

Using Order Constraints in Crowd Data Sourcing

Antoine Amarilli^{1,2}, Yael Amsterdamer^{3,4}, Tova Milo⁴, Pierre Senellart^{1,2} February 12th, 2018

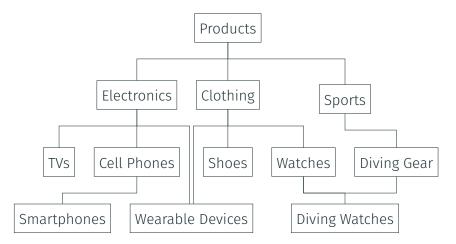
¹Télécom ParisTech

²École normale supérieure

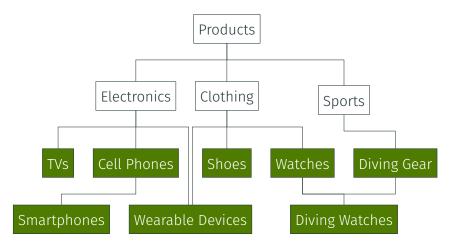
³Bar Ilan University

⁴Tel Aviv University

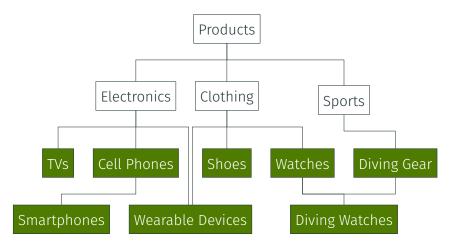
Introduction



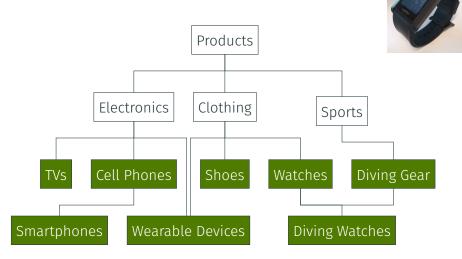
Taxonomy of items for a store



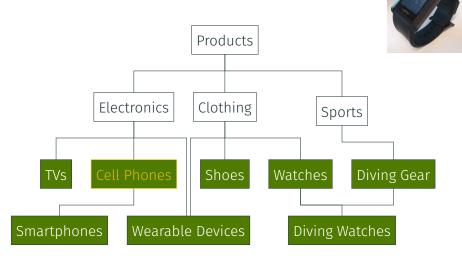
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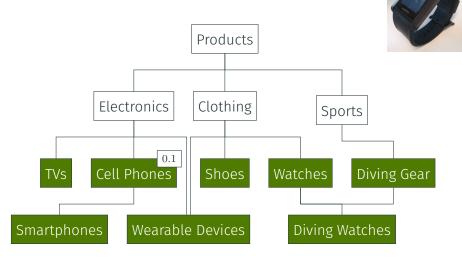
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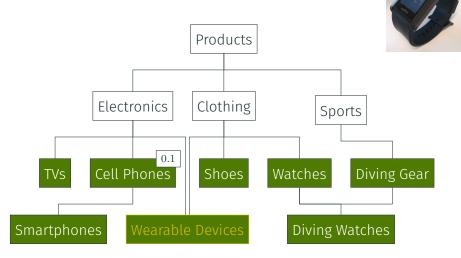
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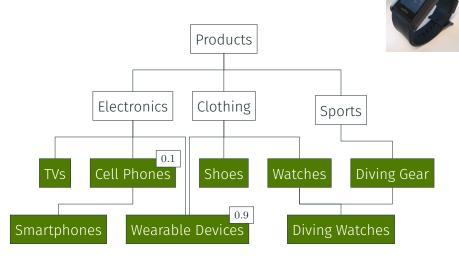
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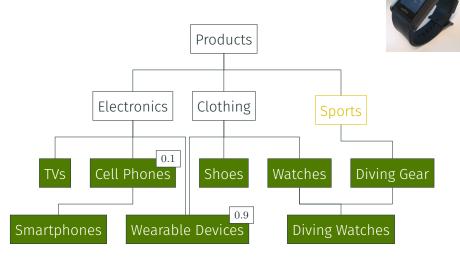
