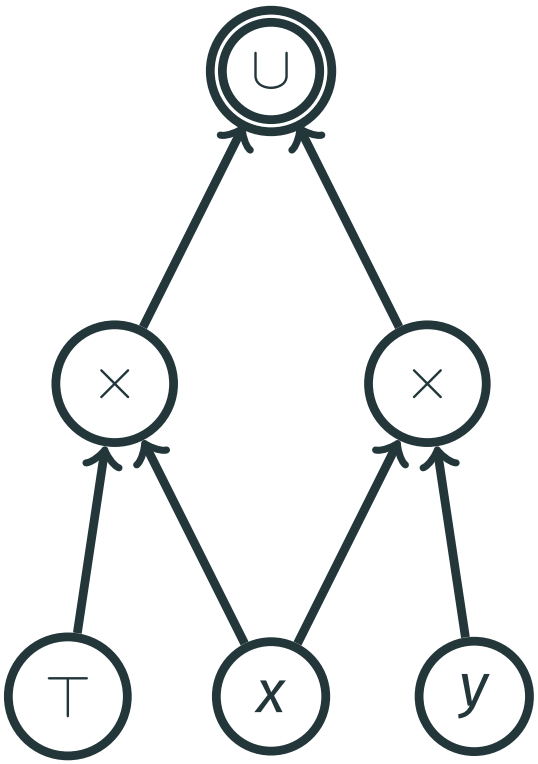

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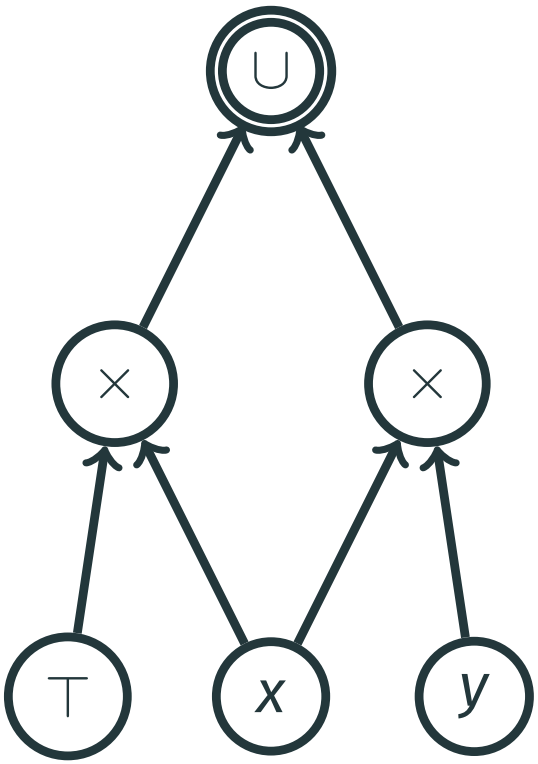

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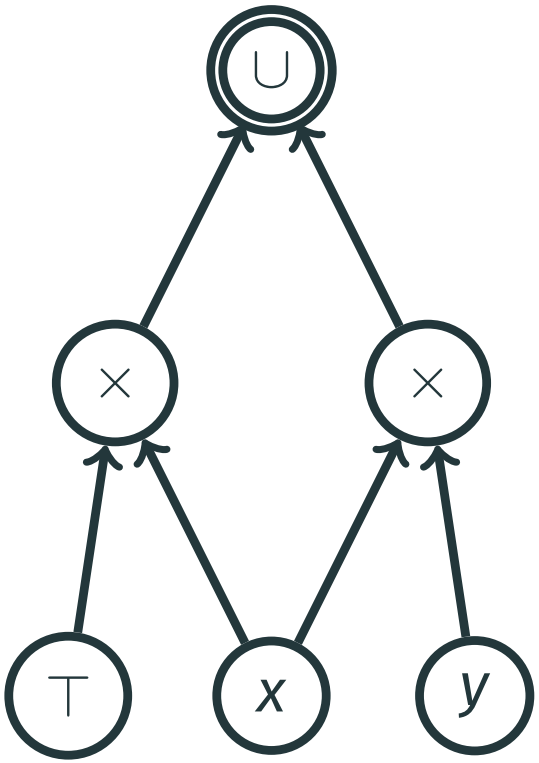

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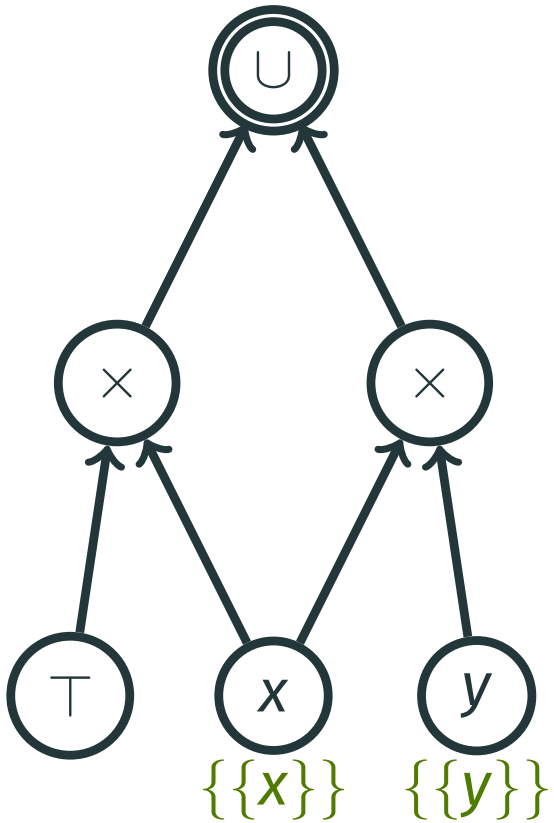

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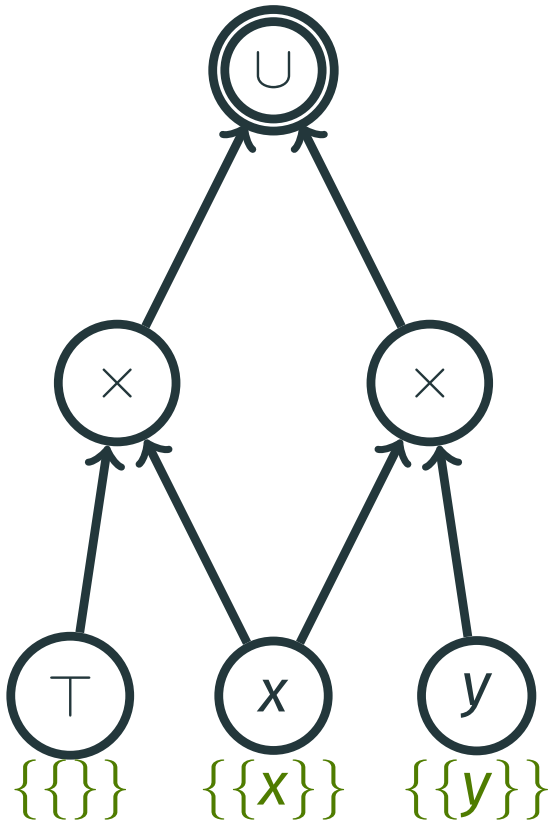

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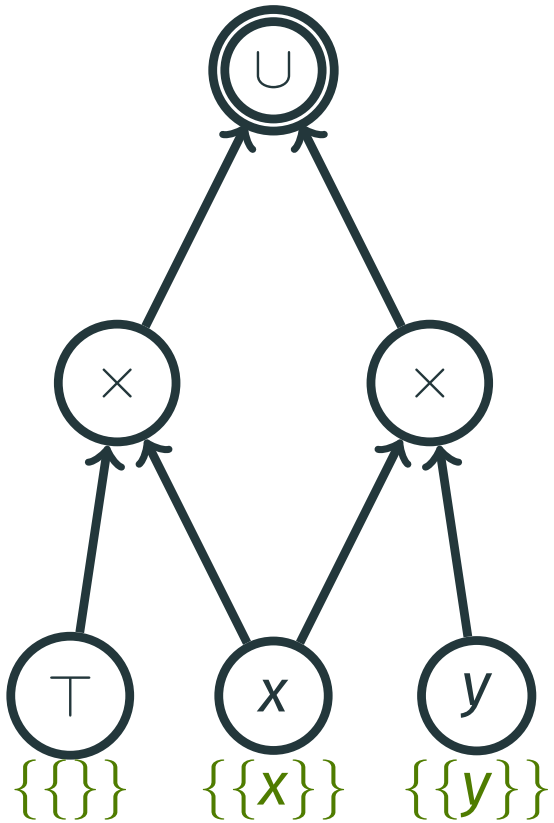

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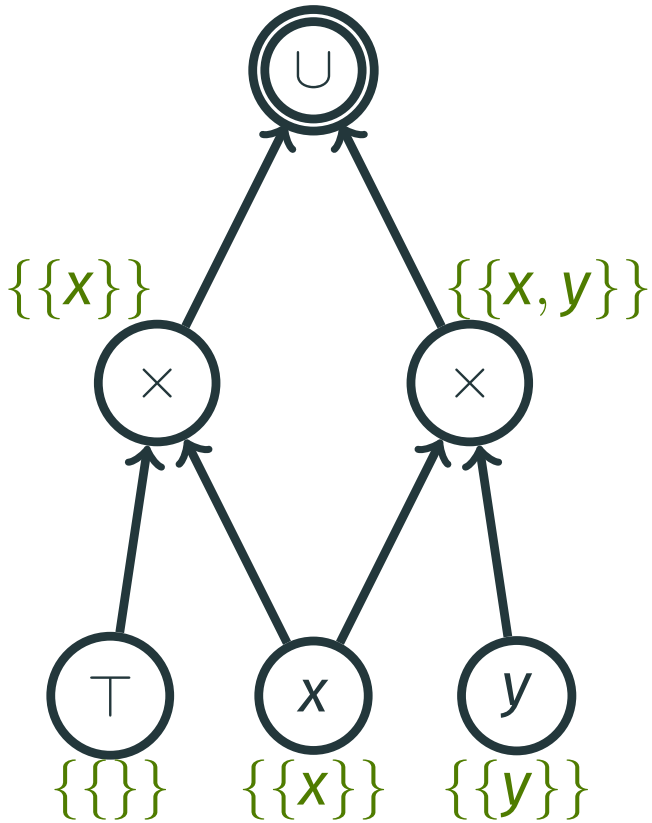

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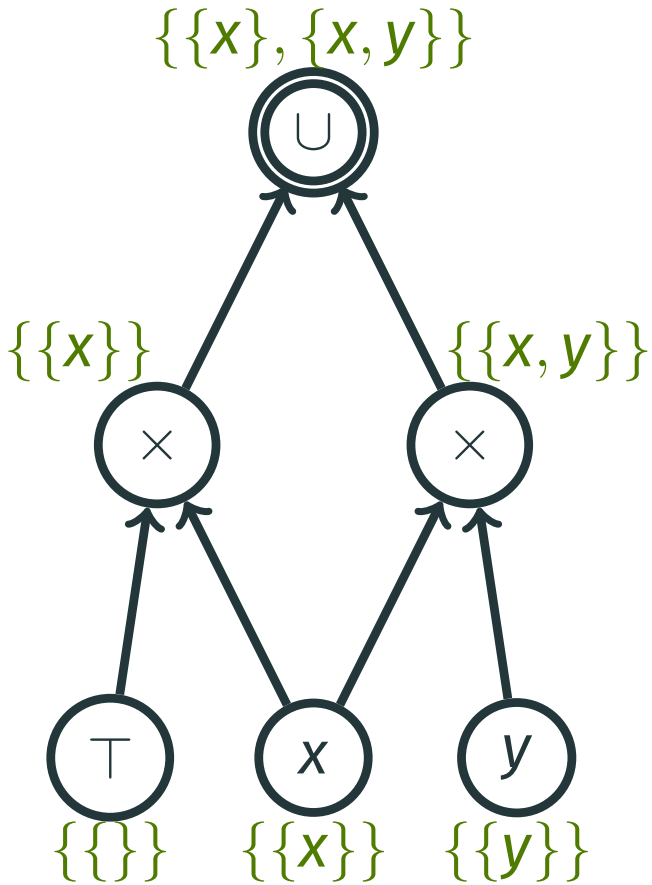

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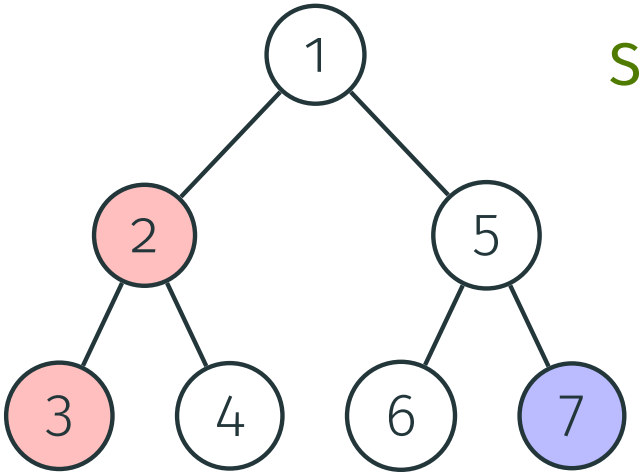

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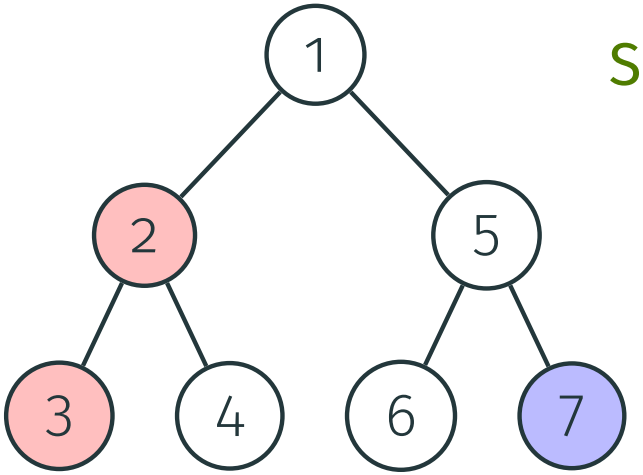

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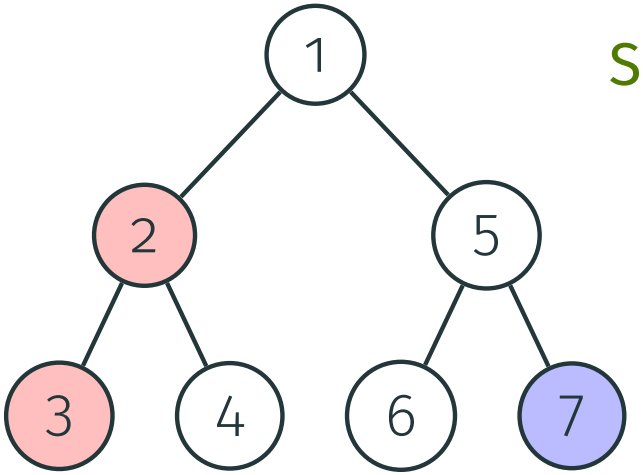

