On the Complexity of Mining Itemsets from the Crowd Using Taxonomies

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Data mining – discovering interesting patterns in large databases

Database – a (multi)set of transactions

Transaction – a set of items (aka. an itemset)

A simple kind of pattern to identify are frequent itemsets

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Human knowledge mining

- Some databases only exist in the minds of people
- Example: popular activities in Athens:
  - t_1: I went to the acropolis and to the museum.
    $\Rightarrow$ $\{\texttt{acropolis}, \texttt{museum}\}$
  - t_2: I visited Piraeus and had some ice cream.
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  - t_3: On Monday I attended the keynote and had coffee.
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  - t_3: On Monday I attended the keynote and had coffee.
    $\Rightarrow$ $\{\texttt{keynote}, \texttt{coffee}\}$
- We want frequent itemsets: frequent activity combinations

$\Rightarrow$ How to retrieve this data from people?

Harvesting the data

- We cannot collect such data in a centralized database:
  1. It's impractical to ask all users to surrender their data

     "Everyone please tell us all you did the last three months."
  2. People do not remember the information

     "What were you doing on August 23th, 2013?"

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     "What were you doing on August 23th, 2013?"
- People remember summaries that we could access

  "Do you often eat ice cream when attending a keynote?"

⇒ We can just ask people if an itemset is frequent

Crowdsourcing

- Crowdsourcing – solving hard problems through elementary queries to a crowd of users
- Find out if an itemset is frequent with the crowd:
  1. Draw a sample of users from the crowd. *(black box)*
  2. Ask: is this itemset frequent? *("Do you often have coffee?")*
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$\Rightarrow$ The crowd is an oracle: given an itemset, say if it is frequent

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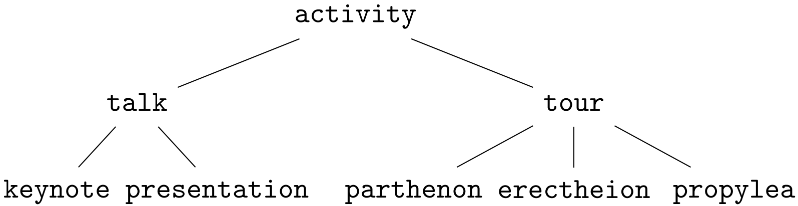
Having a taxonomy over the items can save us work!



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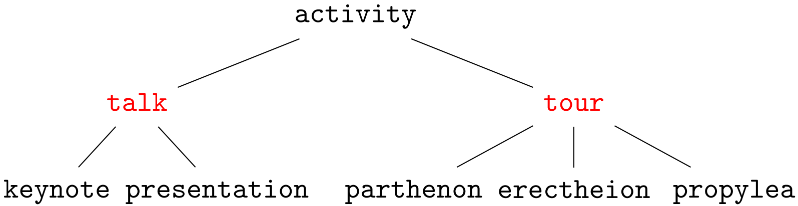

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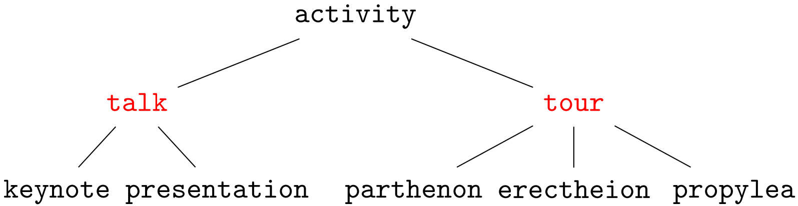

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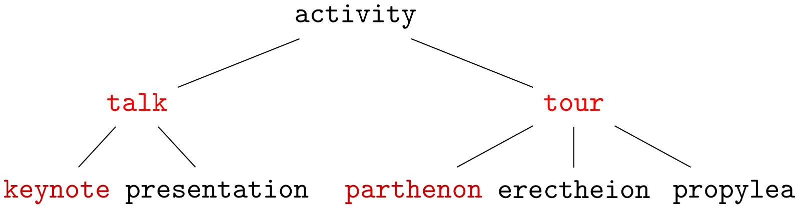

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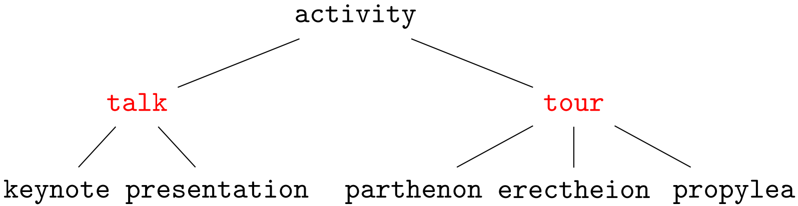

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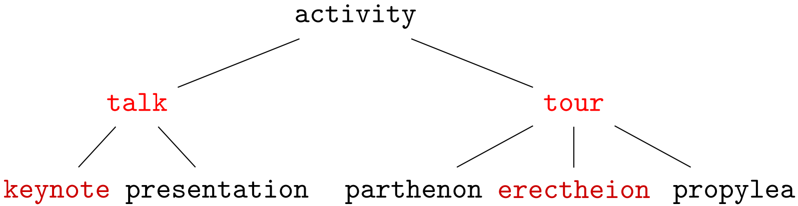

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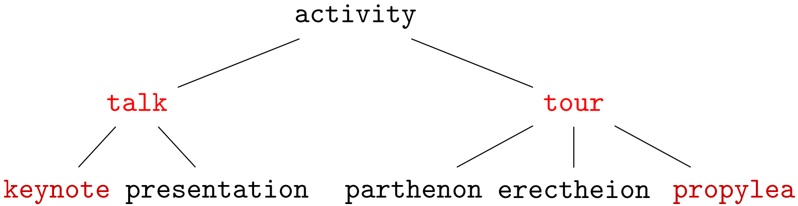

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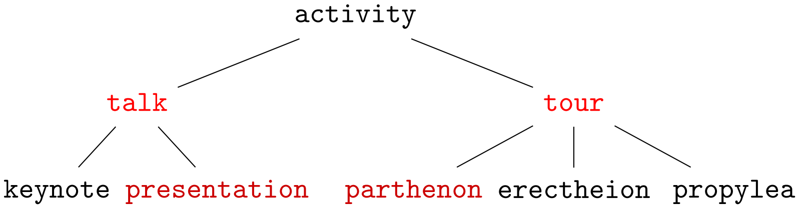

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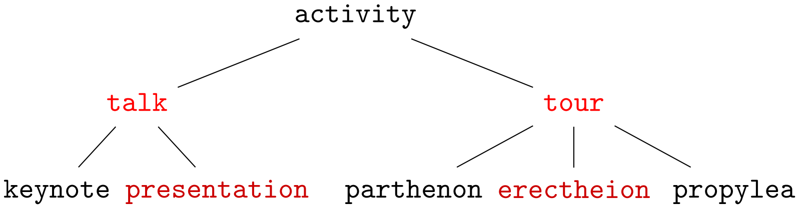

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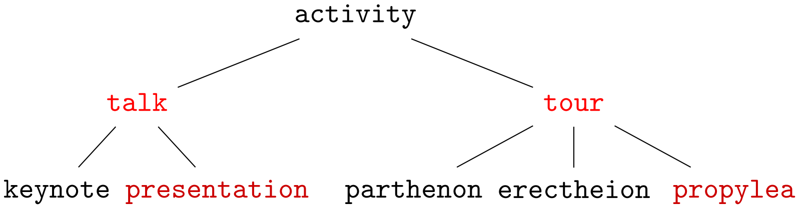