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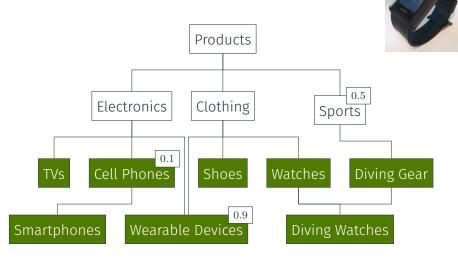
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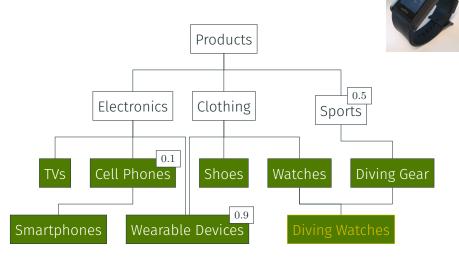
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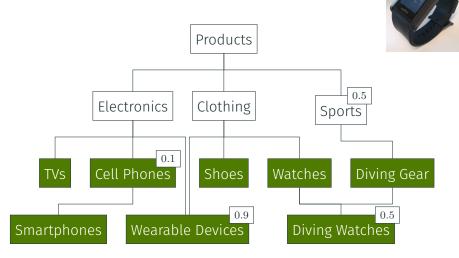
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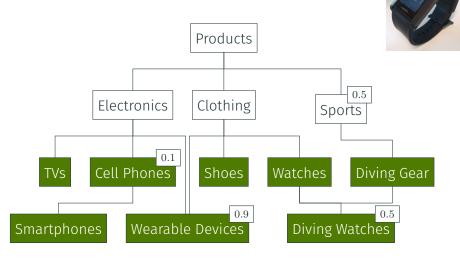
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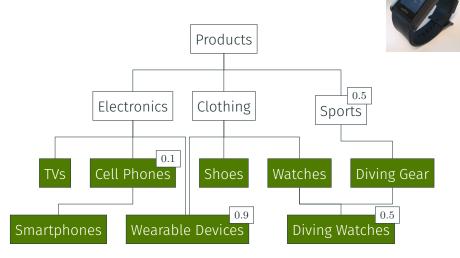
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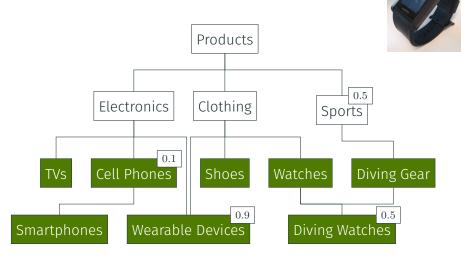


Monotonicity: compatibility increases as we go up.



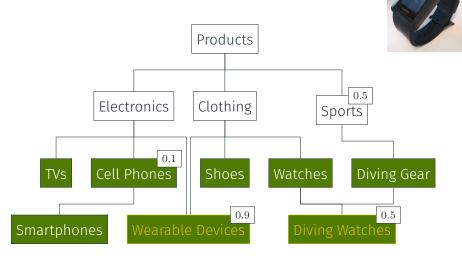
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Best categories?



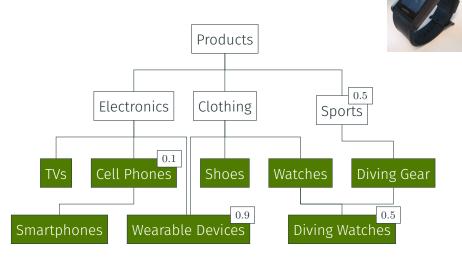
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Best categories? Naive answer...