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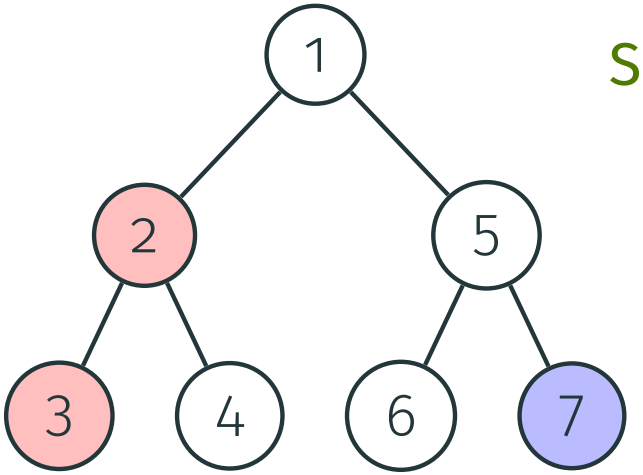

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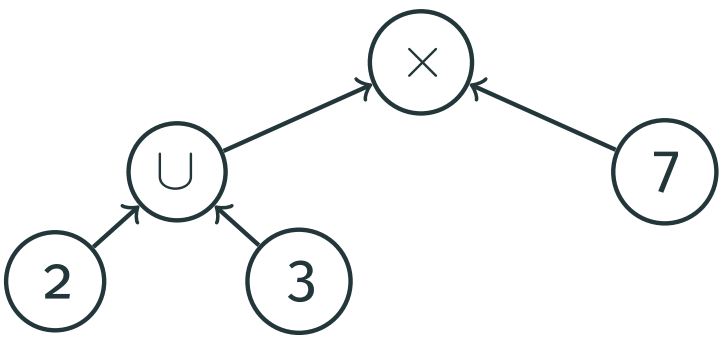

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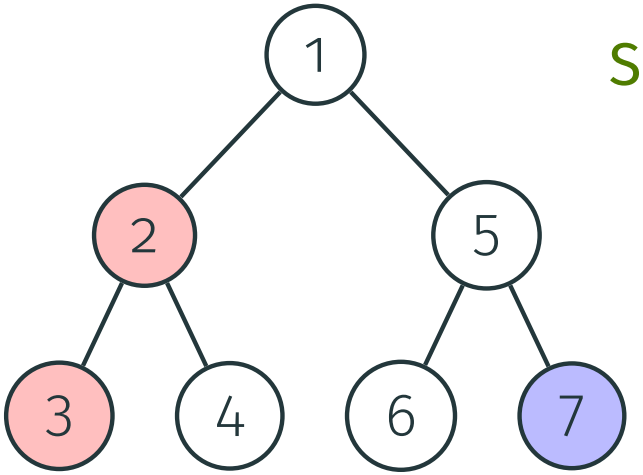

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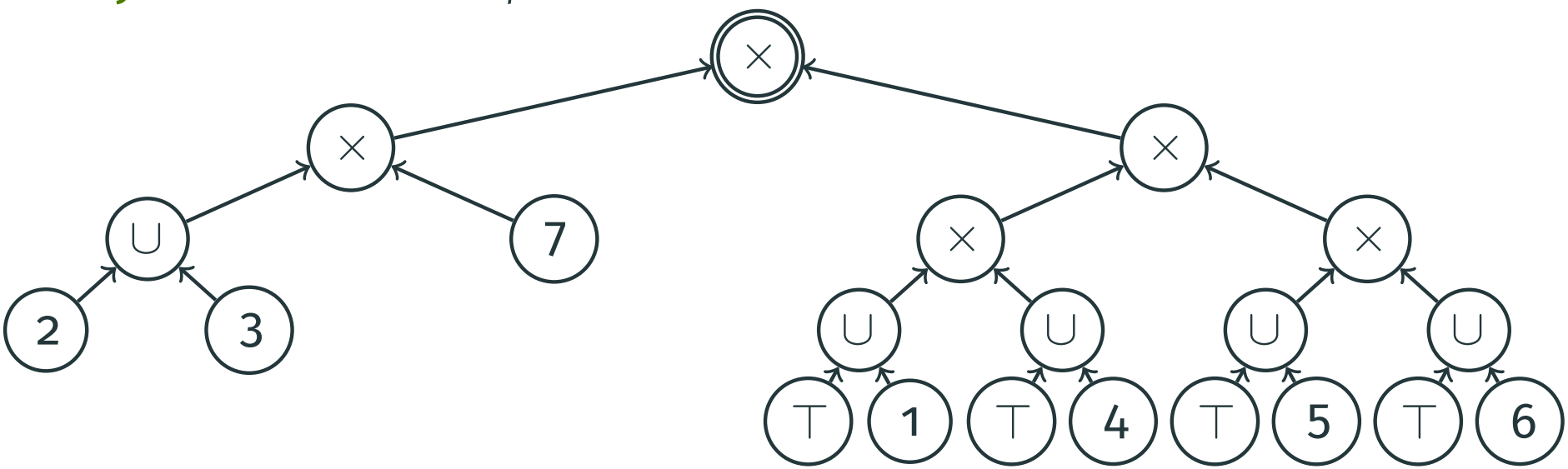

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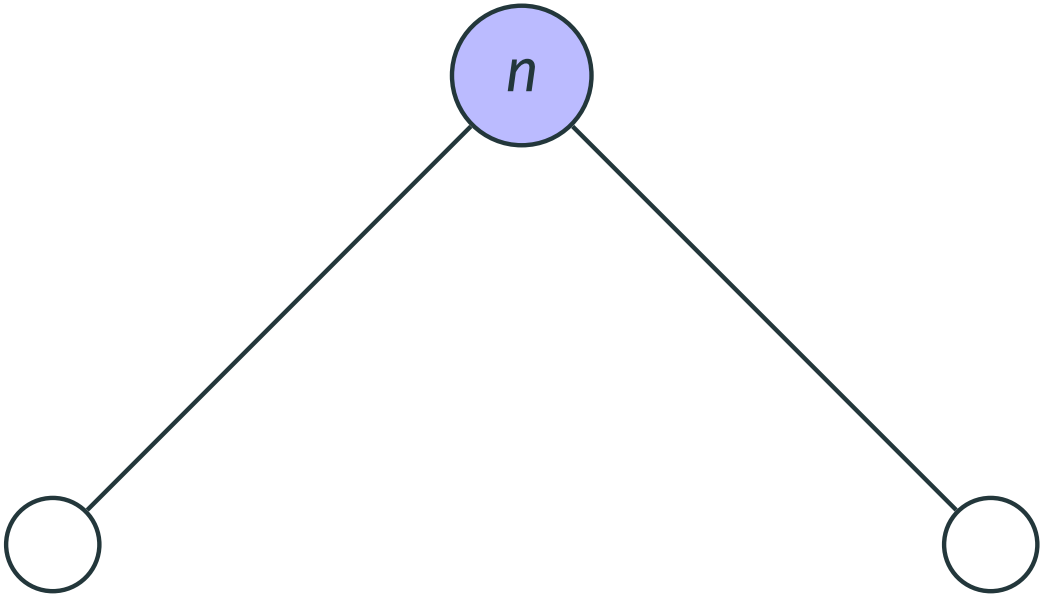

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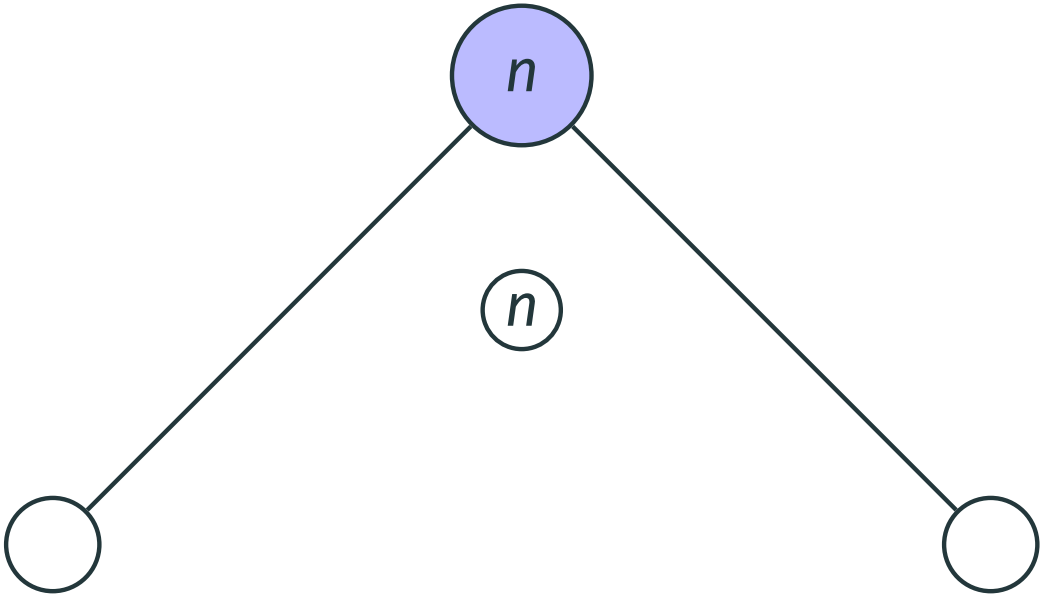

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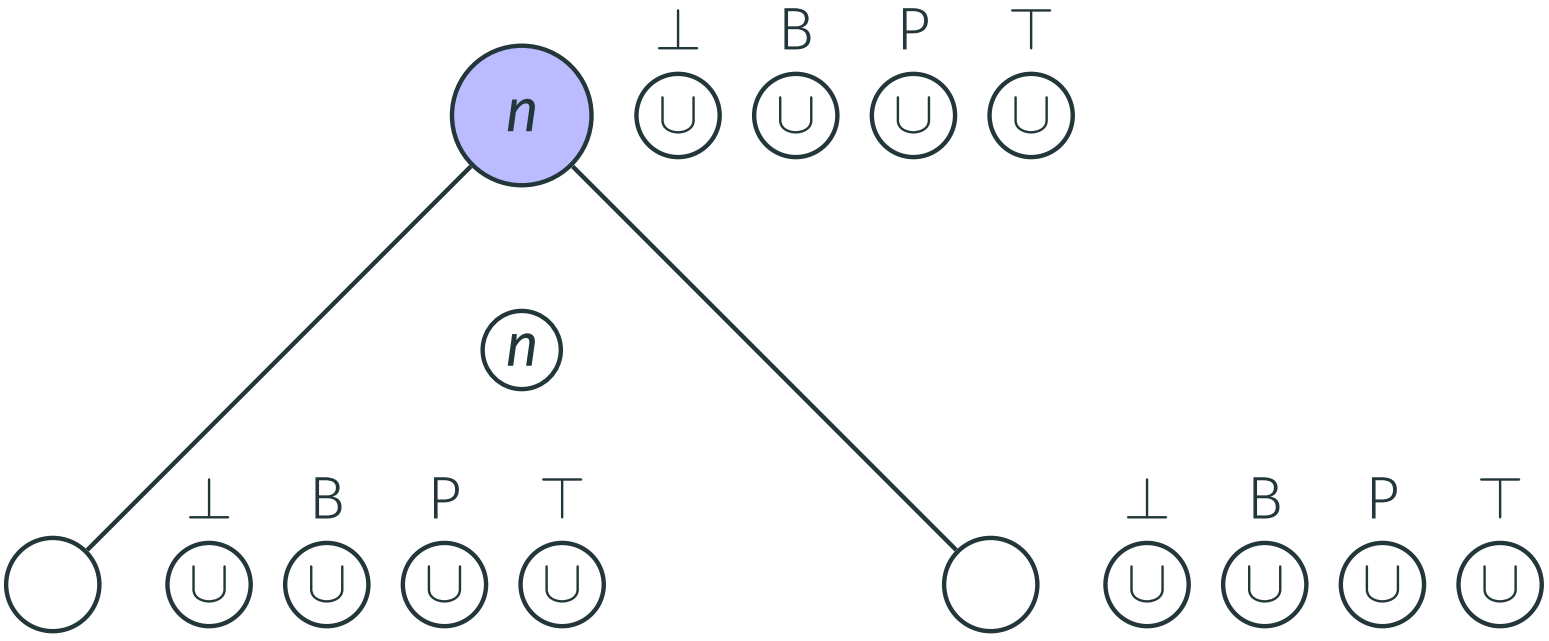

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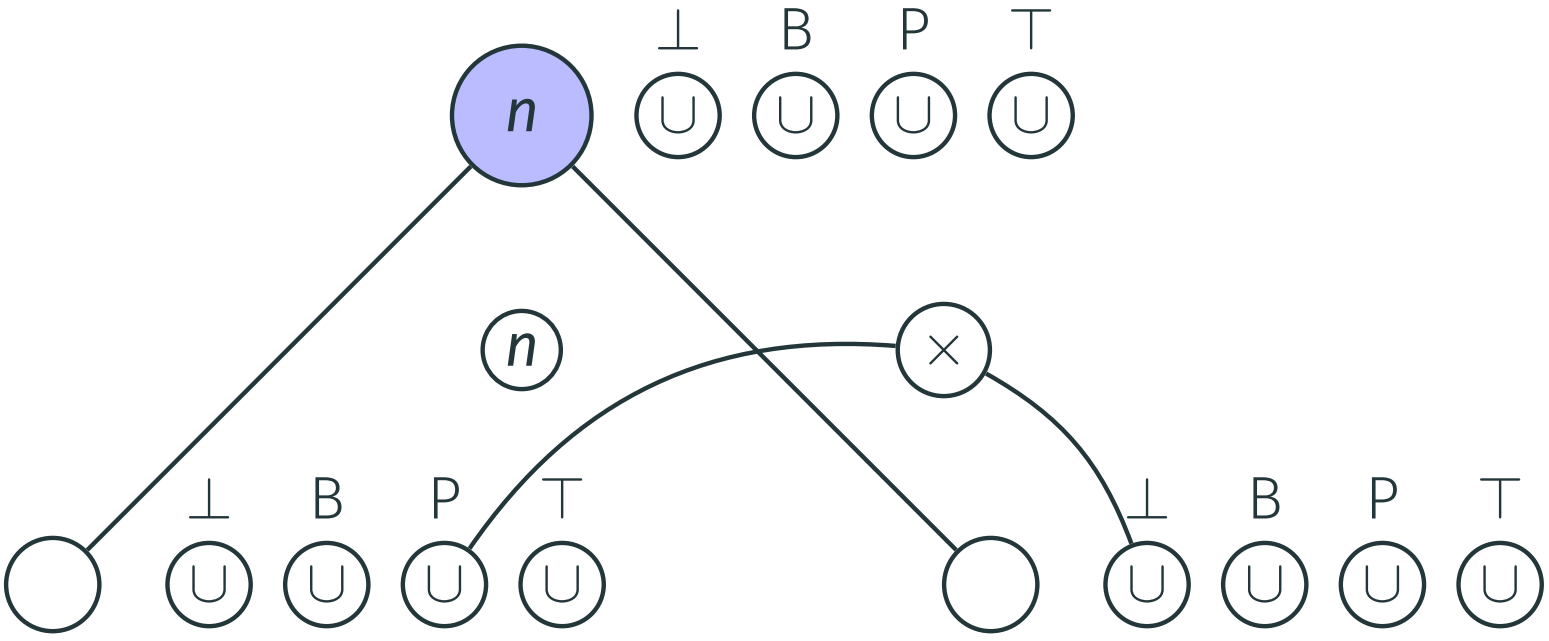