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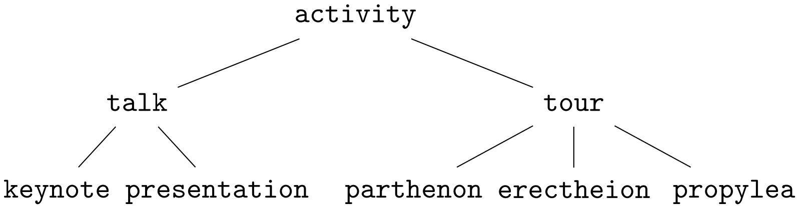

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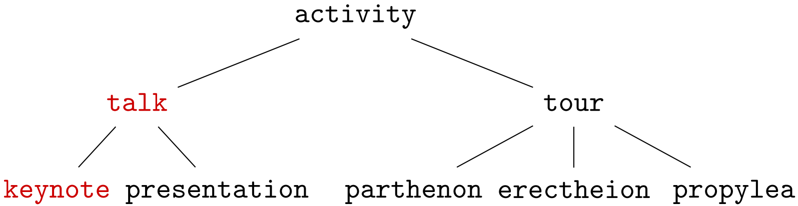

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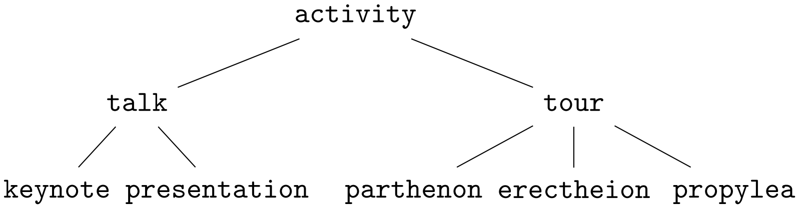

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The problem

We can now describe the problem:

- We have:
  - A known item domain $\mathcal{I}$ (set of items)
  - A known taxonomy Ψ on $\mathcal{I}$ (is-a relation, partial order)
  - A crowd oracle to decide if an itemset is frequent or not
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What is a good algorithm to solve this problem?

Cost

- How to evaluate the performance of a strategy to identify the frequent itemsets?

  Crowd complexity: The number of itemsets we ask about (monetary cost, latency...)

  Computational complexity: The complexity of computing the next question to ask

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  Crowd complexity: The number of itemsets we ask about (monetary cost, latency...)

  Computational complexity: The complexity of computing the next question to ask

- Tradeoff between the two:
  - ⇒ Asking random questions: computationally inexpensive but bad crowd complexity
  - ⇒ Asking clever questions: optimal crowd complexity but computationally expensive

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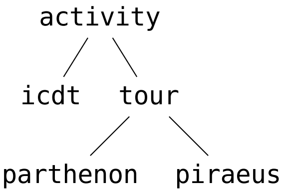
Itemsets



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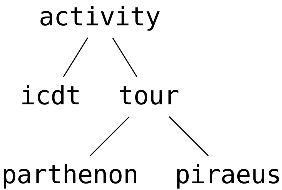

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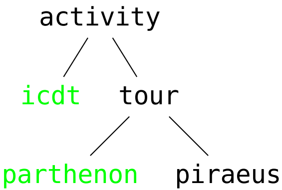

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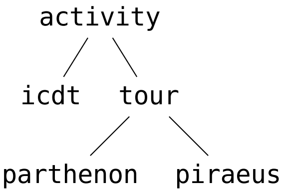

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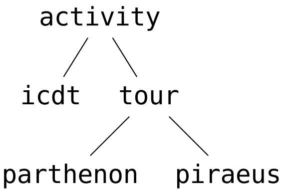

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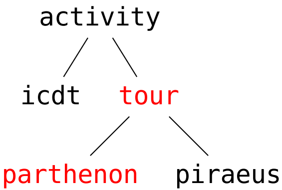

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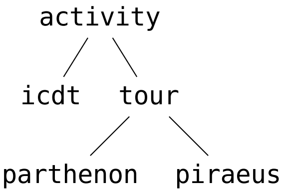

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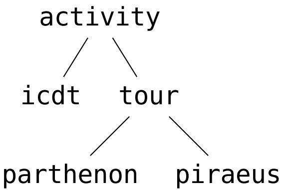

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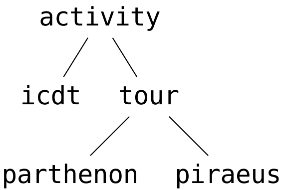

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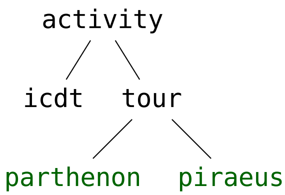

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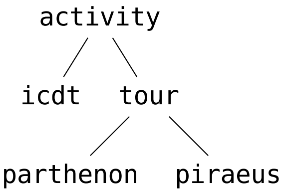

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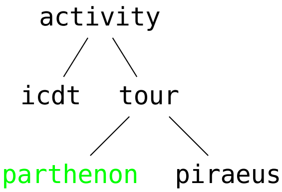

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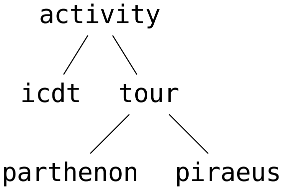

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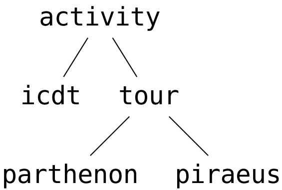

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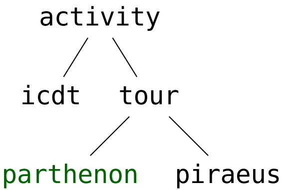

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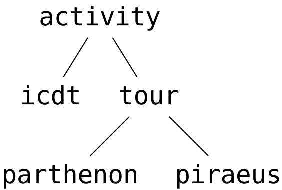

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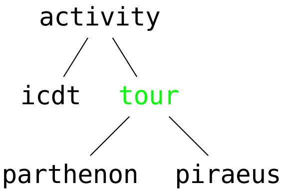

- Itemsets $I(\Psi)$ – the sets of pairwise incomparable items
  - $\Rightarrow$ $\{\texttt{icdt}, \texttt{parthenon}\}$ is an itemset
  - $\Rightarrow$ $\{\texttt{tour}, \texttt{parthenon}\}$ is not
- Order over itemsets:
  - $\{\texttt{parthenon}, \texttt{piraeus}\}$ frequent
    - $\Rightarrow$ $\{\texttt{parthenon}\}$ also frequent
  - $\{\texttt{parthenon}\}$ frequent
    - $\Rightarrow$ $\{\texttt{tour}\}$ also frequent

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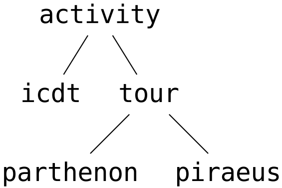

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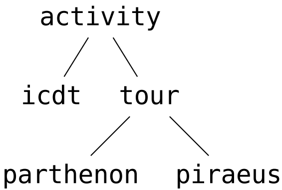
Itemset taxonomy example

Taxonomy Ψ



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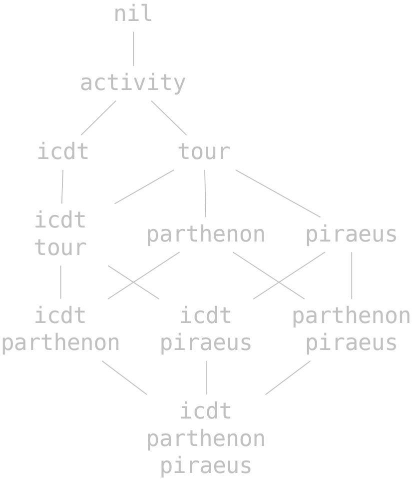