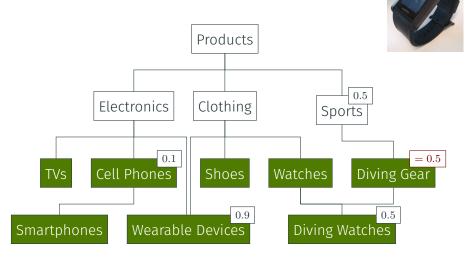


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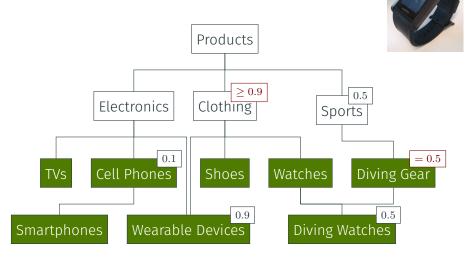
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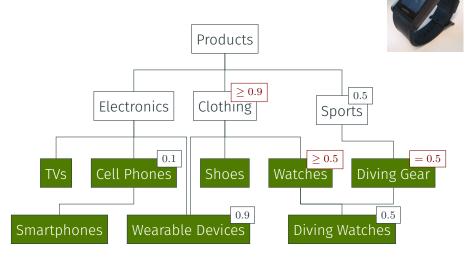
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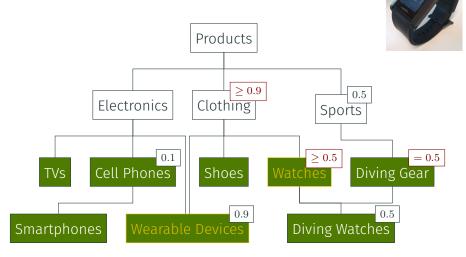
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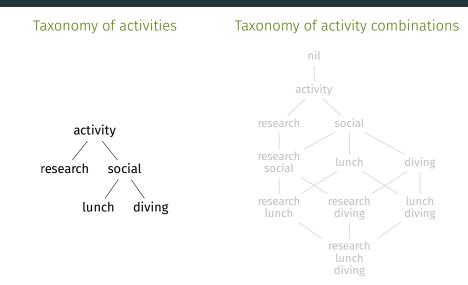
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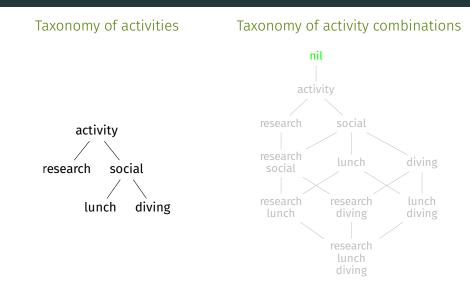


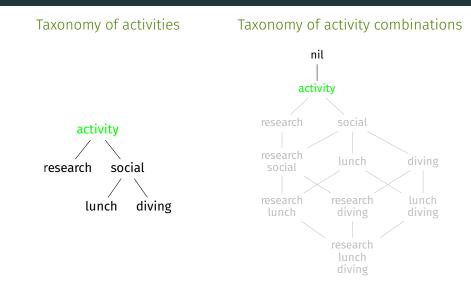
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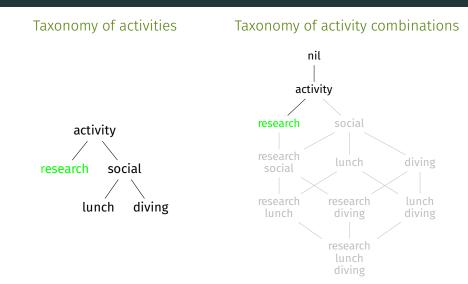


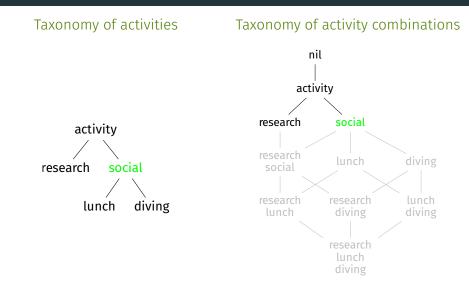
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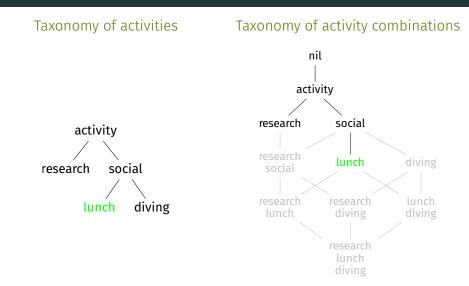