Building set circuits

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For any bottom-up tree automaton A *and input tree* T*, we can build a set circuit of* A *on* T *in* $O(|A| \times |T|)$

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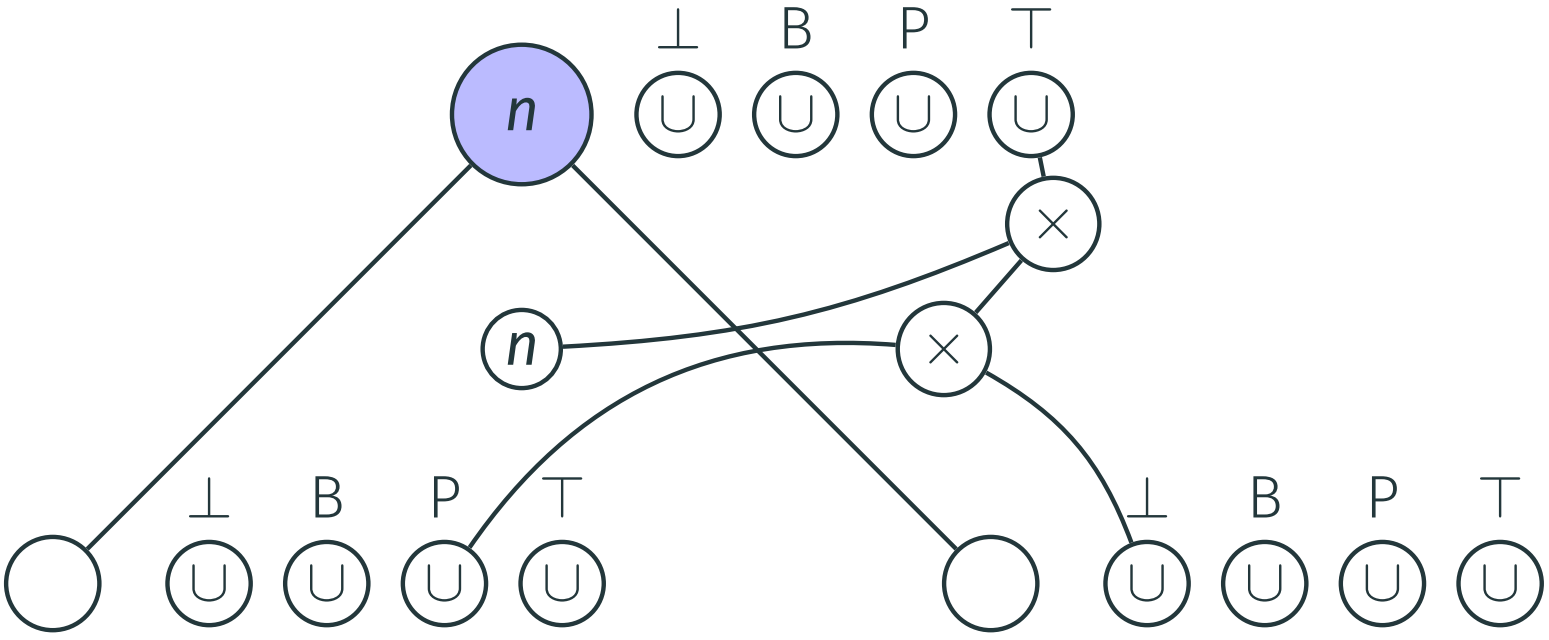

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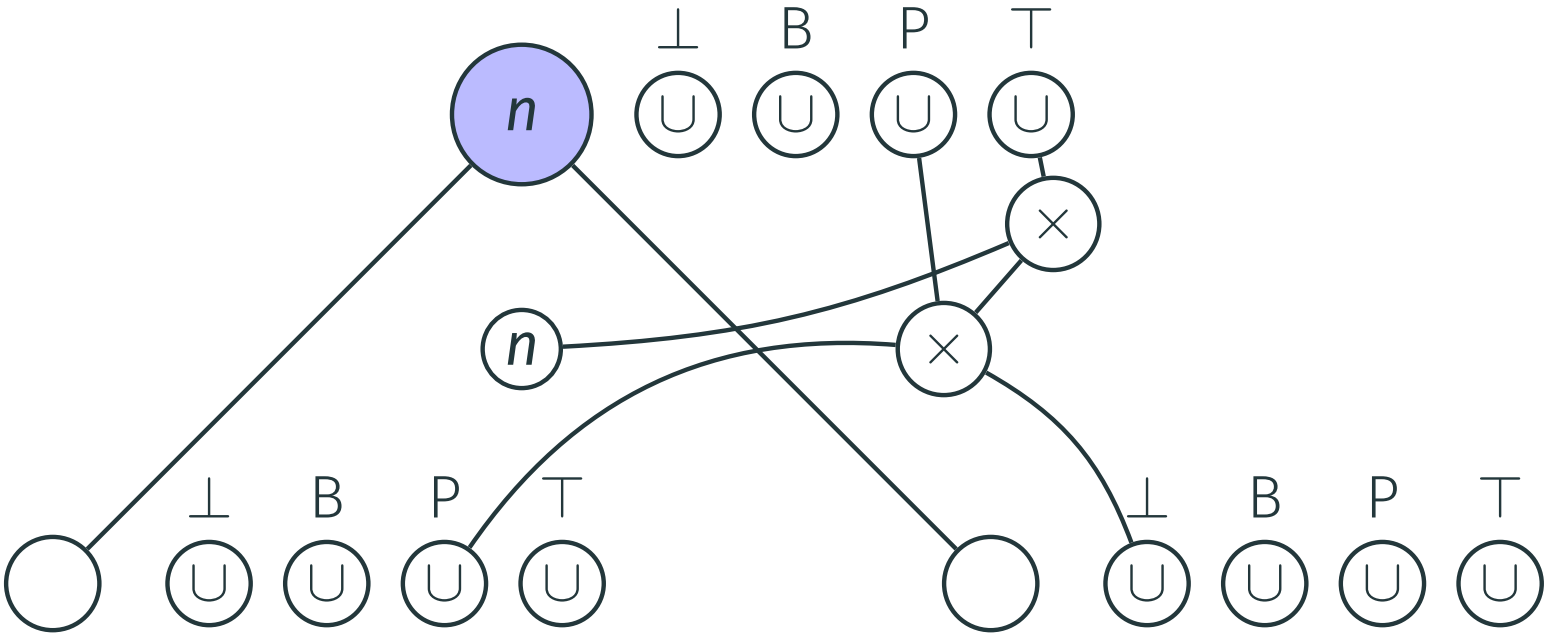


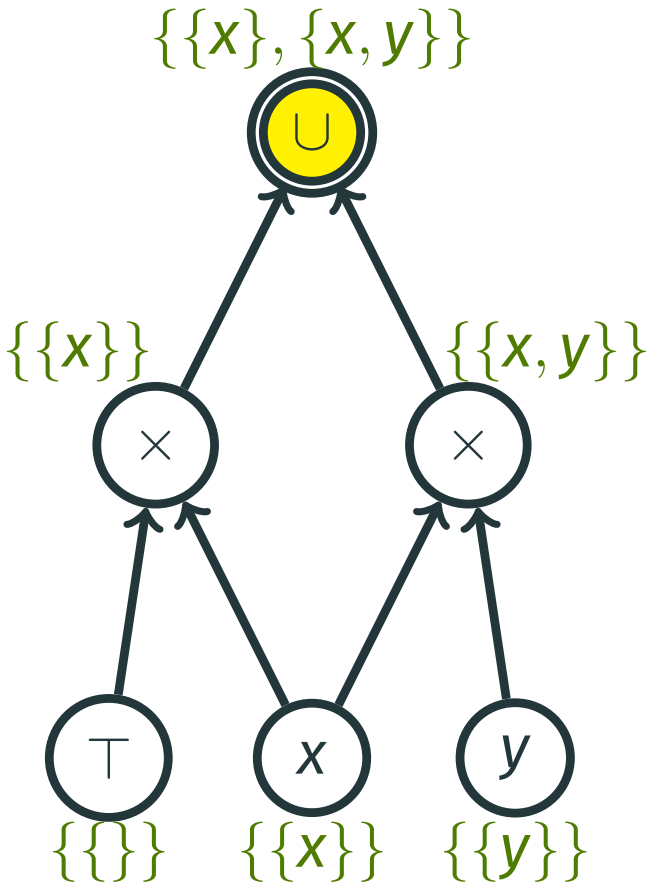
Set circuit restrictions

To have efficient enumeration, we need restrictions on the set circuit:

d-DNNF set circuit:

- ∪ are all deterministic:

The inputs are disjoint
(= no set is captured by two inputs)



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- × are all **decomposable**:

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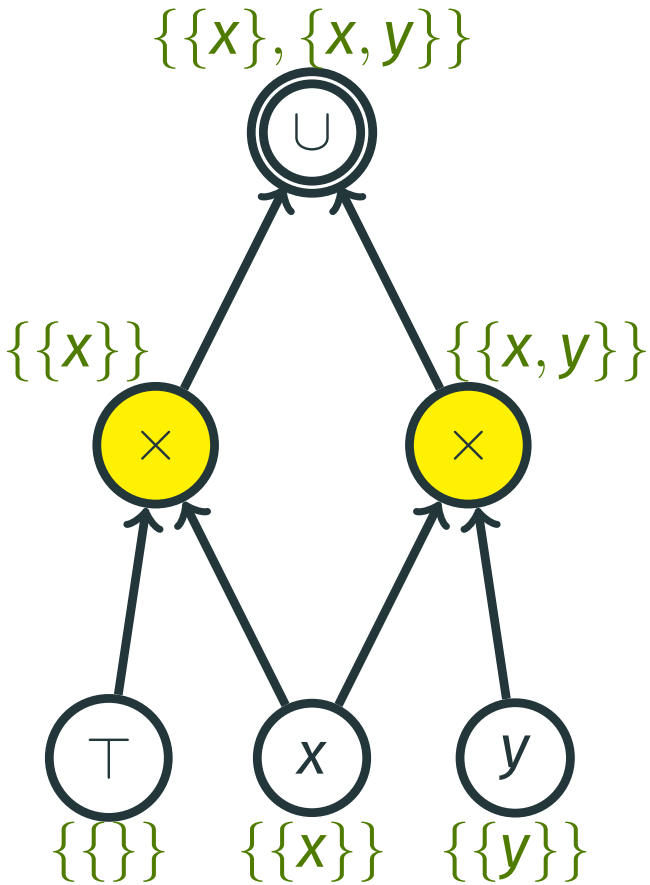

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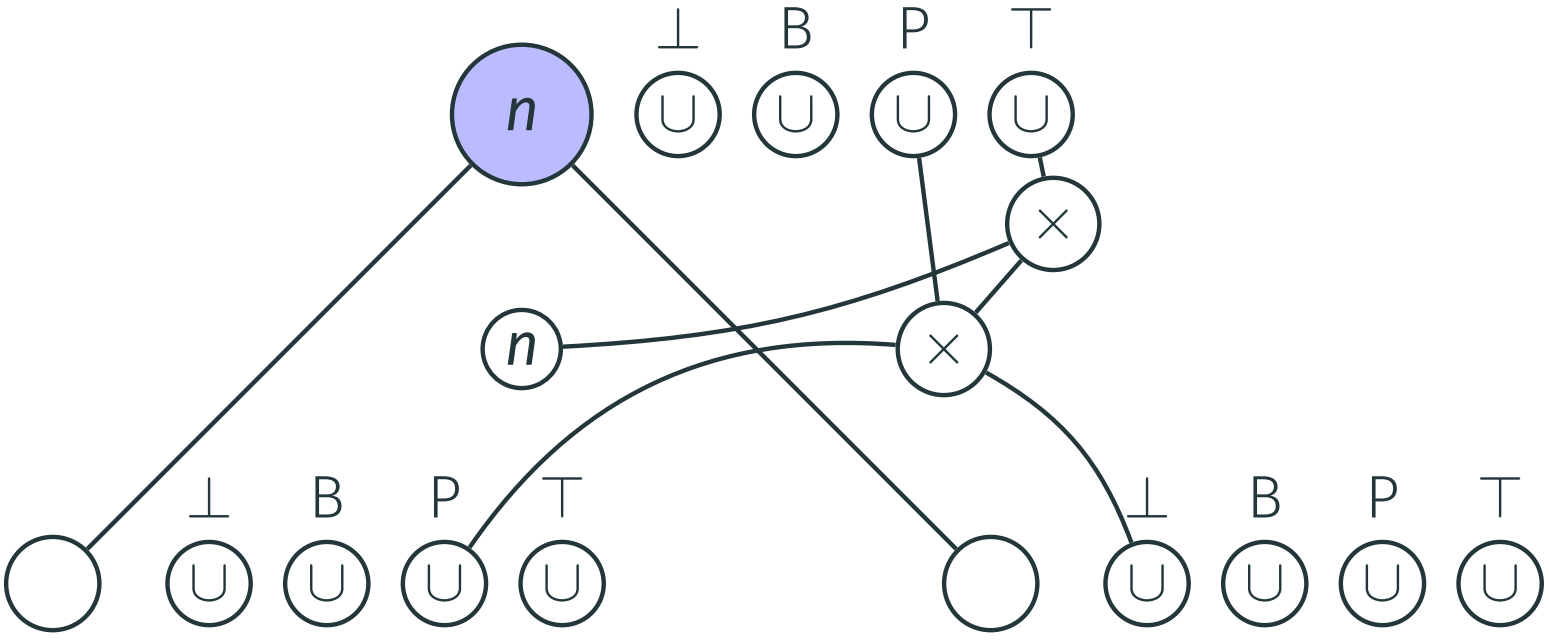

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→ The set circuit of Q is now a factorized representation which describes all the tuples that make Q true

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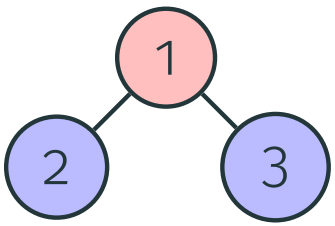

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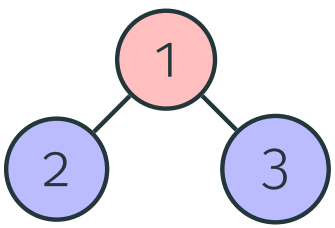

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1	2
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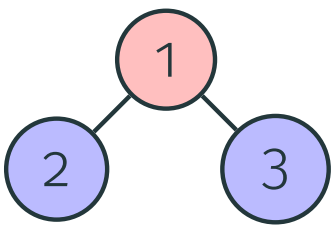

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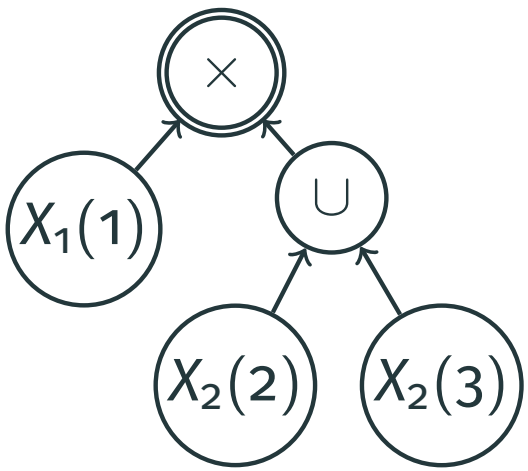