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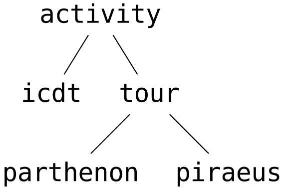

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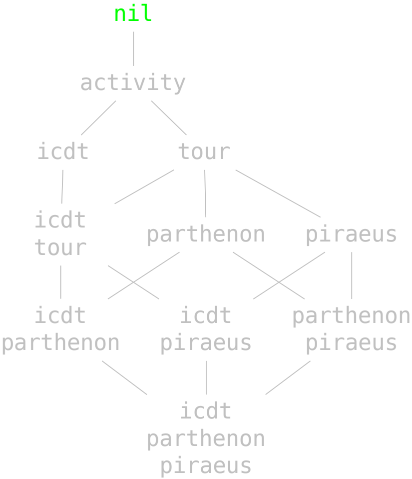

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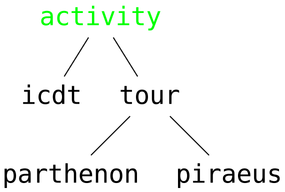

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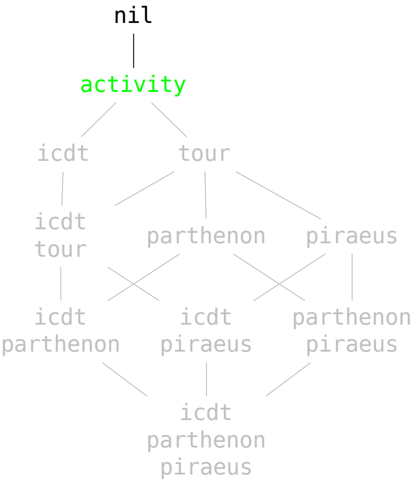

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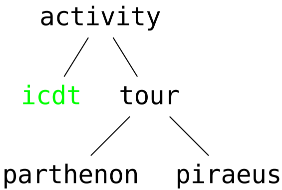

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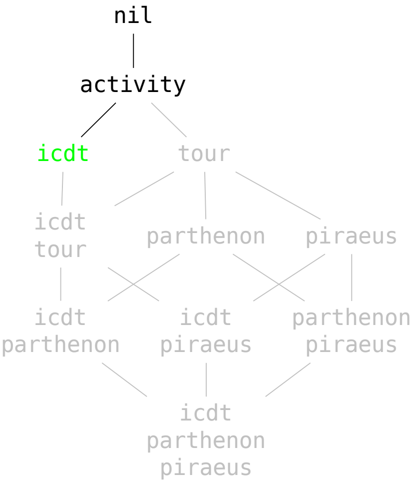

Itemset taxonomy example

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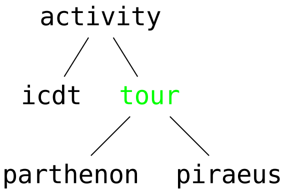

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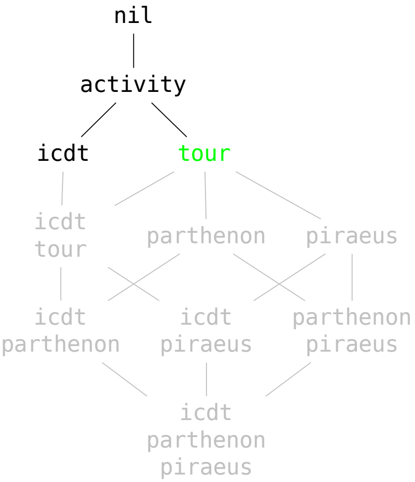

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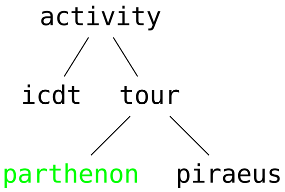
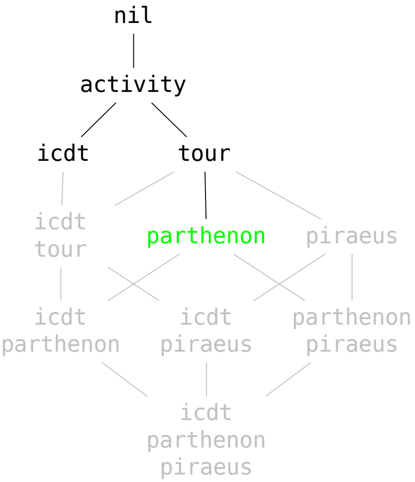


Itemset taxonomy I(Ψ)



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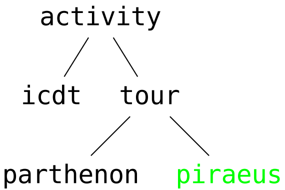

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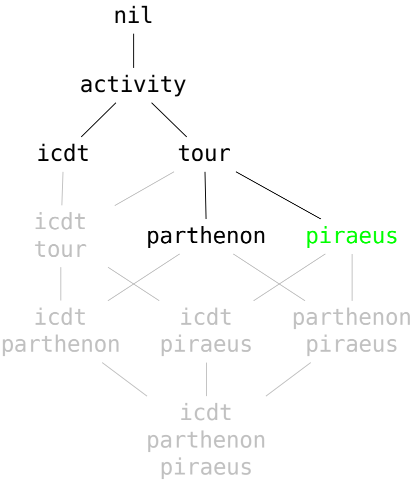

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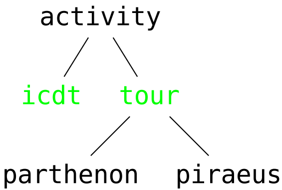

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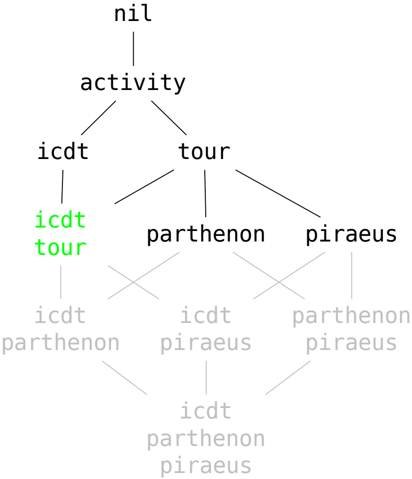

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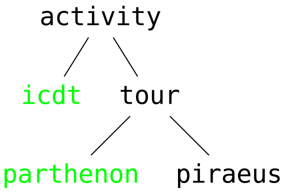

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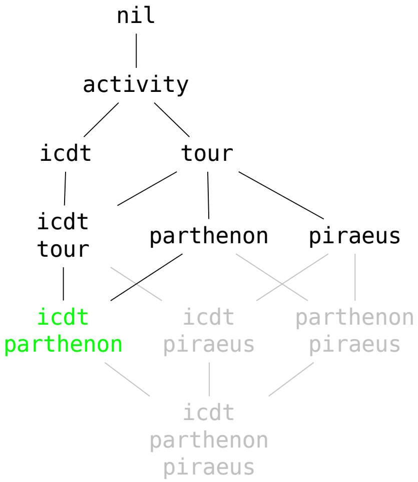

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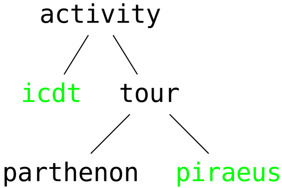