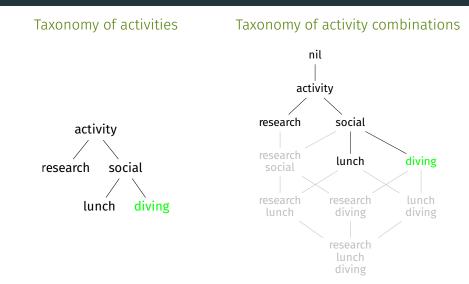


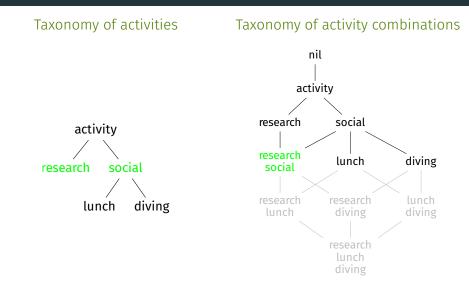


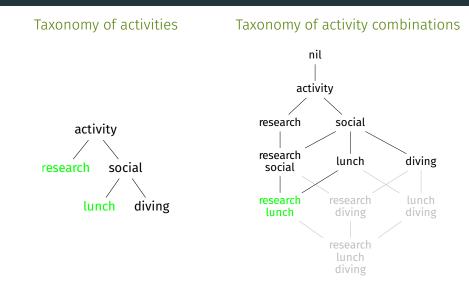


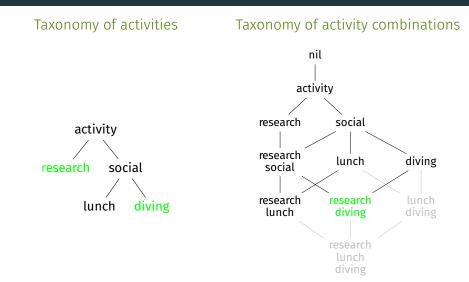


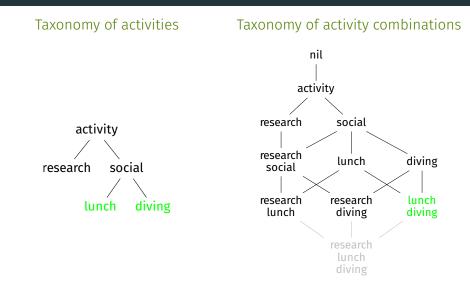


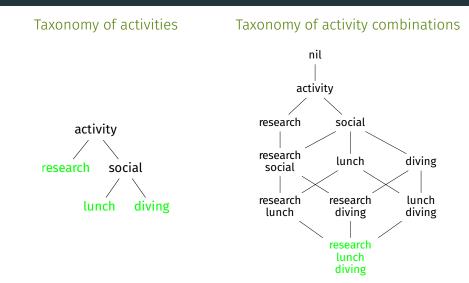


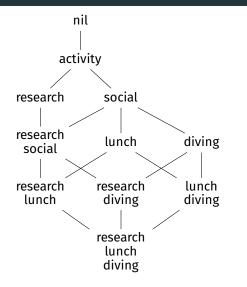


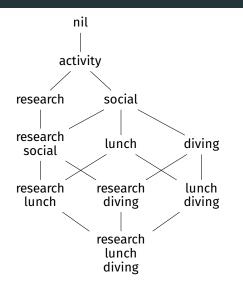




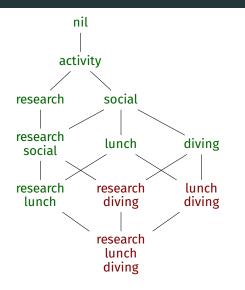




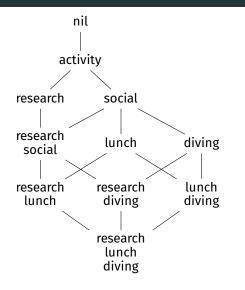




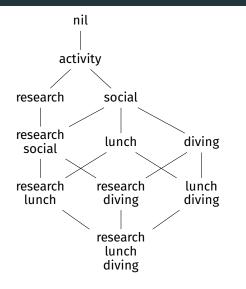
• More specific combinations are less frequent



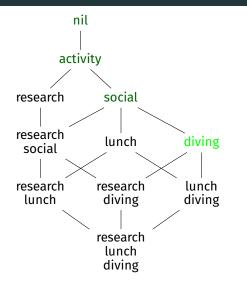
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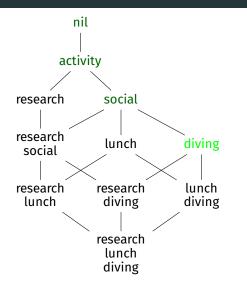
- More specific combinations are less frequent
- Ask crowd questions:



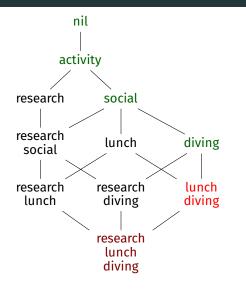
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- Is {diving} frequent?