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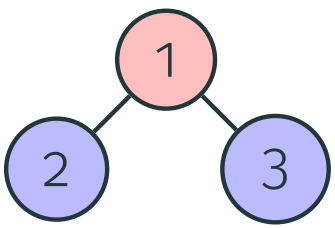

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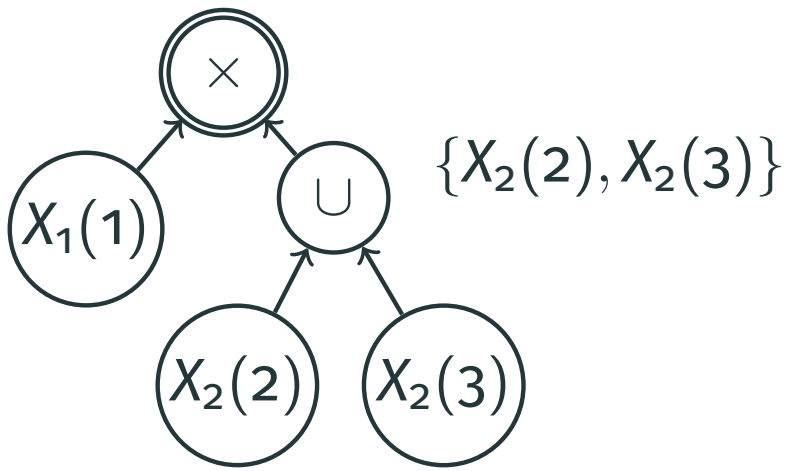

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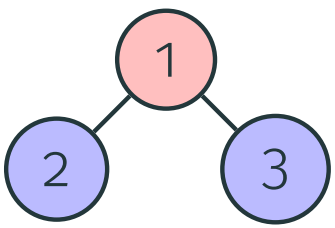

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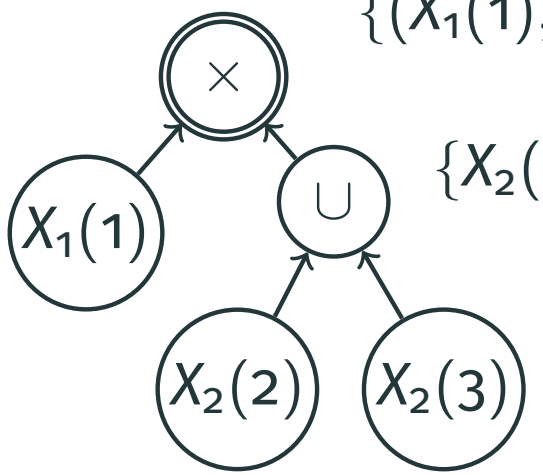
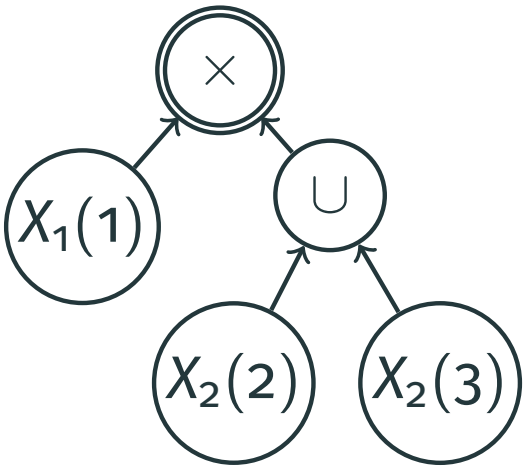


$\{(X_1(1), X_2(2)), (X_1(1), X_2(3))\}$

$\{X_2(2), X_2(3)\}$

Enumeration for set circuits

The situation so far



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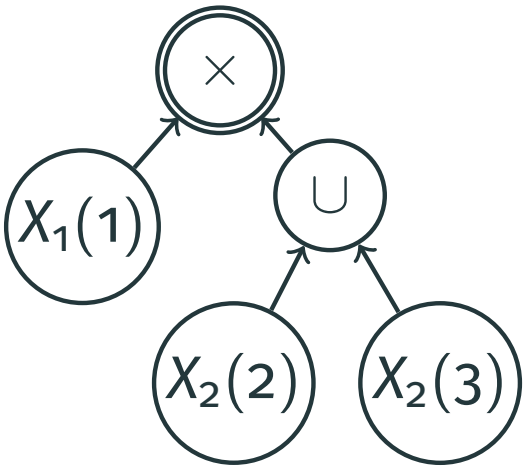

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Given a d-DNNF set circuit C*, we can enumerate its captured sets with preprocessing linear in* $|C|$ *and delay linear in each set*

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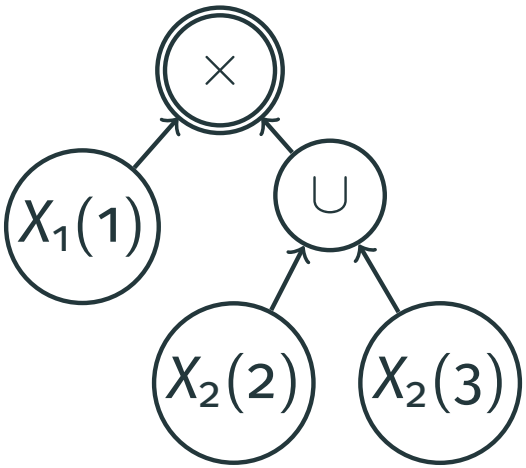

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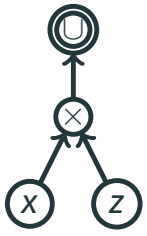
Given a d-DNNF set circuit C, we can enumerate its captured sets with preprocessing linear in $|C|$ and delay linear in each set

→ This is a **generic result** (does not talk about MSO or trees)

→ Any problems whose solutions can be coded as a d-DNNF can be efficiently enumerated via this method

Enumeration proof overview

Preprocessing phase:



d-DNNF

set circuit

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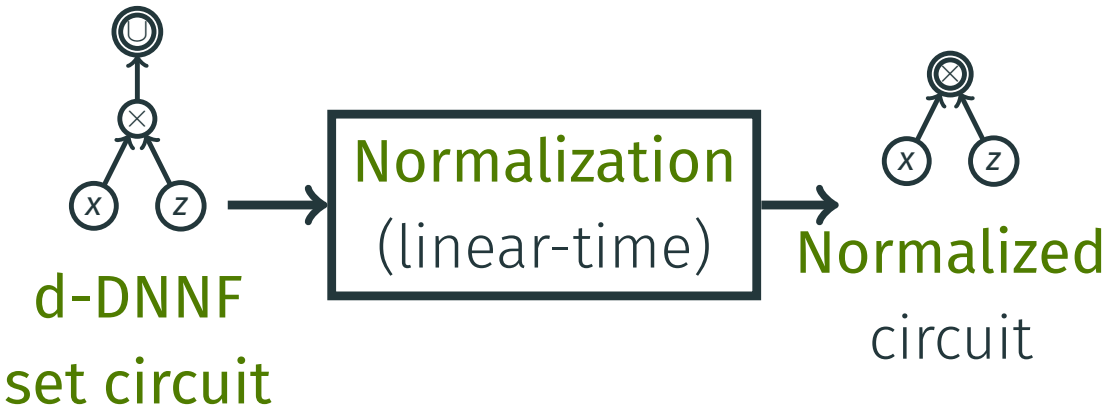

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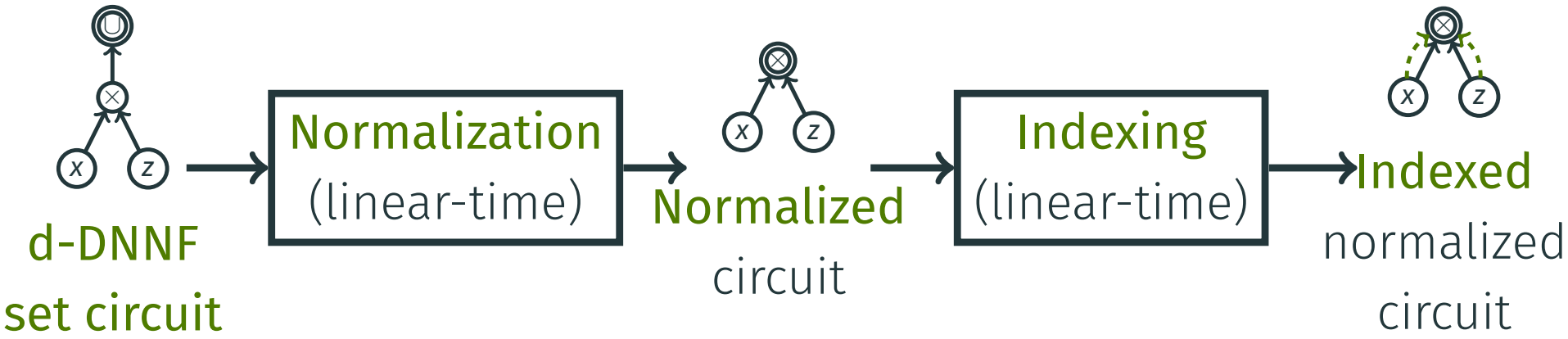

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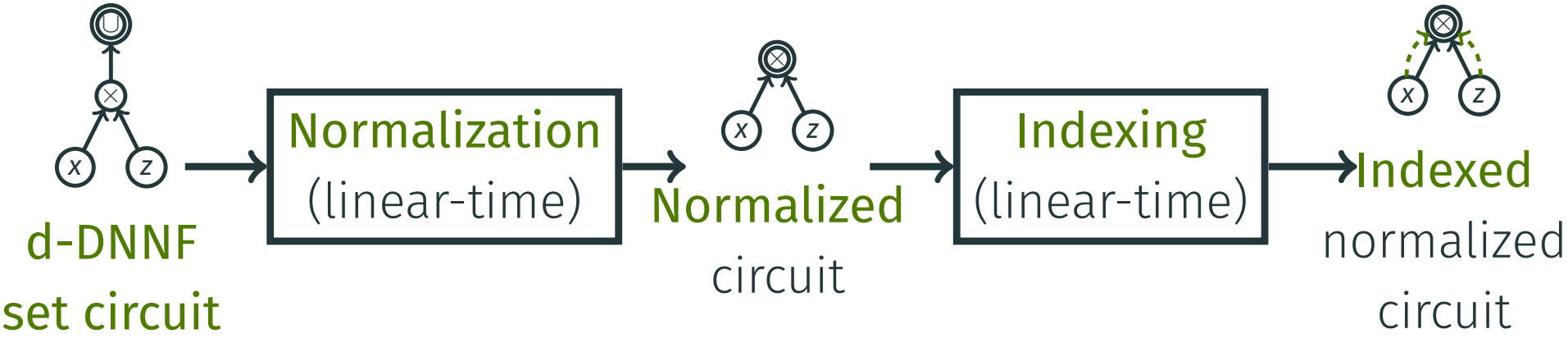

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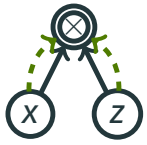

Indexed
normalized
circuit

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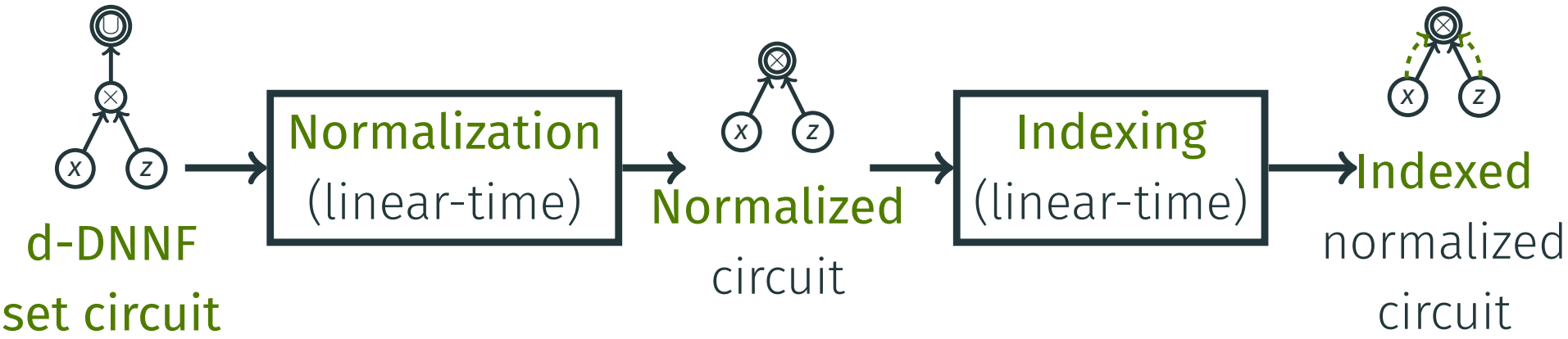

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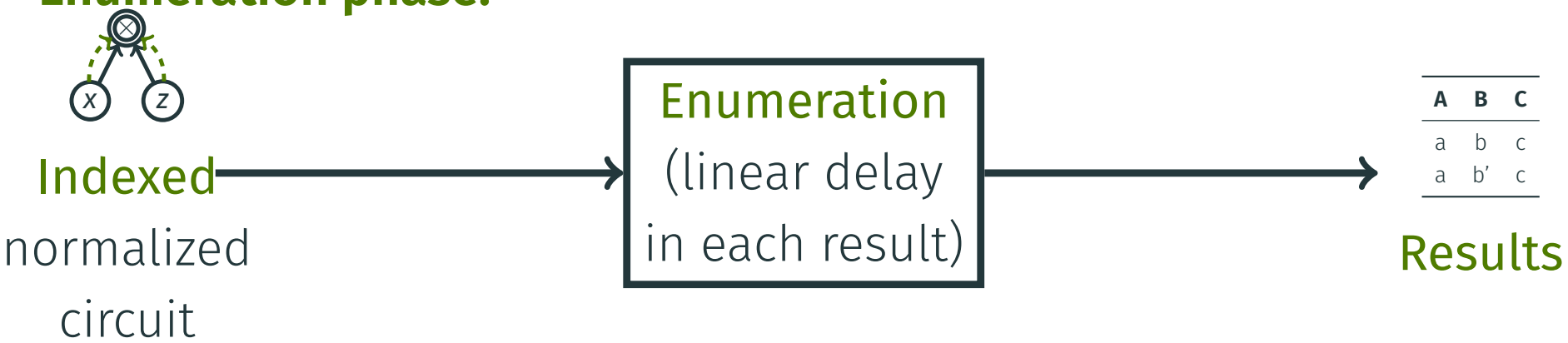

Enumerating captured sets of d-DNNF set circuits

Task: Enumerate the elements of the set $S(g)$ captured by a gate g

$\rightarrow$ E.g., for $S(g) = \{\{x\}, \{x, y\}\}$, enumerate $\{x\}$ and then $\{x, y\}$