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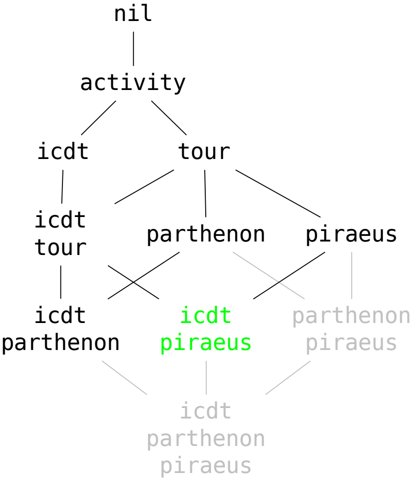

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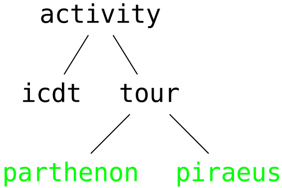

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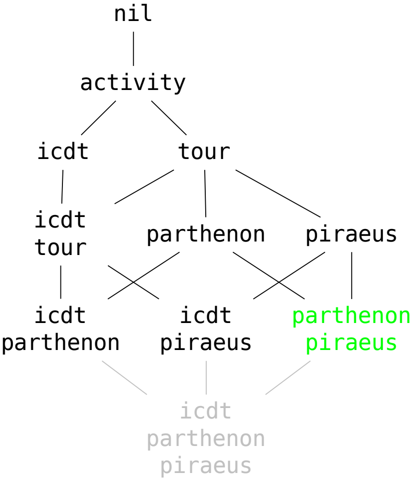

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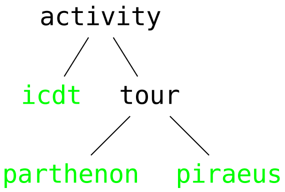

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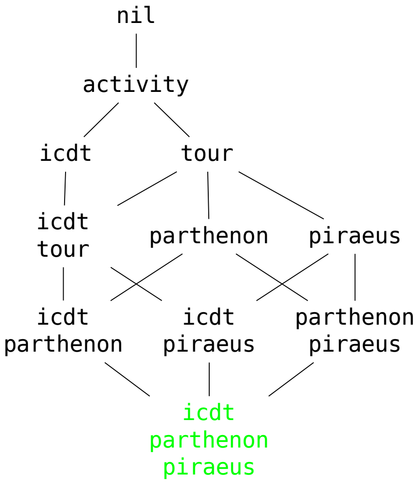
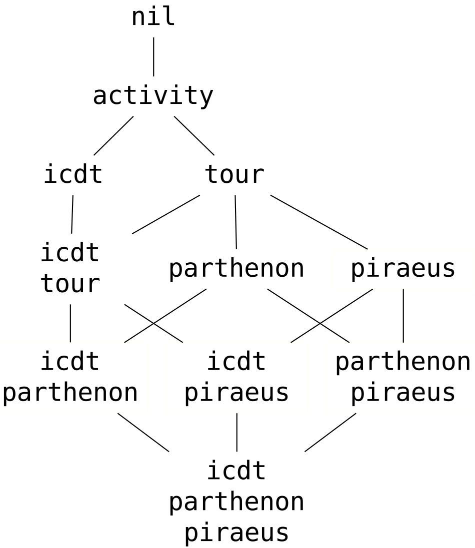
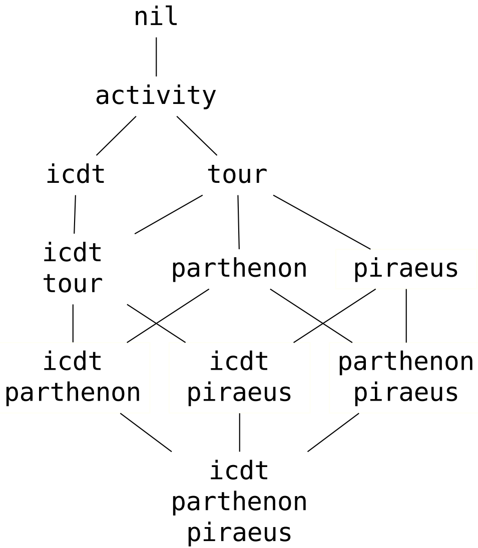


Solutions

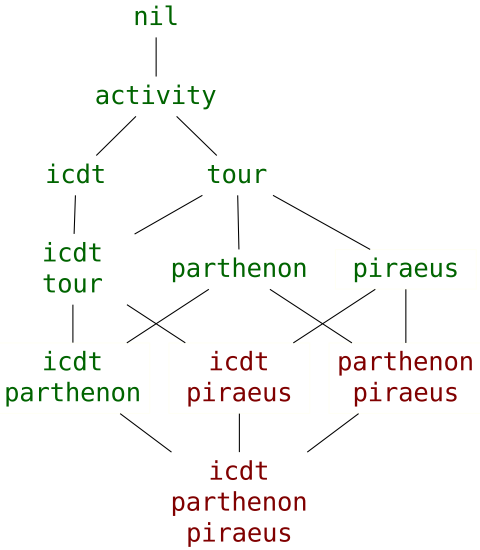

nil
activity
icdt
tour
icdt tour
parthenon
piraeus
icdt parthenon
icdt piraeus
parthenon piraeus
icdt parthenon piraeus

Solutions



- "Being frequent" is a monotone predicate over $\mathbf{I}(\Psi)$

Solutions



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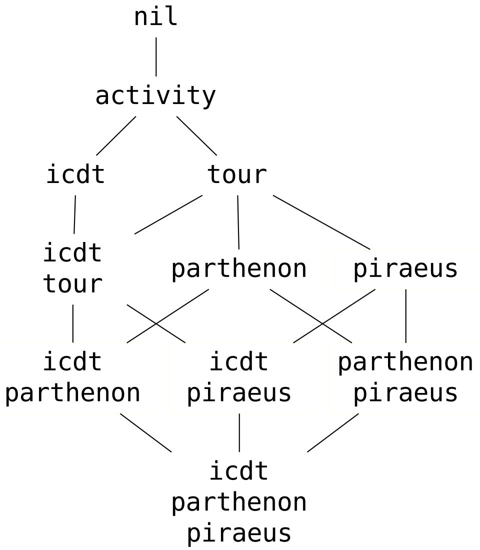
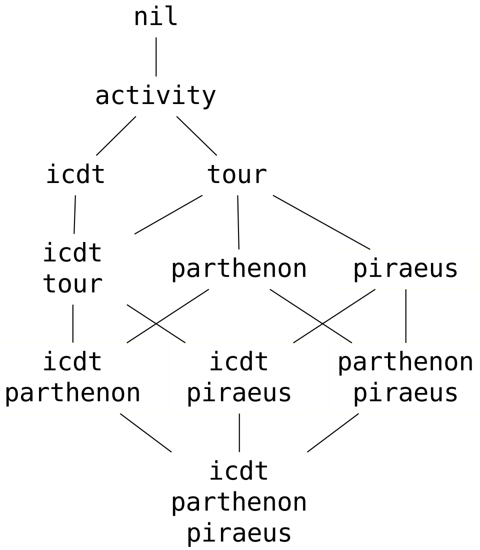


- "Being frequent" is a monotone predicate over $I(\Psi)$
- Ask questions:

Solutions



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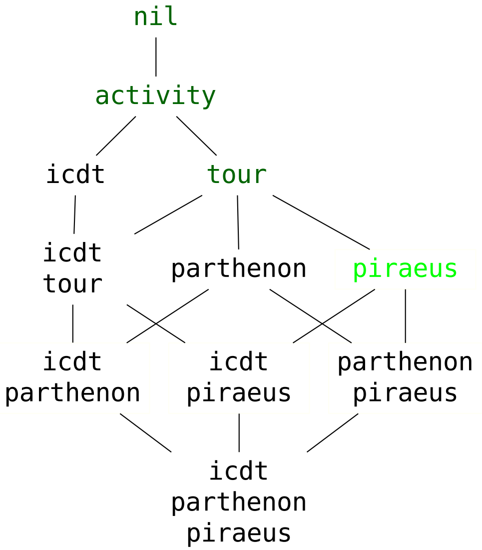
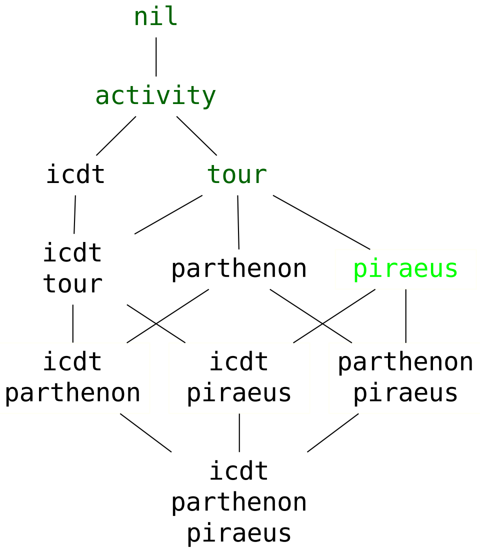
- Ask questions:
- Is $\{\texttt{piraeus}\}$ frequent?

Solutions



- "Being frequent" is a monotone predicate over $\mathbf{I}(\Psi)$
- Ask questions:
- Is {piraeus} frequent?
  $\Rightarrow$ Yes!

Solutions



- "Being frequent" is a monotone predicate over $\mathsf{I}(\Psi)$
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- Is $\{\texttt{piraeus}\}$ frequent?
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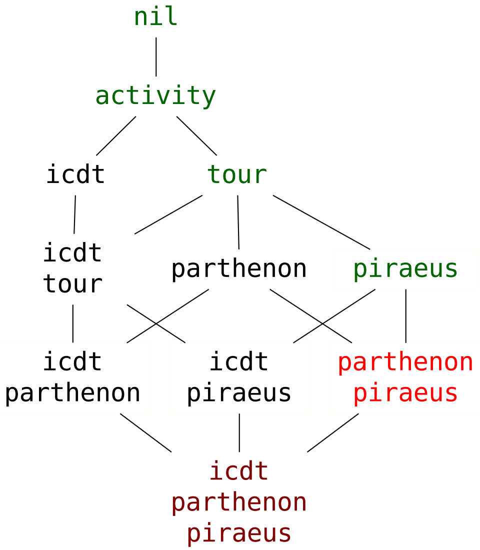
Solutions



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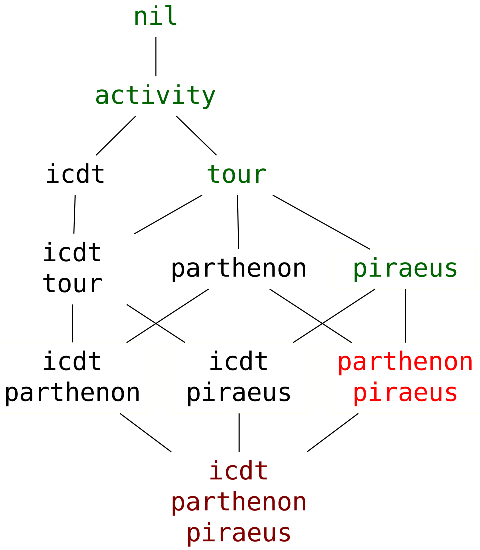
Solutions



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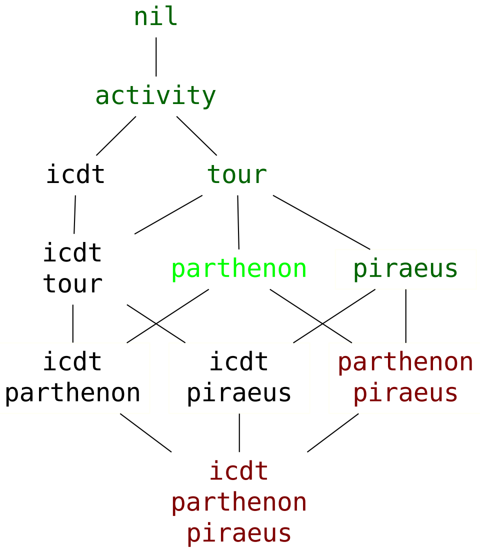

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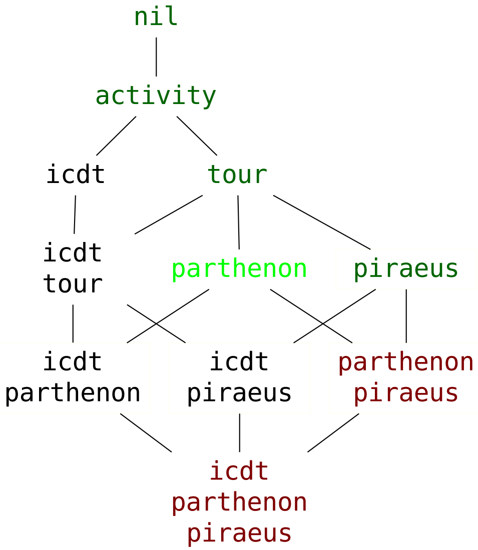
Solutions



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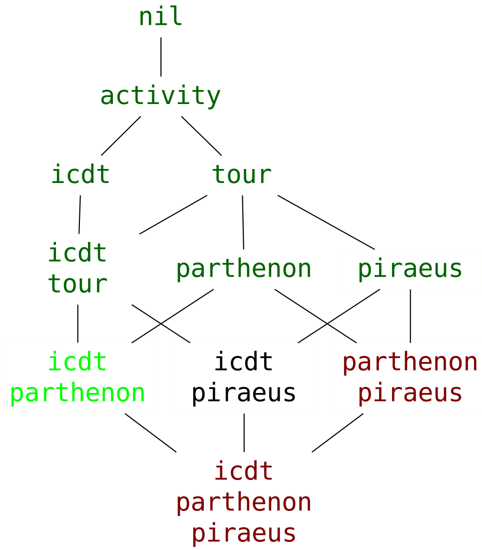

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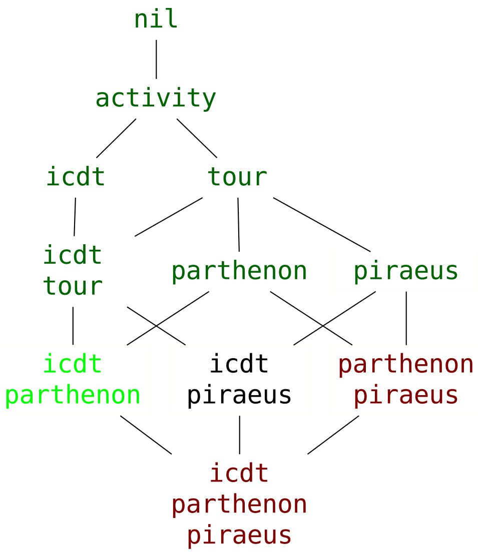

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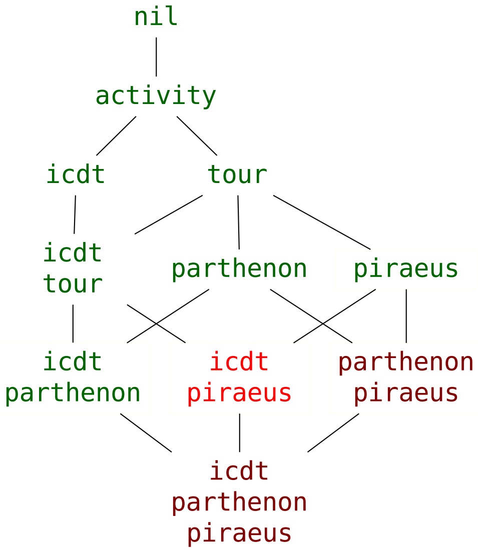