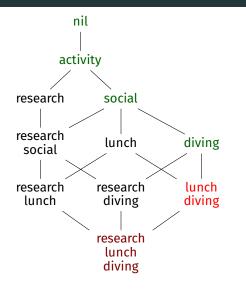
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 - \Rightarrow Yes!



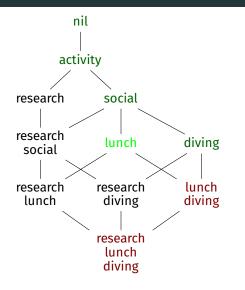
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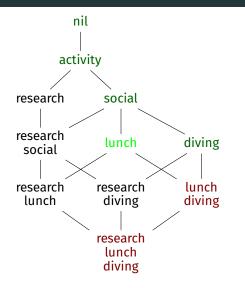


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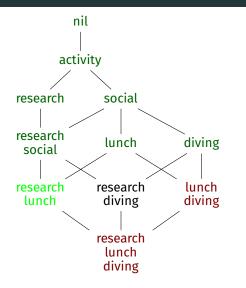


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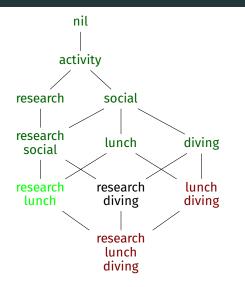
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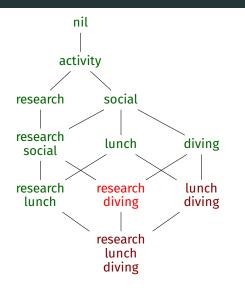
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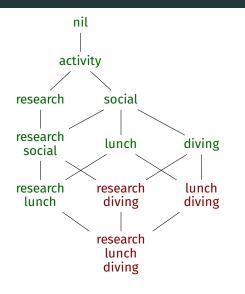
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• How much food do people eat at conference buffets?

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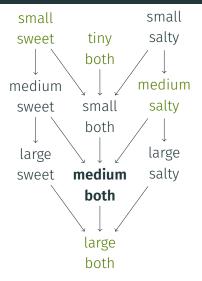
small	tiny	small	
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medium	small	medium	 How much food do people eat at conference buffets? Let's ask fellow organizers!
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large	medium	large	
sweet	both	salty	
	large both		

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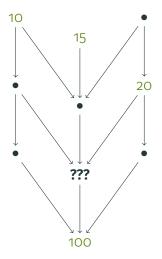
small sweet medium	tiny both	small salty medium	• How much food do pooplo opt at
sweet	small both	salty	 How much food do people eat at conference buffets? Let's ask fellow organizers!
large sweet	medium both	large saltyHow to estimate quantities for my own conference?	How to estimate quantities
	large both		

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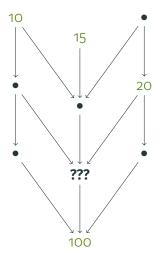
small sweet	tiny both	small salty	
medium sweet large sweet	small both medium	medium salty large salty	 How much food do people eat at conference buffets? Let's ask fellow organizers! How to estimate quantities for my own conference?
	both large both		 Some order relations are implied



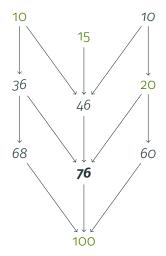
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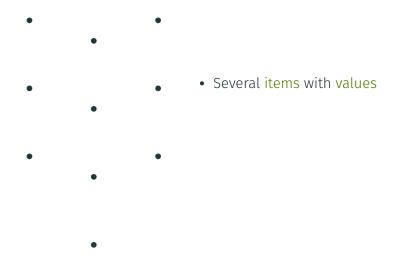


- How much food do people eat at conference buffets?
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- Some order relations are implied
- ightarrow How to complete the missing values?



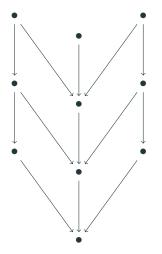
- How much food do people eat at conference buffets?
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- \rightarrow How to complete the missing values?