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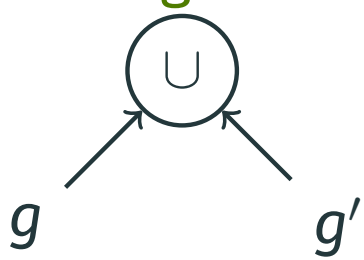

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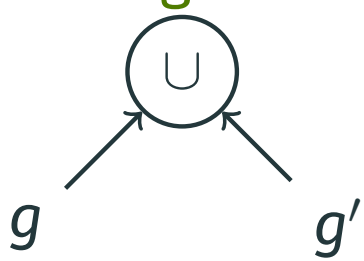

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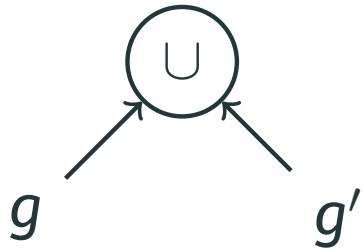
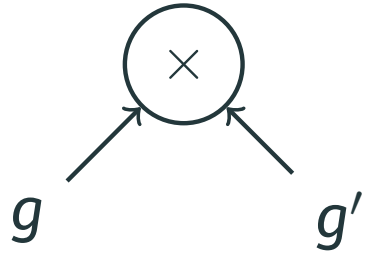


$\times$-gate



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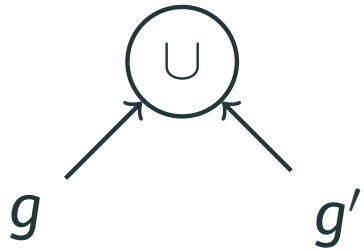

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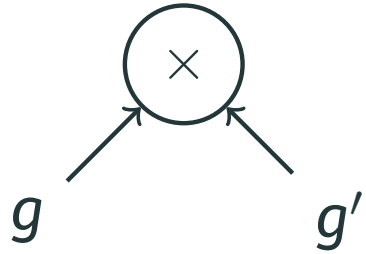

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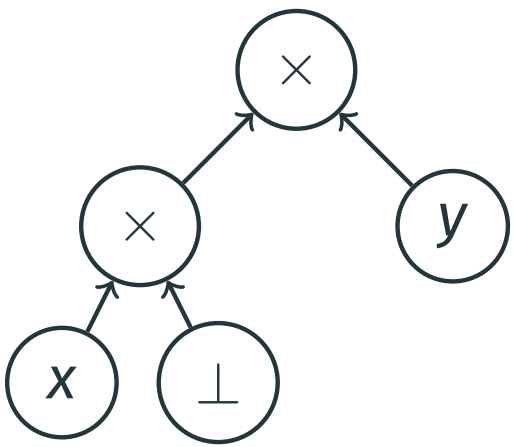
Decomposability: no duplicates

Normalization: handling $\emptyset$



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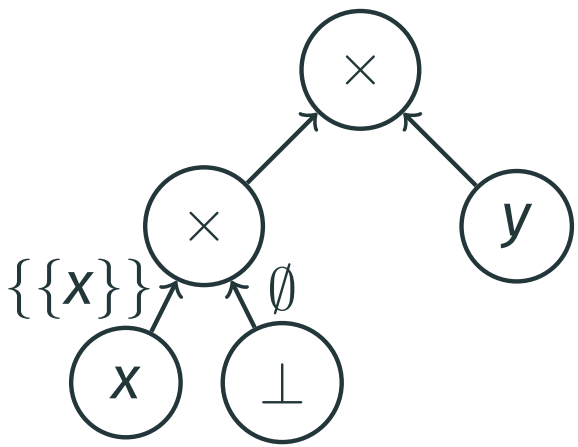

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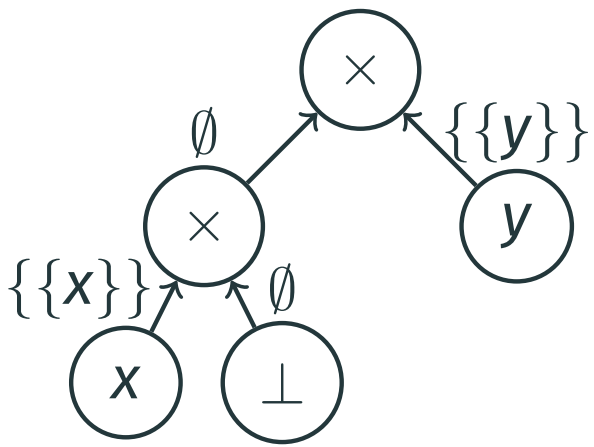

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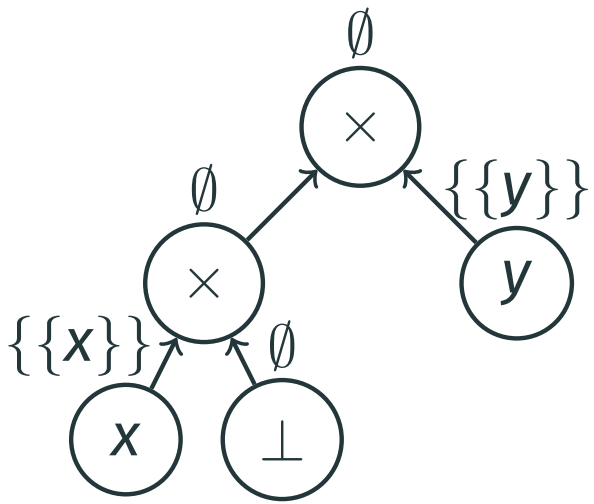

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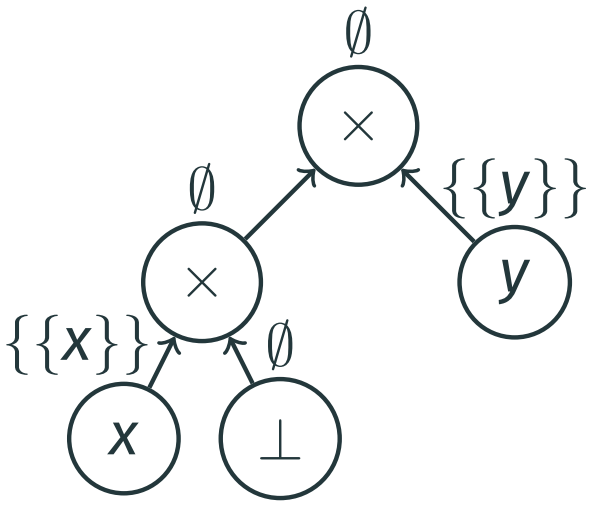

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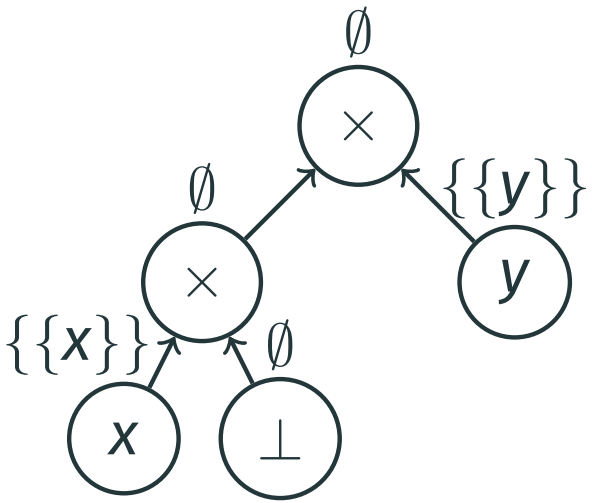

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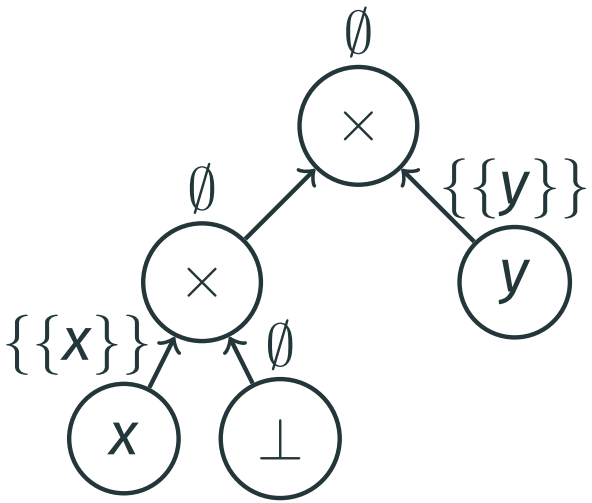

- Problem: if $S(g) = \emptyset$ we waste time
- Solution: in preprocessing
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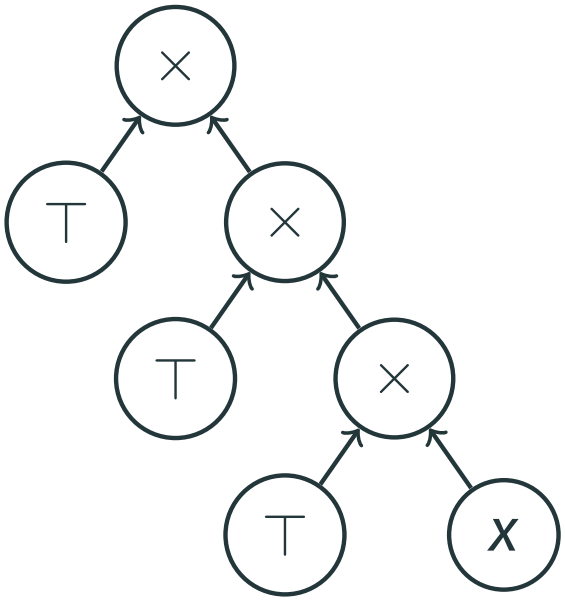
  - then get rid of the gate

Normalization: handling empty sets



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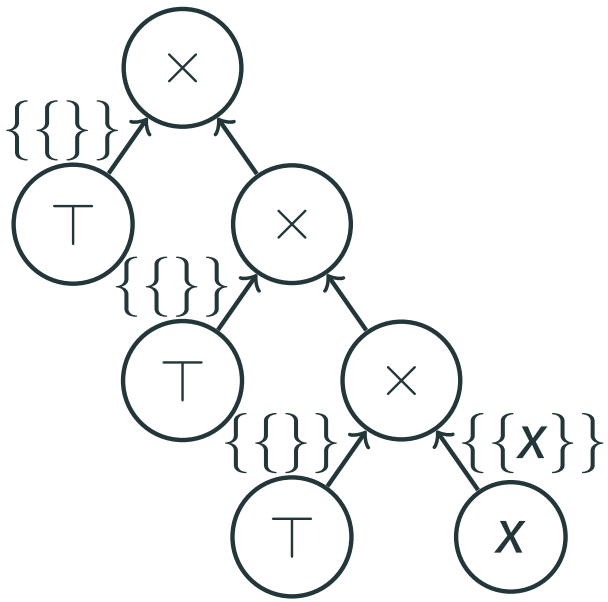

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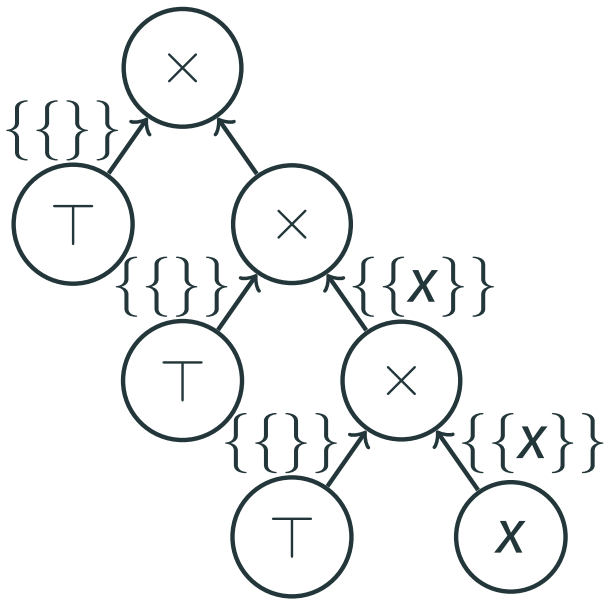

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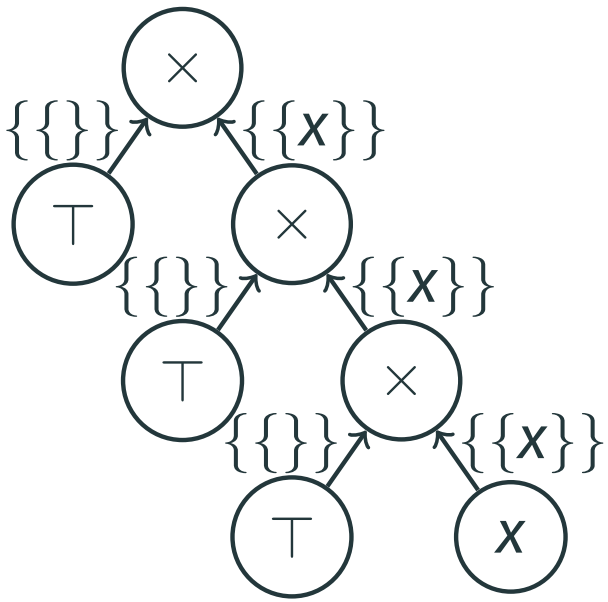

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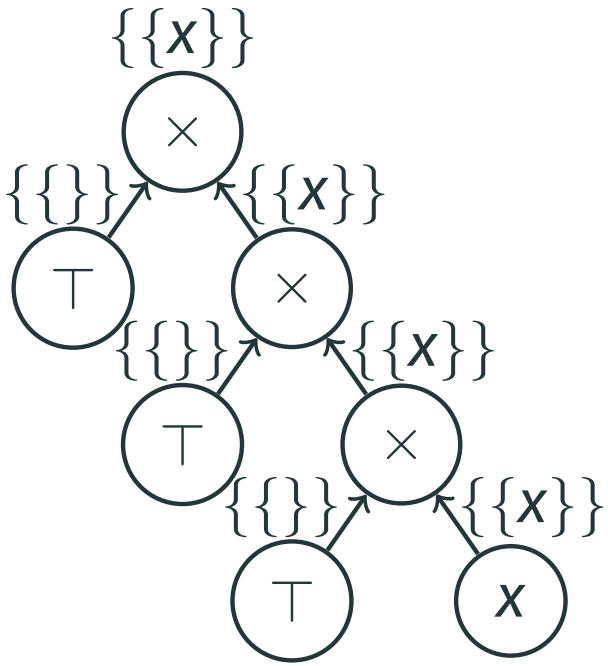