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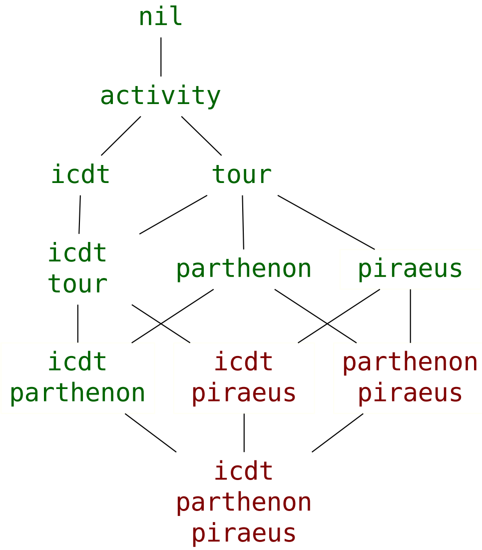
Solutions



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2. Preliminaries
3. Crowd complexity
4. Output crowd complexity
5. Computational complexity
6. Conclusion

Crowd complexity lower bound

- How many questions do we need to ask?
- Each query yields one bit of information

Crowd complexity lower bound

- How many questions do we need to ask?
- Each query yields one bit of information
- Information-theoretic lower bound: at least $\Omega(\log N)$ queries, with N the number of solutions
- $N = \Omega\left(2^{|\mathrm{I}(\Psi)|}\right)$ and $|\mathrm{I}(\Psi)| = \Omega\left(2^{|\Psi|}\right)$
- W.r.t. the original taxonomy Ψ, $\Omega\left(2^{\mathrm{width}(\Psi)} / \sqrt{\mathrm{width}[\Psi]}\right)$

Crowd complexity upper bound

`nil`	$6/7$
`a1`	$5/7$
`a2`	$4/7$
`a3`	$3/7$
`a4`	$2/7$
`a5`	$1/7$

- Query itemsets that are frequent in about half of the solutions

Crowd complexity upper bound

nil	6/7
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- Itemset split: min of proportion where frequent and proportion where infrequent

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- Query itemsets that are frequent in about half of the solutions
- Itemset split: min of proportion where frequent and proportion where infrequent
- Existing result from order theory [Linial and Saks, 1985]: there is a constant $\delta_0 \approx 1/5$ such that some itemset achieves a split $\geq \delta_0$

$\Rightarrow$ The previous bound is tight: we need $\Theta(\log N)$ queries

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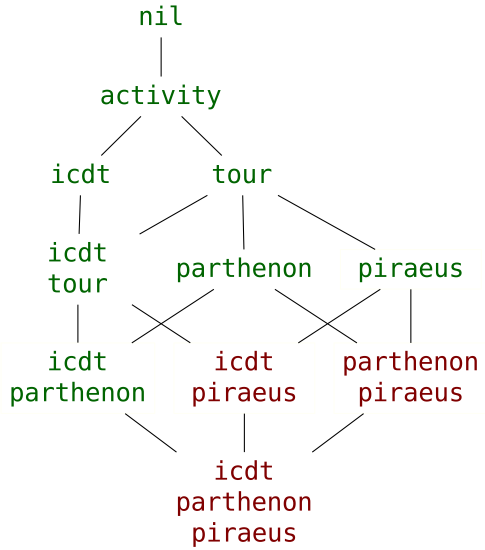
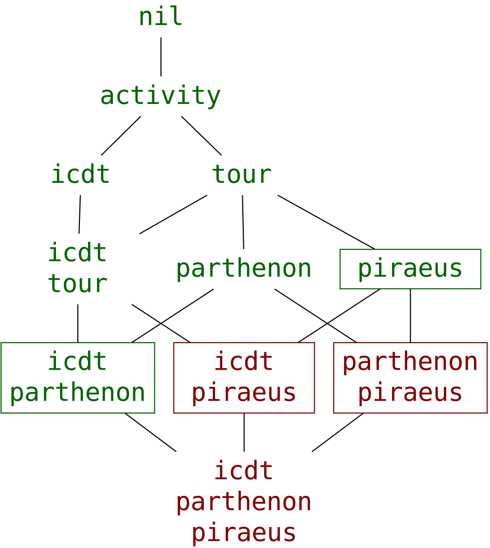
Maximal frequent itemsets



- Complexity with respect to the output size
- Output representation: Maximal frequent itemsets (MFI)
- Minimal infrequent itemset (MII)

Maximal frequent itemsets



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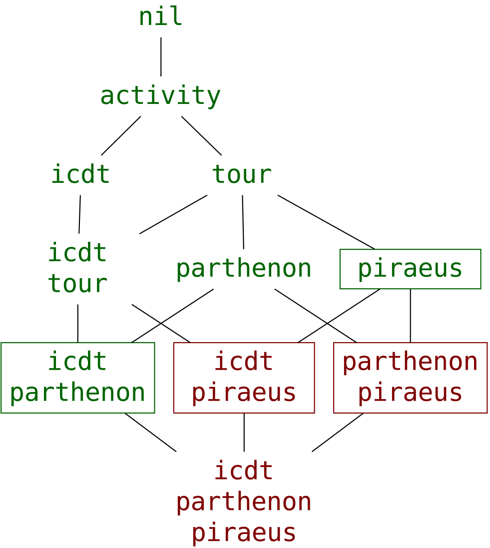
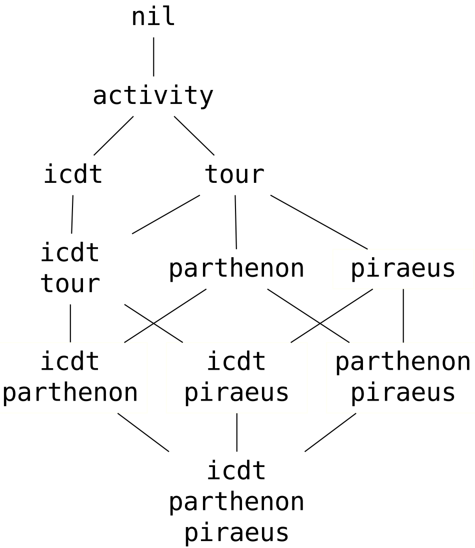


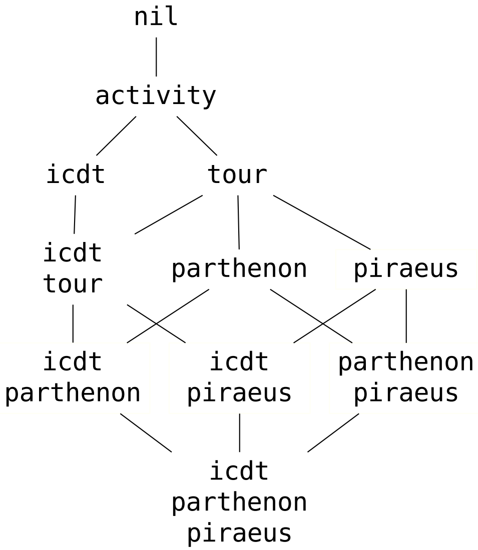
- Complexity with respect to the output size
- Output representation: Maximal frequent itemsets (MFI)
- Minimal infrequent itemset (MII)
- Must query all MFIs and MIIs
- Solutions with few MFIs/MIIs should be easier to find

MFI/MII upper bound



- Explicit algorithm to find each MFI/MII in $\leqslant |\mathcal{I}|$ queries
- Example:

MFI/MII upper bound



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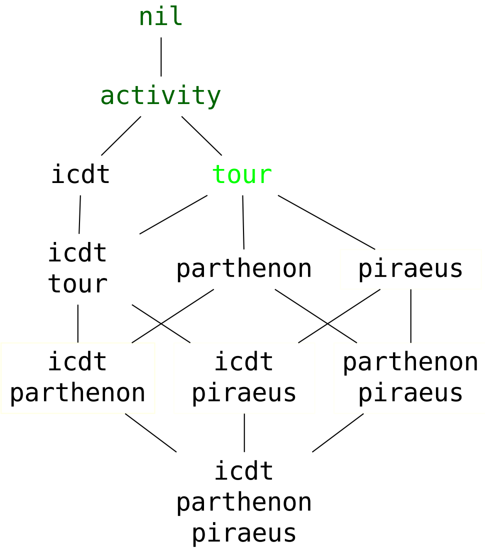
  - Pick an itemset:

MFI/MII upper bound



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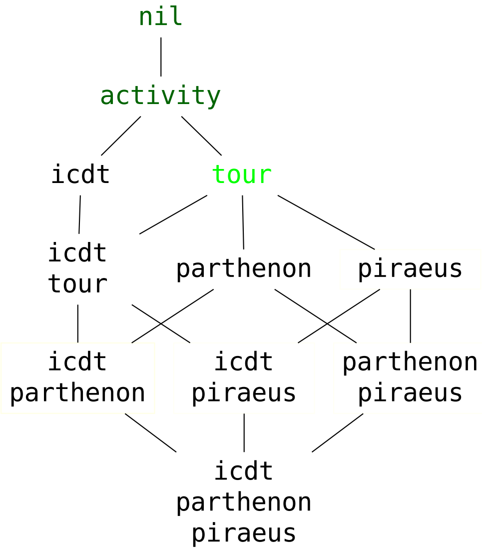

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  - Pick an itemset: $\{\texttt{tour}\}$
  - Specialize it...

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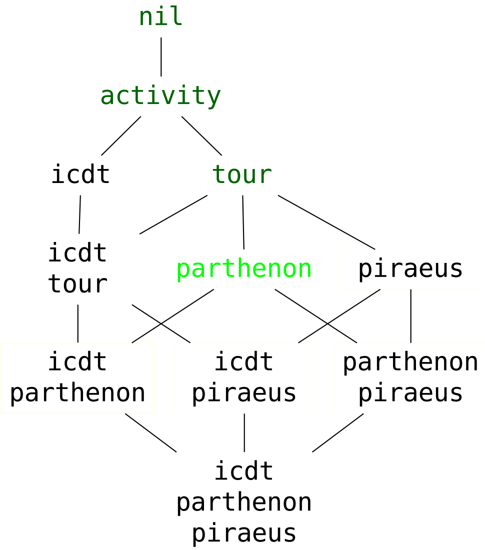

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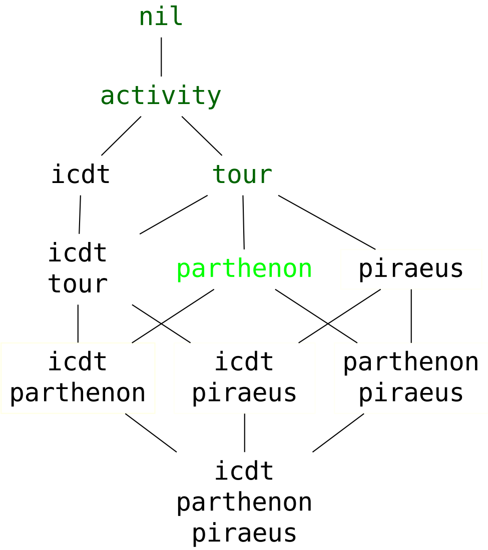
MFI/MII upper bound



- Explicit algorithm to find each MFI/MII in $\leqslant |\mathcal{I}|$ queries
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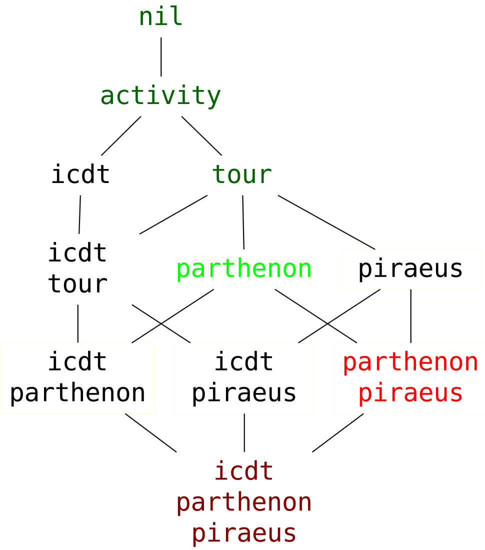

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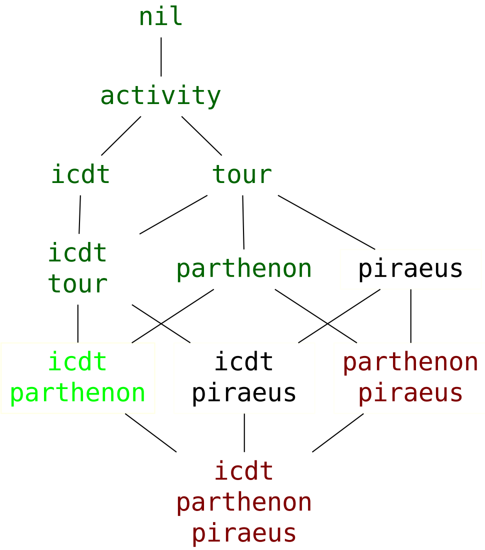

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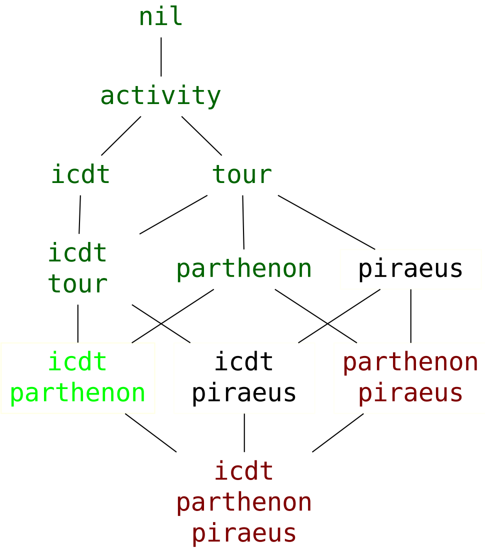

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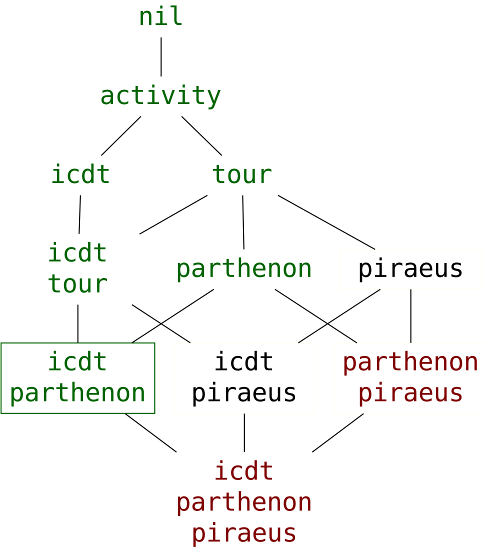