• Several items with values

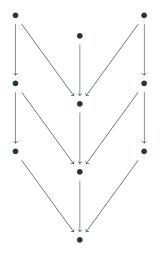


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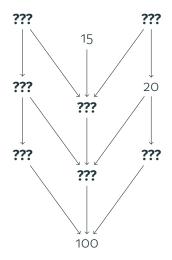
- Several items with values
- Order relations on the values



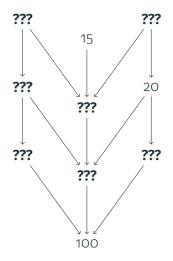
- Several items with values
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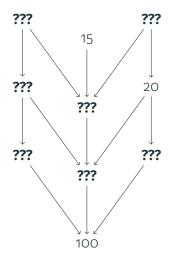
- Several items with values
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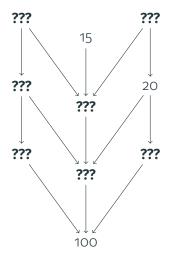
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- Several items with values
- Order relations on the values
- Some values are known and the others are unknown
- \rightarrow Estimate the unknown values
- $\rightarrow\,$ Find the next crowd question to ask to obtain more known values

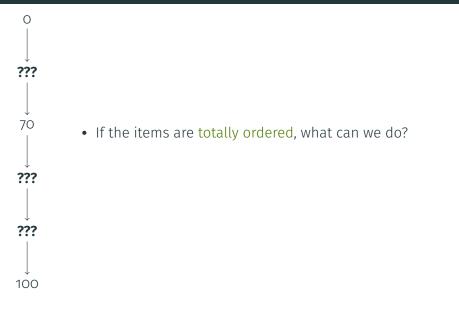
Estimating Missing Values

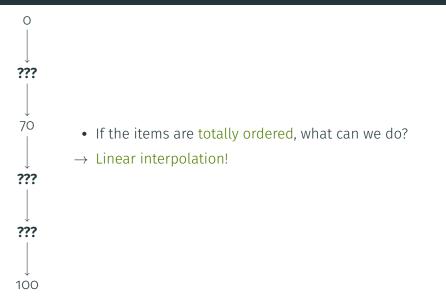
Estimating missing values (Antoine, Yael, Pierre, Tova, ICDT'17)

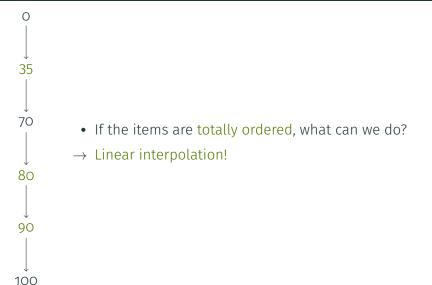


- Known and unknown values with order relation
- Estimate the unknown values without asking more crowd questions

Easy case: total order







 \cap

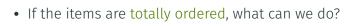
35

70

80

90

100



- \rightarrow Linear interpolation!
- \rightarrow Can we generalize this if the order is not total?

• Introduce one variable per item:

 \rightarrow X, Y, Z, W

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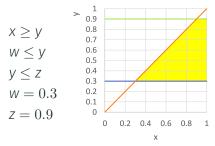
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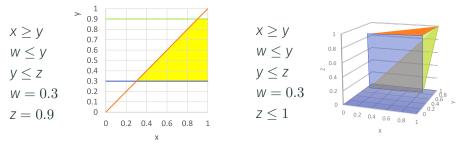
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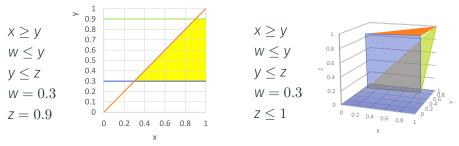
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 \rightarrow Estimate unknown values as the center of mass of the polytope!

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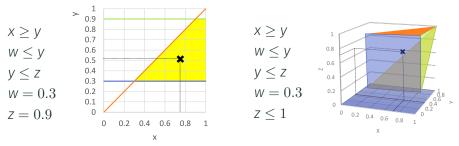
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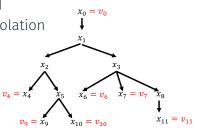
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• For total orders, the polytope method gives the same result as linear interpolation

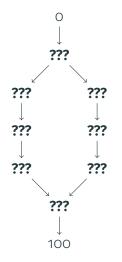
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- Intractability results for this task and for computing the top-*k* items
- Tractable cases when the order is a tree