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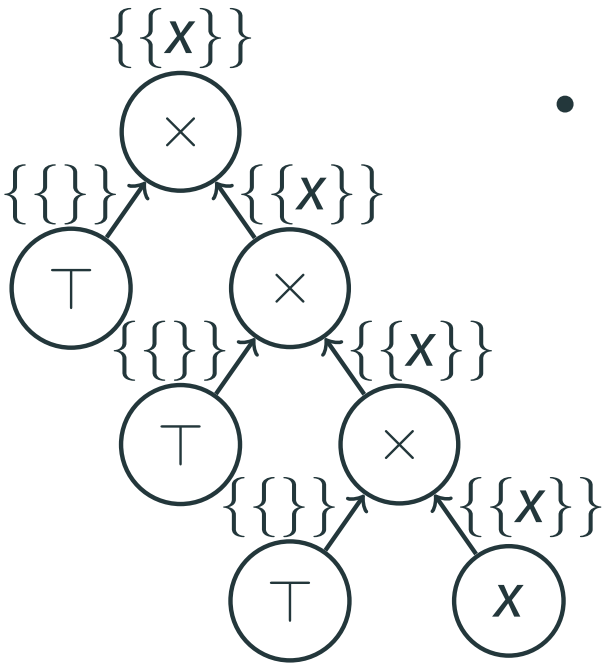
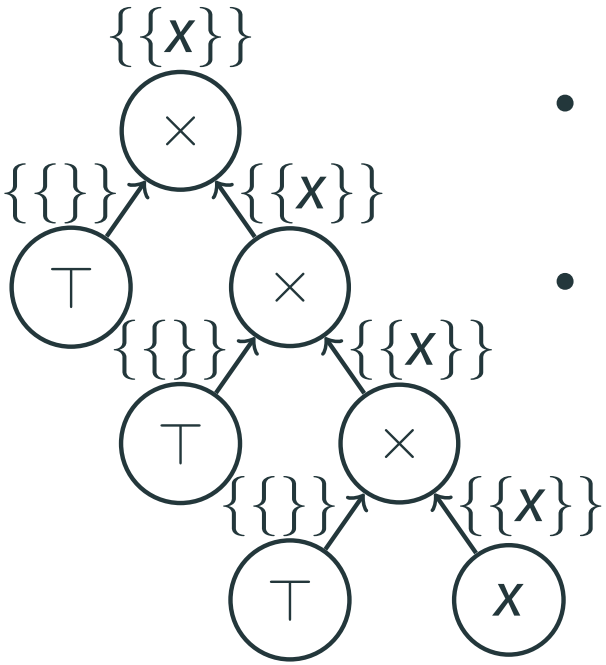


- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of $\times$-gates

Normalization: handling empty sets



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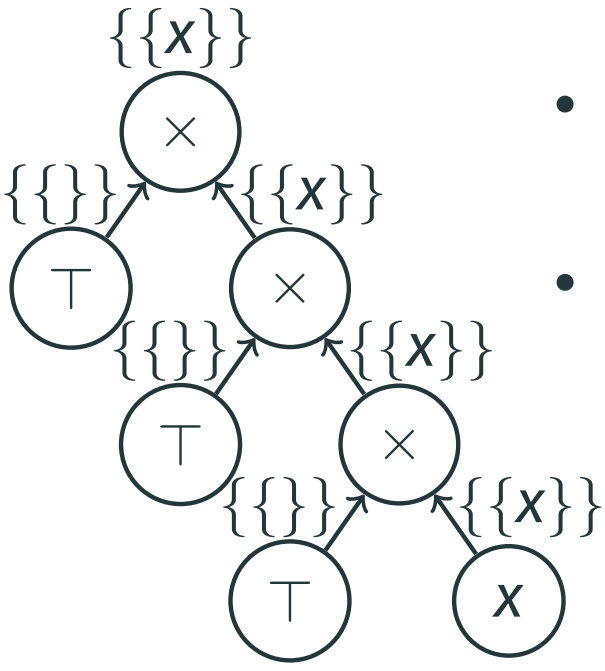

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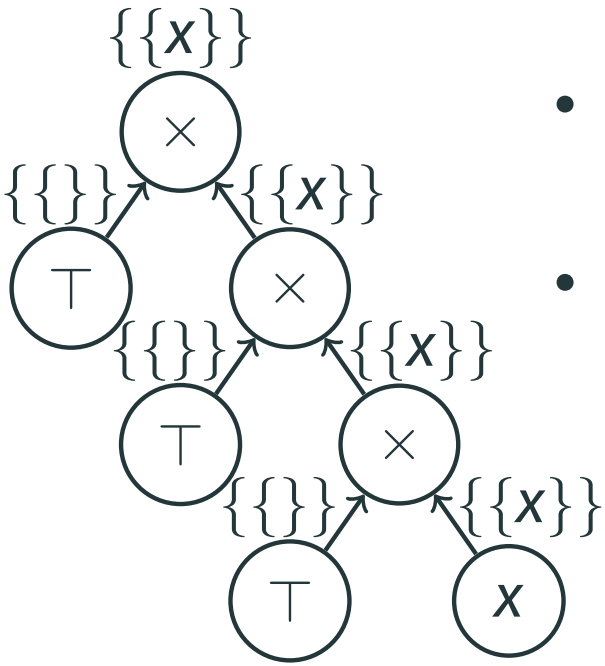

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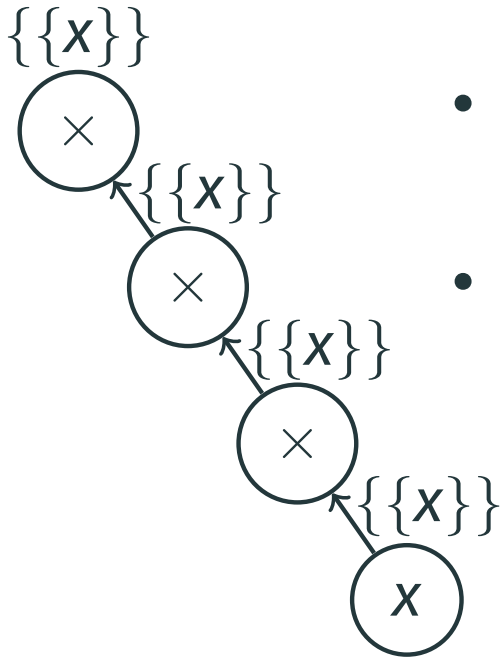

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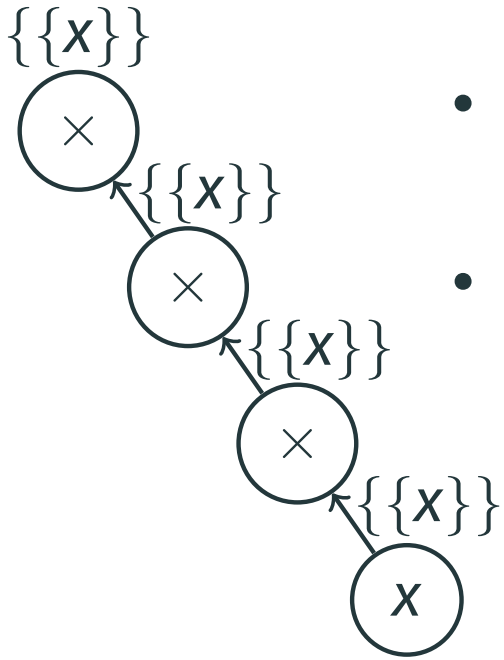

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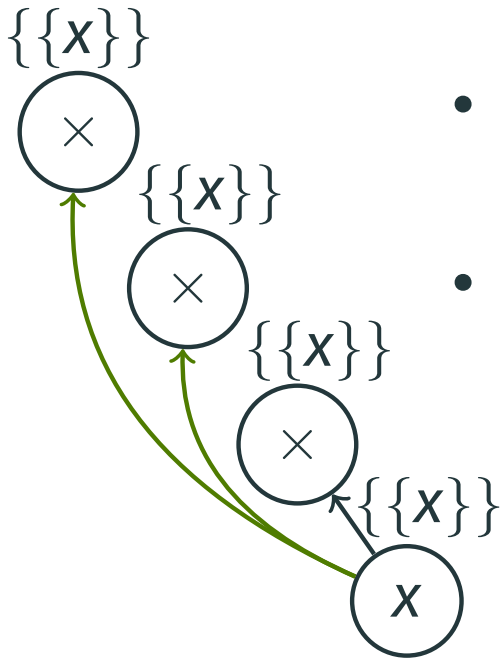

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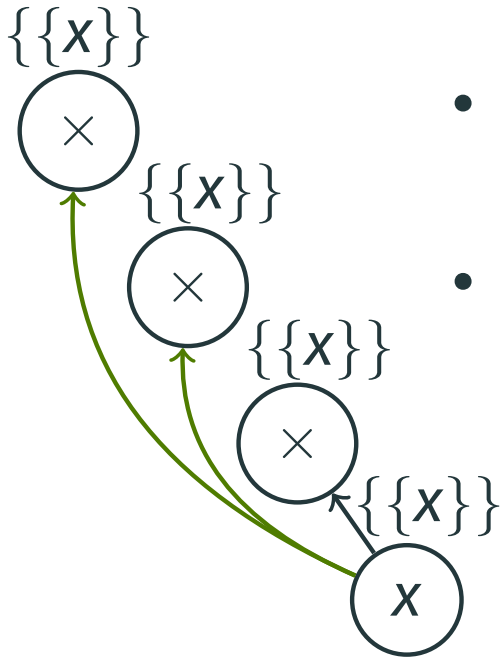

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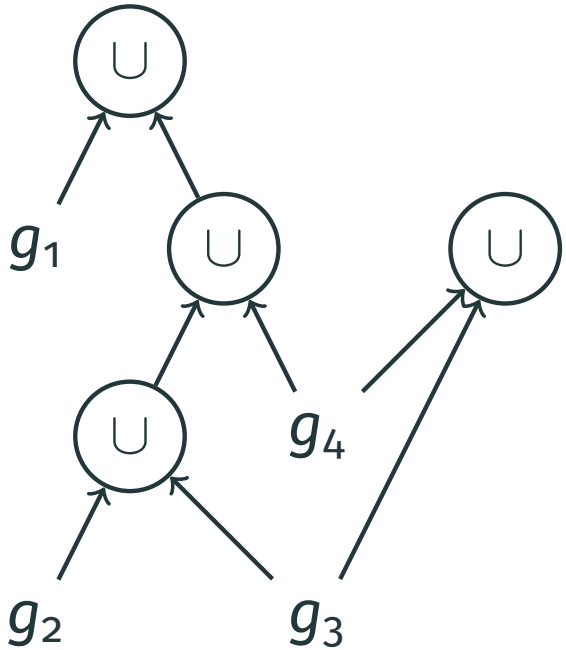
$\rightarrow$ Now, traversing a $\times$**-gate** ensures that we make progress: it **splits** the sets non-trivially

Indexing: handling ∪-hierarchies



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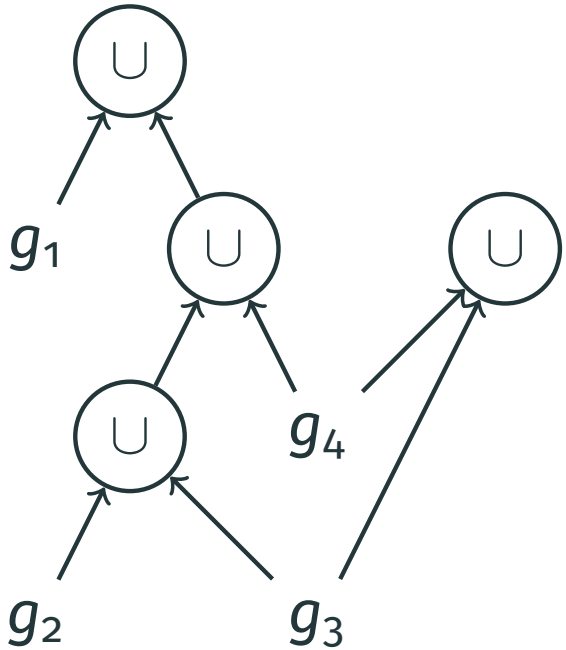

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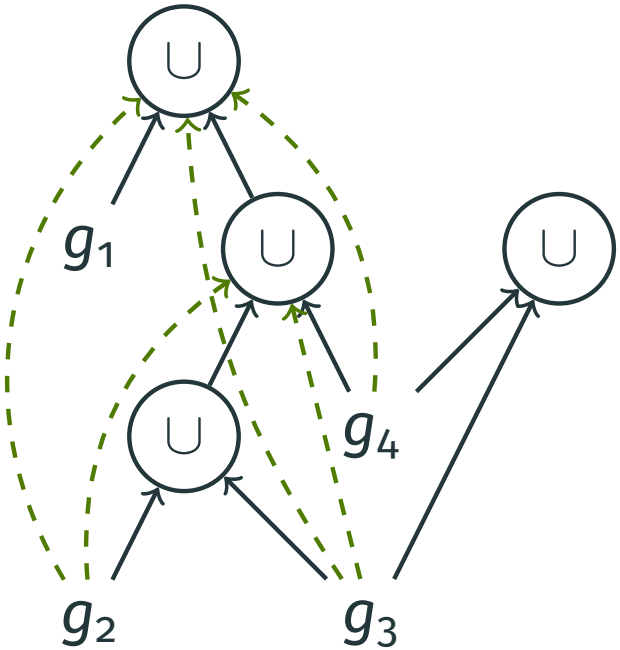

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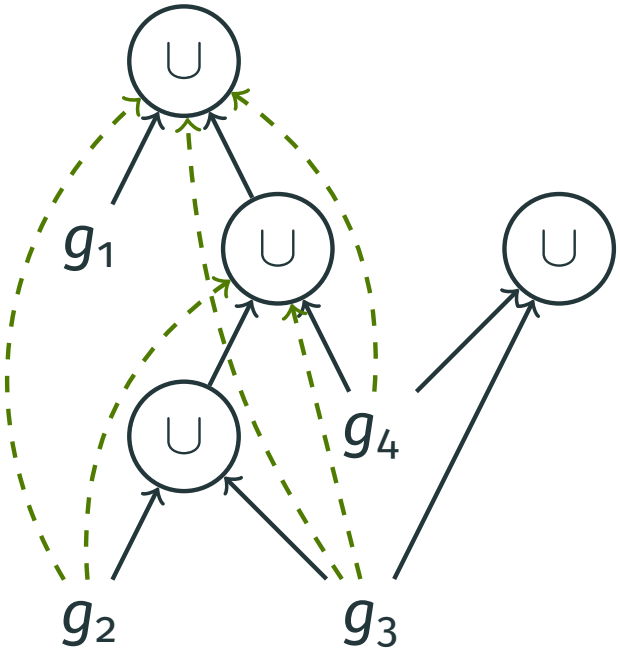

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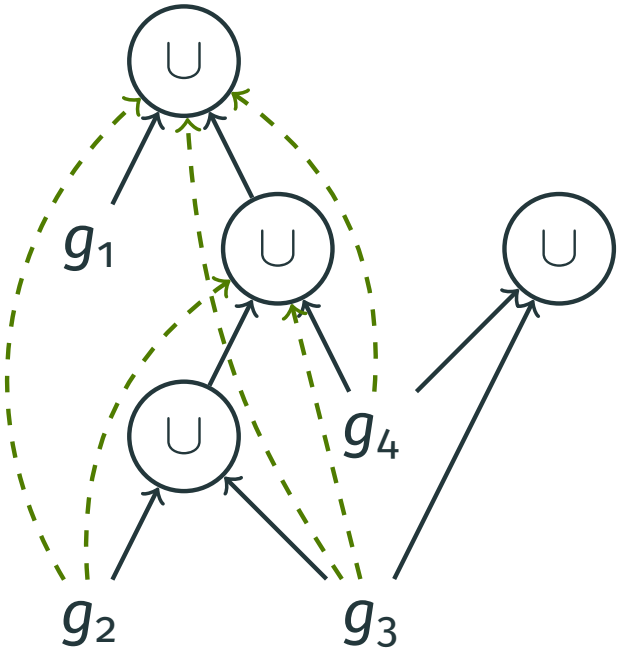

