

## Using Order Constraints in Crowd Data Sourcing

Antoine Amarilli ${ }^{1,2}$, Yael Amsterdamer ${ }^{3,4}$, Tova Milo ${ }^{4}$, Pierre Senellart ${ }^{1,2}$
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${ }^{1}$ Télécom ParisTech
²École normale supérieure
${ }^{3}$ Bar Ilan University
${ }^{4}$ Tel Aviv University

## Introduction

## Example 1: Classifying products



Taxonomy of items for a store

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Taxonomy of items for a store with categories.

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Ask the crowd to classify items

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Best categories?

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## Example 3: Estimating unknown values

- How much food do people eat at conference buffets?


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- Let's ask fellow organizers!


## Example 3: Estimating unknown values

| small |  |  |
| :--- | :---: | :---: |
| sweet | tiny <br> both | sally |
|  |  |  |

medium
sweet small
both
medium
salty

- How much food do people eat at conference buffets?
- Let's ask fellow organizers!
large

sweet \begin{tabular}{c}
medium <br>
both

$\quad$

salty <br>
\end{tabular}

large both

## Example 3: Estimating unknown values

| small |  |
| :--- | :--- | :--- |
| sweet | tiny small |
|  | both |

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large large sweet medium salty both
medium salty

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| :--- | :--- | :--- |

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## General problem

- Several items with values


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- Some values are known and the others are unknown
$\rightarrow$ Estimate the unknown values
$\rightarrow$ Find the next crowd question to ask to obtain more known values


## Estimating Missing Values

## Estimating missing values (Antoine, Yael, Pierre, Tova, ICDT’17)



- Known and unknown values with order relation
- Estimate the unknown values without asking more crowd questions


## Easy case: total order



- If the items are totally ordered, what can we do?


## Easy case: total order



- If the items are totally ordered, what can we do? $\rightarrow$ Linear interpolation!


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- If the items are totally ordered, what can we do?
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\(\rightarrow\) Can we generalize this if the order is not total?

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- Introduce one variable per item:
\(\rightarrow x, y, z, w\)

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- For total orders, the polytope method gives the same result as linear interpolation

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- Fixing one value to its interpolated value can change other interpolated values! (unlike linear interpolation)

\section*{Asking Questions}

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- Unknown values are Boolean: either o or 1
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- Ask the right questions to determine the Boolean function completely

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- Matching upper bound: we can always query an item that splits the remaining Boolean functions evenly
- Output-sensitive complexity: we can determine the function in a number of crowd queries linear in the "frontier"
- Computational hardness, e.g., of finding the best-split element

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Worst number of questions needed to learn a monotonic predicate over a poset predicate over \(X\)（i．e．，for any \(x, y \in X\) ，if \(P(x)\) and \(x \leq y\) then \(P(y)\) ）．I can
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Worst number of questions needed to learn a monotonic predicate over a poset

Consider \((X, \leq)\) a finite poset over \(n\) items，and \(P\) an unknown monotonic predicate over \(X\)（i．e．，for any \(x, y \in X\) ，if \(P(x)\) and \(x \leq y\) then \(P(y)\) ）．I can
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asked Jan 28 ＇13 at 14：58

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\(\qquad\)
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Minimal elements of a monotonic predicate over the powerset \(2^{|n|}\)Consider a monotonic predicate \(P\) over the powerset \(2^{|n|}\)（ordered by inclusion）． By＂monotonic＂I mean：\(\forall x, y \in 2^{|n|}\) such that \(x \subset y\) ，if \(P(x)\) then \(P(y)\) ．I am

\section*{Conclusion}

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- Order relation between them


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(ask the same question multiple times to get a better estimate)

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\section*{References}
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\section*{Image credits}
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