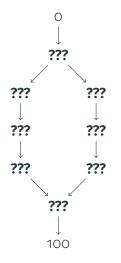


Open problems



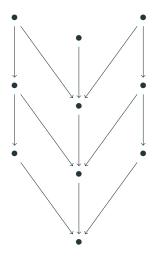
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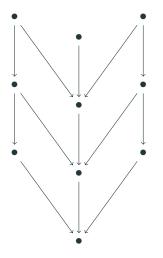


- Are there more general posets/polytopes where we can interpolate efficiently? (generalizing trees, e.g., treelike posets)
- Fixing one value to its interpolated value can change other interpolated values! (unlike linear interpolation)

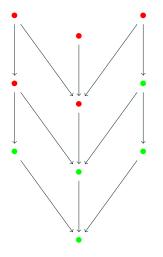
Asking Questions



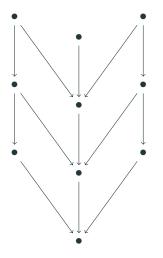
• Unknown values are Boolean: either O or 1



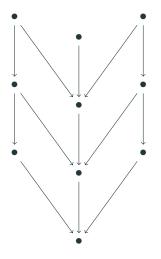
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- Unknown values are Boolean: either O or 1
- The Boolean function is monotone with respect to the order
- Ask the right questions to determine the Boolean function completely

• If the items are totally ordered, what can we do?

•

•



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- \rightarrow Binary search!
- $\rightarrow\,$ Can we generalize this if the order is not total?

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- Computational hardness, e.g., of finding the best-split element

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Worst number of questions needed to learn a monotonic predicate over a poset

- Consider (X, \leq) a finite poset over *n* items, and *P* an unknown monotonic predicate over *X* (i.e., for any *x*, *y* \in *X*, if *P*(*x*) and *x* \leq *y* then *P*(*y*)). I can
- 15 evaluate *P* by providing one node $x \in X$ and finding out if P(x) holds or not. My doal is to determine exactly the set of nodes $x \in X$ such that P(x) holds using as

asked Jan 28 '13 at 14:58 a3nm 2,906 • 10 • 46

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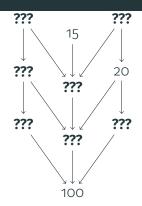
Minimal elements of a monotonic predicate over the powerset $2^{|n|}$

- Consider a *monotonic* predicate *P* over the powerset $2^{|n|}$ (ordered by inclusion). By "monotonic" I mean: $\forall x, y \in 2^{|n|}$ such that $x \subset y$, if P(x) then P(y). I am
- 12 looking for an algorithm to find all the minimal elements of *P*, i.e., the $x \in 2^{|n|}$ such that P(x) but $\forall y \in x = P(y)$. Since the width of $2^{|n|}$ is $\binom{n}{2}$, there could be



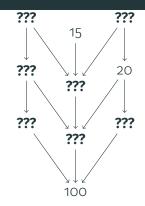
Conclusion

- General problem:
 - Known and unknown values
 - Order relation between them



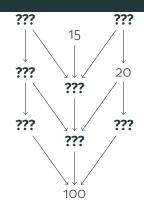
• General problem:

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- $\rightarrow~$ Estimate the missing values



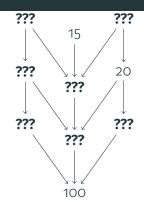
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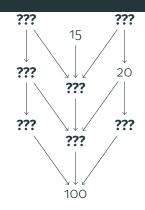
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 $\rightarrow\,$ Can we combine the two approaches?

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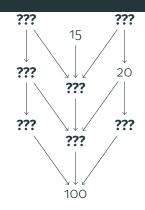
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Thanks for your attention!



Amarilli, A., Amsterdamer, Y., and Milo, T. (2014). On the Complexity of Mining Itemsets from the Crowd Using Taxonomies.

In ICDT.

Amarilli, A., Amsterdamer, Y., Milo, T., and Senellart, P. (2017). **Top-k Queries on Unknown Values under Order Constraints.** In *ICDT*.

- Title slide, Bar-Ilan logo, https: //en.wikipedia.org/wiki/File:Bar_Ilan_logo2.svg
- Slide 9: picture by Yael Amsterdamer, https://a3nm.net/ work/talks/icdt2017/amarilli2017top_slides.pdf, slide 7
- Slide 10: picture by Yael Amsterdamer, https://a3nm.net/ work/talks/icdt2017/amarilli2017top_slides.pdf, slide 14