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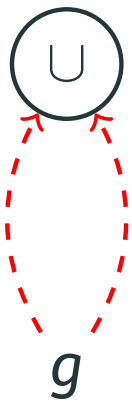

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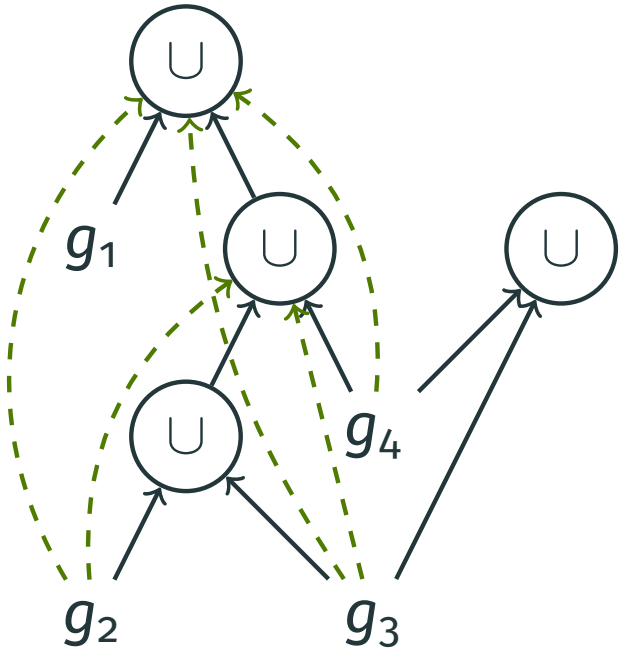

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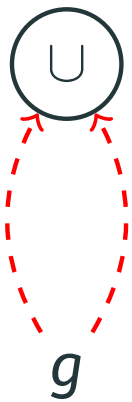

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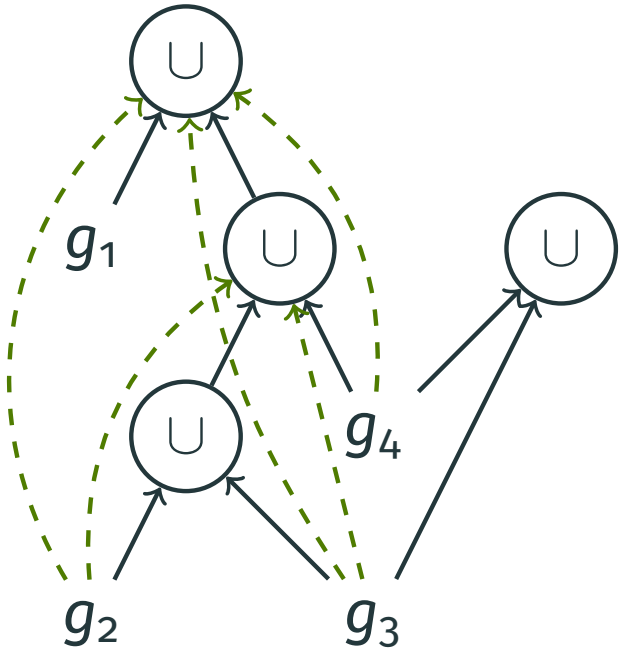

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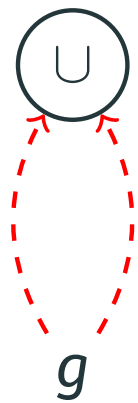
- For **MSO query evaluation**: **upwards-deterministic** circuit so we have a **tree**: simpler constant-memory index



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So we have re-proved:

Theorem [Bagan, 2006, Kazana and Segoufin, 2013]

For any fixed MSO query Q, given a tree T, we can preprocess T in linear time in T and then enumerate each result in linear time in the result

Application to text and combined complexity

Problem statement: Pattern matching in texts

Data: a text T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
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Goal:

- be very efficient in T (constant-delay)
- be reasonably efficient in P (polynomial-time)

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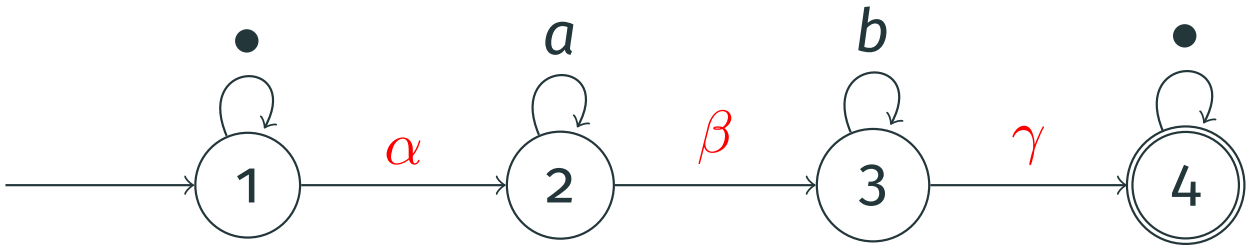

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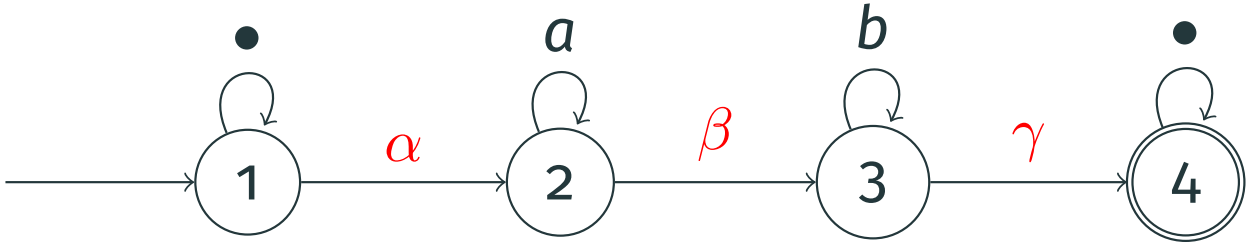

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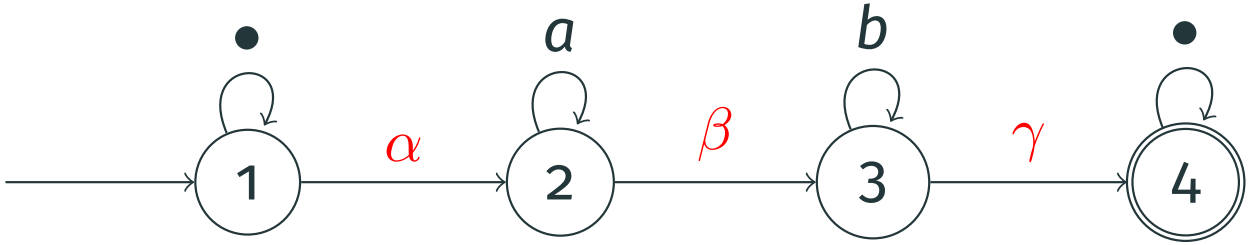

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→ The resulting set circuit is a **binary decision diagram**, i.e., each $\times$-gate has only one input which is not a variable

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Corollary

Given a regular expression pattern P and text T, we can enumerate all matches of P on T with the complexity above

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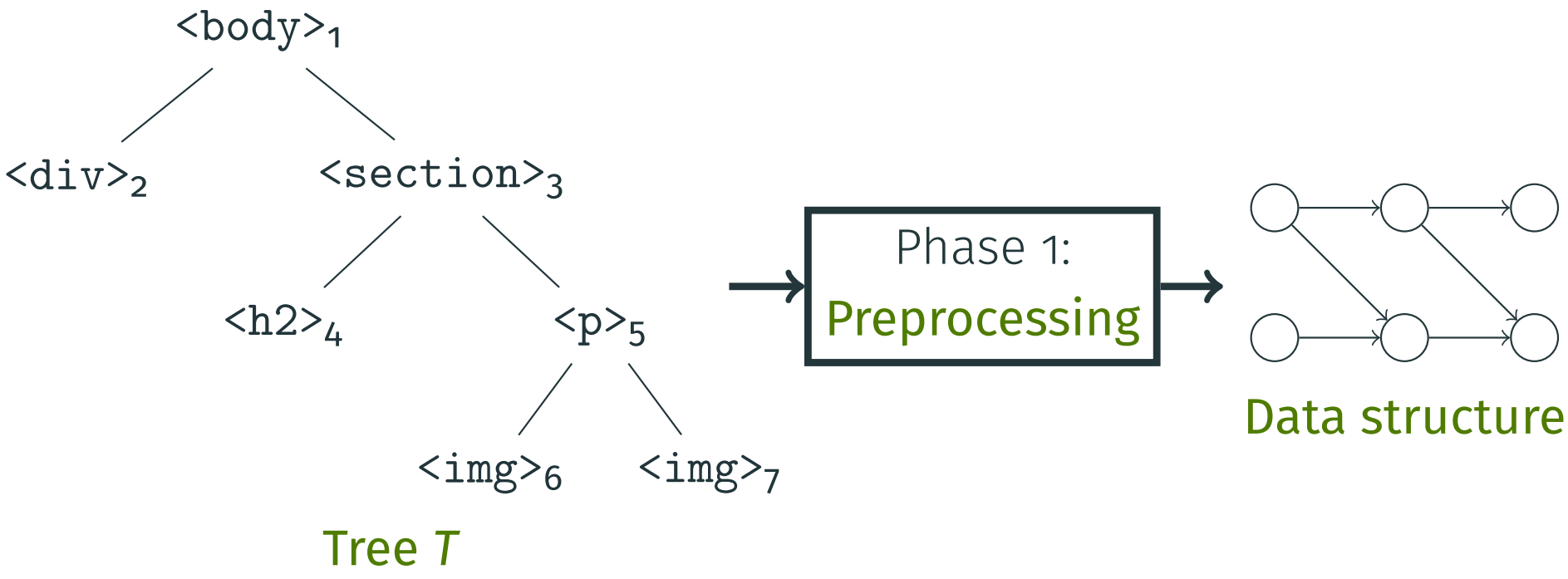
Handling updates

Updates



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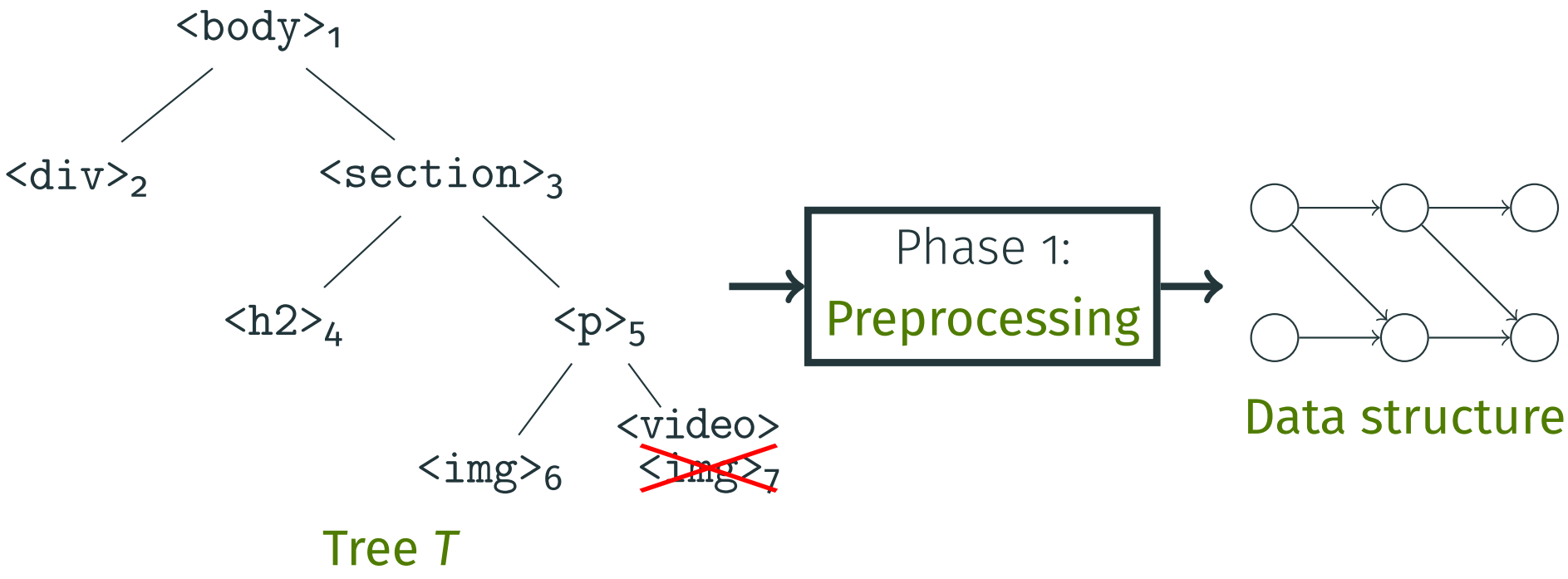

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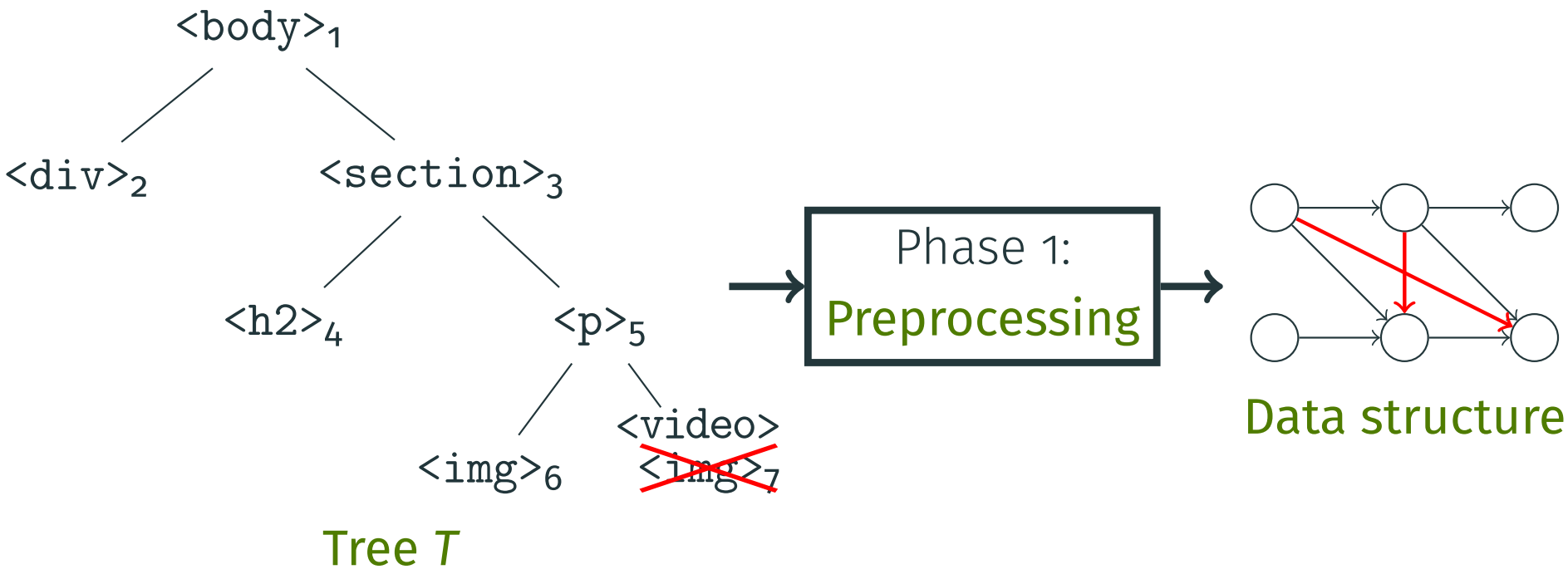