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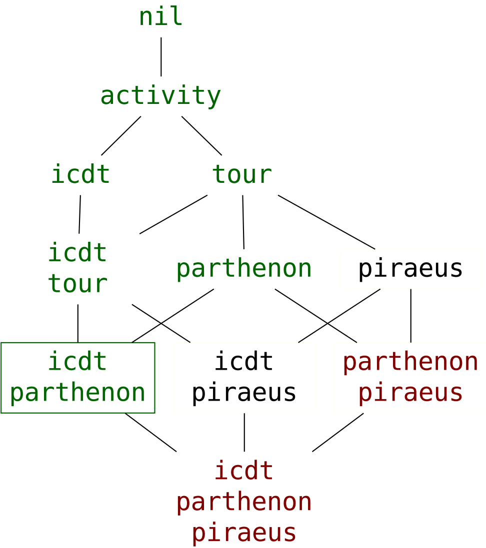

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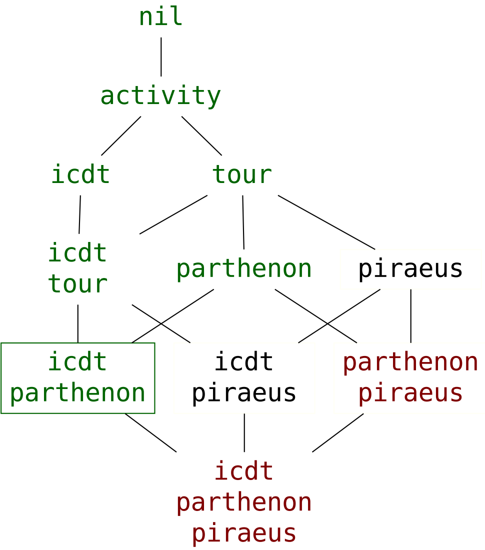

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$\Rightarrow$ Complexity: $O(|\mathcal{I}| \cdot (|\mathrm{MFI}| + |\mathrm{MII}|))$

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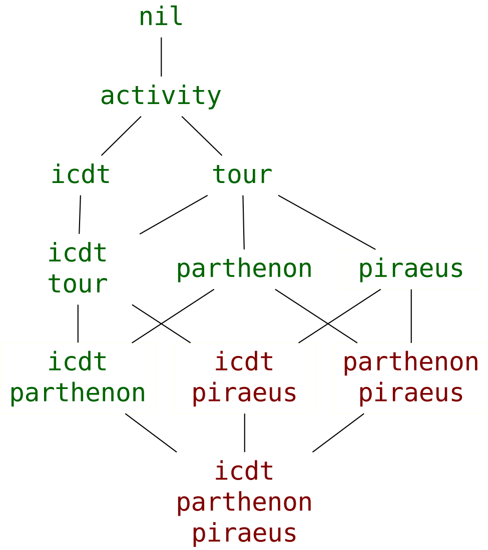
Output computational complexity lower bound



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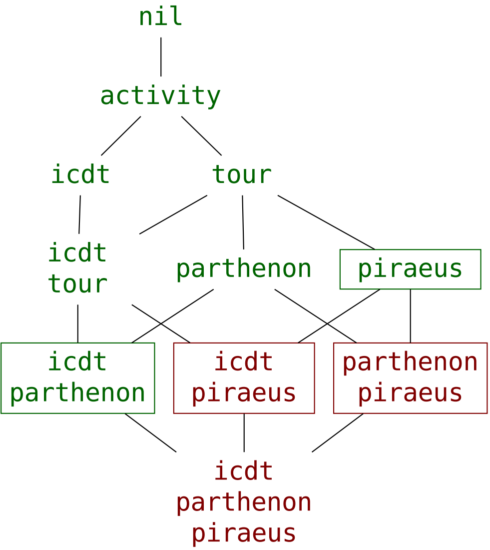

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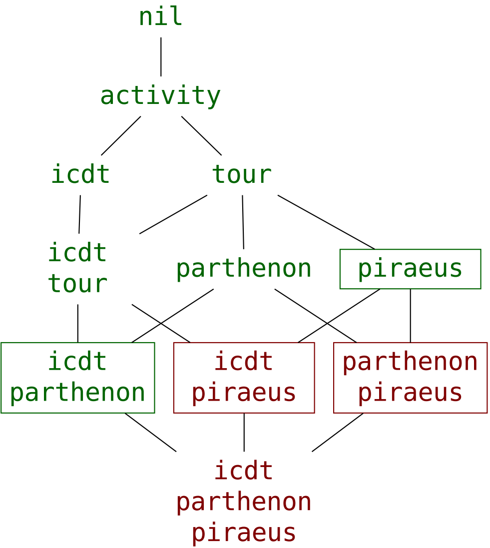

- Previous algorithm assumes $|I(\Psi)|$ is materialized
- Do we need to?
- Decide if finished: do the MFIs/MIIs cover all itemsets?
- This is EQ-hard, for problem EQ [Bioch and Ibaraki, 1995] (exact complexity open)

Computational complexity lower bound

- Find an unclassified itemset of $\mathsf{I}(\Psi)$ frequent for about half of the possible solutions
- We can count the possible solutions (exponential in $|\mathsf{I}(\Psi)|$)
- A solution is an "itemset" of $\mathsf{I}(\Psi)$, an antichain, and counting the antichains of $\mathsf{I}(\Psi)$ is $\mathsf{FP}^{\#\mathsf{P}}$-complete.

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$\Rightarrow$ Finding the best-split element in $I(\Psi)$ is $FP^{\#P}$-hard in $|I(\Psi)|$?

- Problem: $I(\Psi)$ is not a general DAG, so we only show hardness in $|\Psi|$ for restricted (fixed-size) itemsets
- Intuition: count antichains by comparing to a known poset; use a best-split oracle to compare; perform a binary search

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Thanks for your attention!

References

Bioch, J. and Ibaraki, T. (1995).
Complexity of identification and dualization of positive Boolean functions.
Inf. Comput., 123(1).

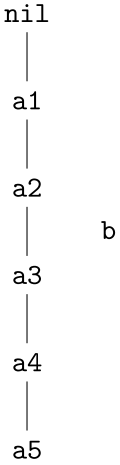
Linial, N. and Saks, M. (1985).
Every poset has a central element.
J. Combinatorial Theory, 40(2).

Greedy algorithms

- Querying an element of the chain may remove $< 1/2$ possible solutions
- Querying the isolated element b will remove exactly $1/2$ solution
- However, querying b classifies far less itemsets

$\Rightarrow$ Classifying many itemsets isn't the same as eliminating many solutions

Finding the greedy-best-split item is $FP^{\#P}$-hard



Restricted itemsets

- Asking about large itemsets is irrelevant.

  "Do you often go cycling and running while drinking coffee and having lunch with orange juice on alternate Wednesdays?"

- If the itemset size is bounded by a constant, $\mathrm{I}(\Psi)$ is tractable

$\Rightarrow$ The crowd complexity $\Theta(\log |\mathrm{S}(\Psi)|)$ is tractable too

Chain partitioning

- Optimal strategy for chain taxonomies: binary search
- We can determine a chain decomposition of the itemset taxonomy and perform binary searches on the chains
- Optimal crowd complexity for a chain, performance in general is unclear
- Computational complexity is polynomial in the size of $I(\Psi)$ (which is still exponential in Ψ)

Lower bound, MFI/MII

- To describe the solution, we need the MFIs or the MIIs.
- However, we need to query both the MFIs and the MIIs to identify the result uniquely: $\Omega(|\mathrm{MFI}| + |\mathrm{MII}|)$ queries.
- We can have $|\mathrm{MFI}| = \Omega\left(2^{|\mathrm{MII}|}\right)$ and vice-versa.
- This bound is not tight (e.g., chain).