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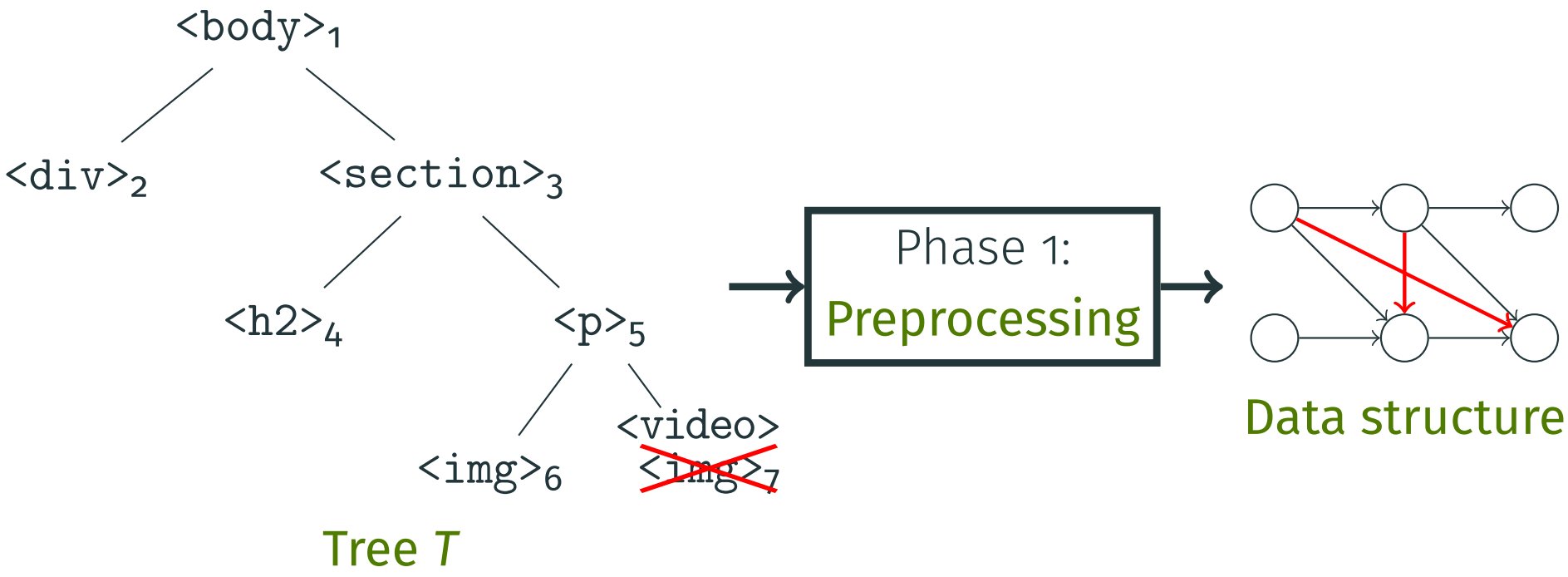

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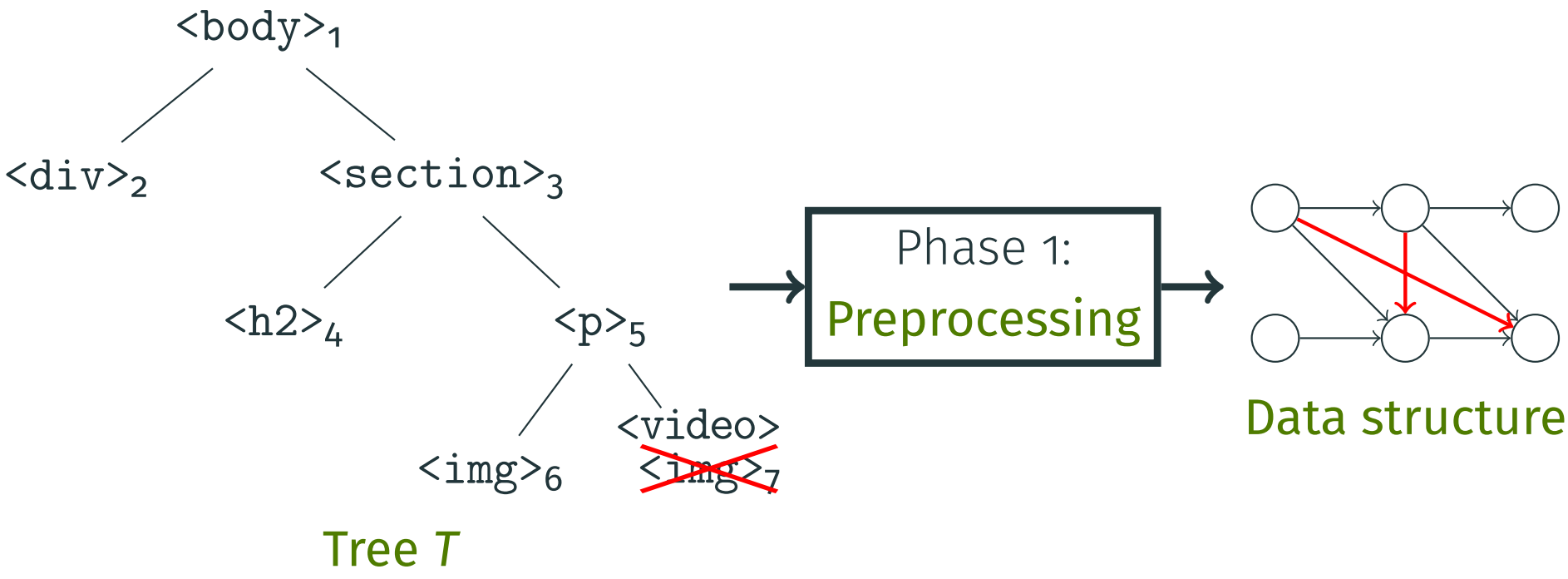

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→ Can we **do better**?

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[Niewerth and Segoufin, 2018]	text	$O(T)$	$O(1)$	$O(\log T)$
[Amarilli et al., 2019b]	trees	$O(T)$	$O(1)$	$O(\log T)$

Summary and open problems

Summary and conclusion

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Given a deterministic tree automaton A and a tree T, we can build in $O(|A| \times |T|)$ a d-DNNF set circuit capturing the results of A on T.

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Open problems:

- Implementation use cases?
- Lower bounds?
- Enumeration with order?
- Memory usage?
- Connection to tuple testing?
- Generic indexes?

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Hack: adding tree nodes to express the variable assignments

- Query: $Q(x_1, \ldots, x_n)$ with free variables $x_1, \ldots, x_n$

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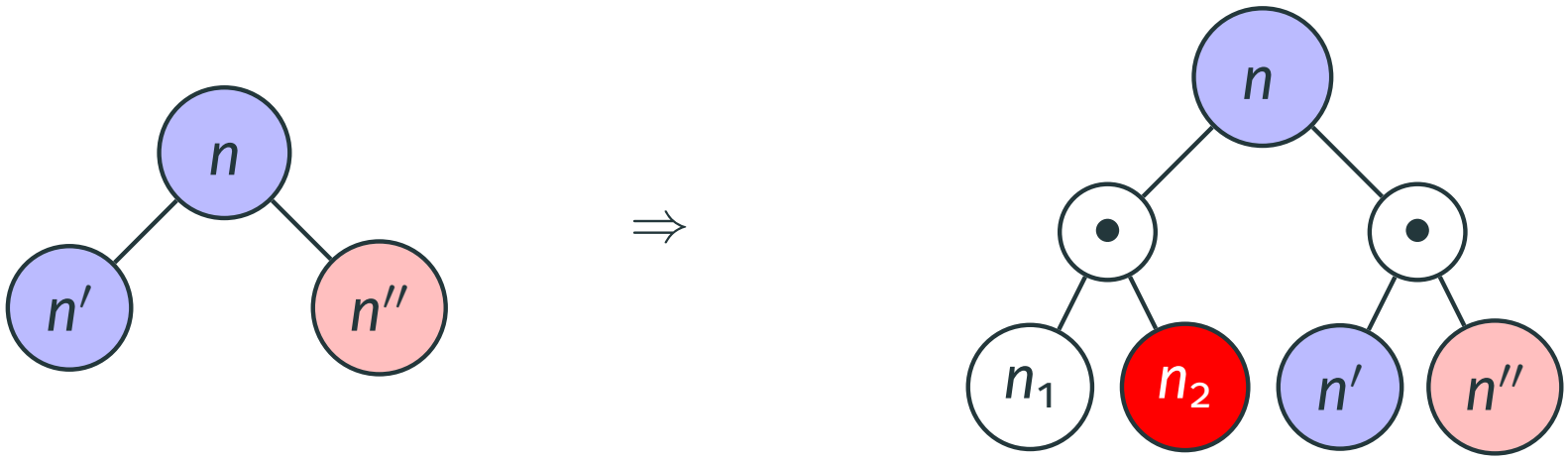
$\rightarrow$ Add special nodes: for each node n and variable x_i, add a node n_i which is colored red iff x_i is the node n



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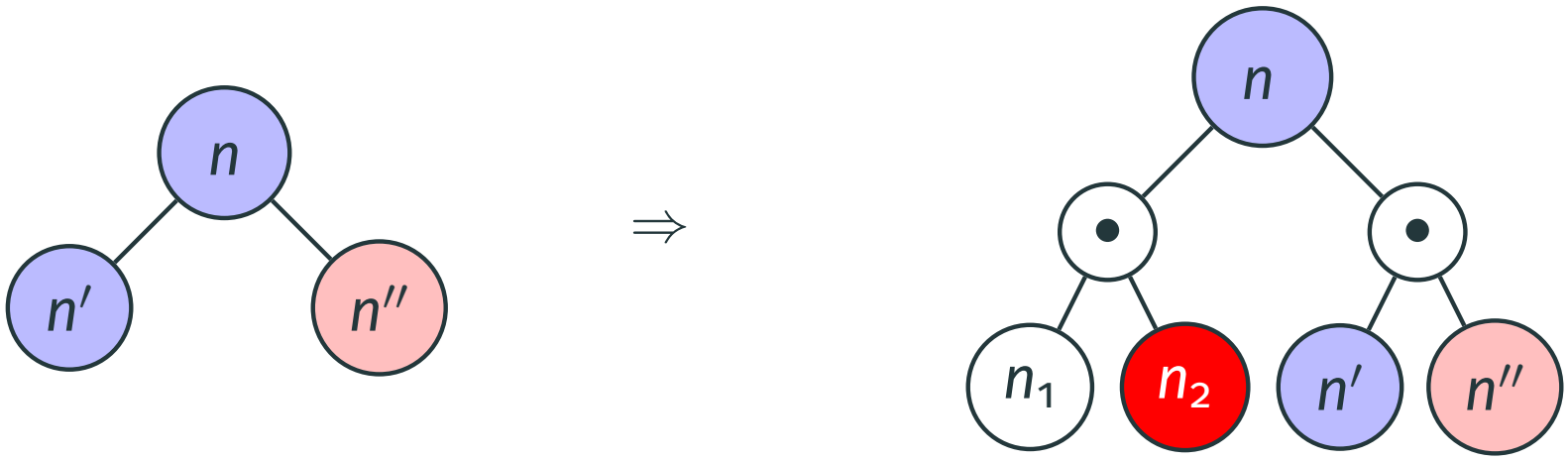

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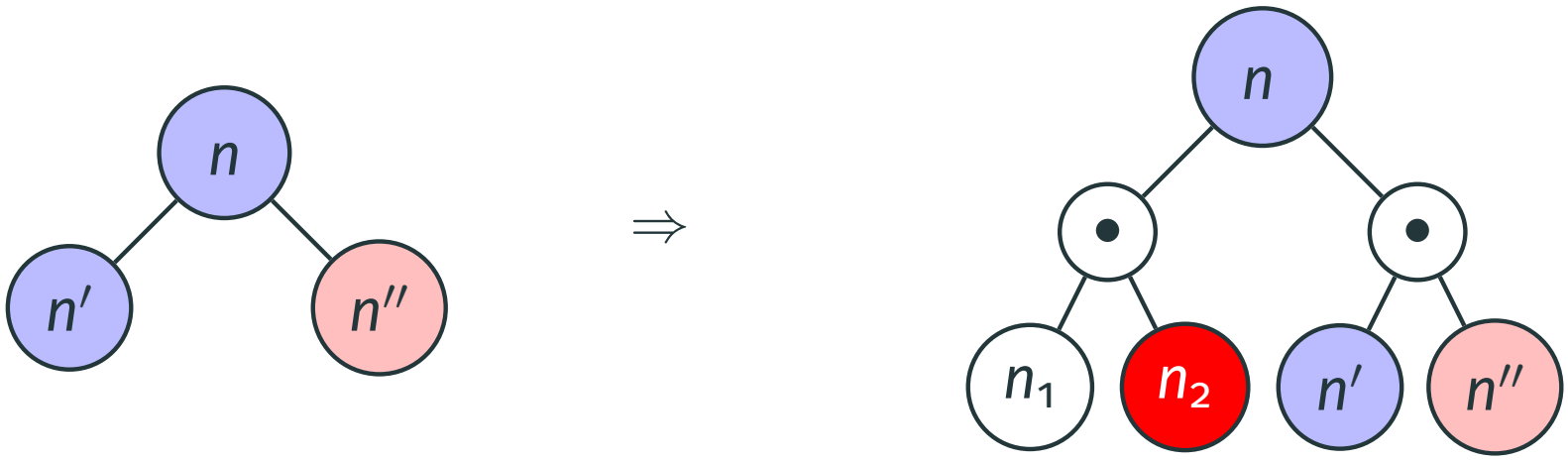

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→ Now, the results are all ways to color the special nodes red and make the Boolean query true

Lower Bound

Existential Marked Ancestor Queries

Given:	Tree t with some marked nodes
Query:	Does node v have a marked ancestor?
Updates:	Mark or unmark a node

Theorem

$$t_{query} \quad \in \quad \Omega\left(\frac{\log(n)}{\log(t_{update}\log(n))}\right)$$

Lower Bound

Reduction to Query Enumeration

Fixed Query Q: Return all special nodes with a marked ancestor

For every marked ancestor query v:

1. Mark node v special
2. Enumerate Q and return “yes”, iff Q produces some result
3. Mark v as non-special again

Theorem

$$\max(t_{delay}, t_{update}) \quad \in \quad \Omega\left(\frac{\log(n)}{\log\log(n)}\right)$$

Image credits

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