

Efficient Enumeration via Factorized Representations

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- Amarilli, A., Bourhis, P., Jachiet, L., and Mengel, S.
 A Circuit-Based Approach to Efficient Enumeration. ICALP 2017.
- Amarilli, A., Bourhis, P., and Mengel, S. Enumeration on Trees under Relabelings. ICDT 2018.
- 📄 Niewerth, M.

MSO Queries on Trees: Enumerating Answers under Updates Using Forest Algebras. LICS 2018.

📄 Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M.

Constant-Delay Enumeration for Nondeterministic Document Spanners. ICDT 2019.

- Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M.
 Enumeration on Trees with Tractable Combined Complexity and Efficient Updates. PODS 2019
- 📔 Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M.

Constant-Delay Enumeration for Nondeterministic Document Spanners. TODS 2020.

Amarilli, A., Jachiet, L., Muñoz, M., and Riveros, C.
 Efficient Enumeration for Annotated Grammars. PODS 2022

Problem statement





















Currently:



Currently: $\overbrace{Input}^{A \ B \ c}$ Enumeration $\overbrace{A \ b \ c}^{A \ B \ c}$ Results $\overbrace{A \ b \ c}^{A \ B \ c}$ Results











5/21



• Directed acyclic graph of **gates**



- Directed acyclic graph of **gates**
- Output gate:



• Directed acyclic graph of gates

(x)

- Output gate:
- Variable gates:



• Directed acyclic graph of gates

(x)

 (\perp)

- Output gate:
- Variable gates:
- Constant gates:



• Directed acyclic graph of gates

(x

(±)

- Output gate:
- Variable gates:
- Constant gates:
- Internal gates:





Every gate *g* captures a set *S*(*g*) of sets (called assignments)

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- \top -gates: $S(g) = \{\{\}\}$
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- \times -gate with children g_1, g_2 : $S(g) := \{s_1 \cup s_2 \mid s_1 \in S(g_1), s_2 \in S(g_2)\}$



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Task: Enumerate the assignments of the set S(g) captured by a gate $g \rightarrow$ E.g., for $S(g) = \{\{x\}, \{x, y\}\}$, enumerate $\{x\}$ and then $\{x, y\}$

d-DNNF set circuit:

- are all **deterministic**:
- The inputs are **disjoint**

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(= no assignment is captured by two inputs)



d-DNNF set circuit:

- (U) are all **deterministic**:
- The inputs are **disjoint**
- (= no assignment is captured by two inputs)
 - are all **decomposable**:
- The inputs are **independent**
- (= no variable **x** has a path to two different inputs)



Set circuits vs factorized representations

 A
 B
 C

 a
 b
 c

 a1
 b'
 c'

 a2
 b'
 c'



- Set circuits can be seen as factorized representations
 - ightarrow Not necessarily well-typed, height and/or assignment size may be non-constant
- Determinism: unions are disjoint
- Decomposability: no duplicate attribute names in products
- Structuredness: always the same decomposition of the attributes
Theorem

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Also: restrict to assignments of **constant size** $k \in \mathbb{N}$

Theorem

Given a *d-DNNF* set circuit *C*, we can enumerate its captured assignments of size $\leq k$ with preprocessing linear in |C| and constant delay

Proof techniques

Preprocessing phase:









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 - \cdot then get rid of the gate













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 \rightarrow Now, when traversing a \times -gate we make progress: non-trivial split of each set

Indexing: handling U-hierarchies



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- Custom constant-delay reachability index for multitrees



Applications

Application 1: MSO query evaluation on trees





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Data: a tree T where nodes have a color from an

Query Q in monadic second-order logic (MSO)

- $\cdot P_{\odot}(x)$ means "x is blue"
- $\cdot x \rightarrow y$ means "x is the parent of y"

5 3

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results: (2,7), (3,7)

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Data complexity: Measure efficiency as a function of T (the query Q is fixed)

We can enumerate the answers of MSO queries on trees with **linear-time preprocessing** and **constant delay**.

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We can prove this with our methods:

Theorem (A., Bourhis, Jachiet, Mengel, ICALP'15, ICALP'17)

For any bottom-up deterministic **tree automaton A** and input **tree T**, we can build a **d-DNNF set circuit** capturing the results of **A** on **T** in $O(|A| \times |T|)$

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- Can be extended to support relabeling updates to the tree in O(log n) time (A., Bourhis, Mengel, ICDT'18)
- Same result for leaf **insertion/deletion** (A., Bourhis, Mengel, Niewerth, PODS'19) up to **fixing a buggy result** [Niewerth, 2018]



Data: a text T

Antoine Amarilli Description Name Antoine Amarilli. Handle: a3nm. Identity Born 1990-02-07. French national. Appearance as of 2017. Auth OpenPGP. OpenId. Bitcoin. Contact Email and XMPP a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France. Studies PhD in computer science awarded by Télécom ParisTech on March 14, 2016. Former student of the École normale supérieure. test@example.com More Résumé Location Other sites Blogging: a3nm.net/blog Git: a3nm.net/git ...



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Query: a pattern P given as a regular expression

 $P := \Box [a-z0-9.]^* @ [a-z0-9.]^* \Box$



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Output: the list of **substrings** of **T** that match **P**:

 $[186, 200\rangle, [483, 500\rangle, \dots]$



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Goal:

- be very efficient in T (constant-delay)
- be reasonably efficient in P (polynomial-time)

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 - Does not really use d-DNNFs, but bounded-width structured DNNFs
- ightarrow Actually equivalent to MSO evaluation on text; generalizes to trees



Data: a text T, e.g., source code

long elt, prev, elt2, prev2=-1; int ret = fscamf(fi, "%ld%ld", &elt, &prev); if (ret != 2) { fprintf(stderr, "Bad offsets after position %ld in index!\n", pi); exit(1);



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Theorem (A., Jachiet, Muñoz, Riveros, PODS'22)

Given an **unambiguous annotation grammar** G and input text w, we can enumerate the **matches** with preprocessing $O(|G| \times |w|^3)$ and delay **linear in each assignment**



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- Improves on a **quintic** preprocessing result [Peterfreund, 2021]
- **Quadratic** and **linear** preprocessing for **subclasses** (rigid grammars, deterministic pushdown annotators)

Conclusion

Summary and conclusion

- Enumerate the captured assignments of d-DNNF set circuits
 - $\rightarrow\,$ with preprocessing linear in the d-DNNF
 - \rightarrow in delay **linear** in each assignment
 - \rightarrow in **constant** delay for constant size
- $\rightarrow\,$ Applies to MSO enumeration on words and trees
- \rightarrow Applies to enumeration of the matches of **annotated context-free grammars** (with more expensive preprocessing)

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Future work:

- In-order enumeration
- Linear-time preprocessing on more general context-free grammar classes
- Connect results on **updates** to **incremental maintenance for regular languages** (A., Jachiet, Paperman, ICALP'21)

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Thanks for your attention!

- Amarilli, A., Bourhis, P., Jachiet, L., and Mengel, S. (2017).
 A Circuit-Based Approach to Efficient Enumeration.
 In ICALP.
- Amarilli, A., Bourhis, P., and Mengel, S. (2018). **Enumeration on Trees under Relabelings.**
- Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M. (2019a).
 Constant-Delay Enumeration for Nondeterministic Document Spanners.
 In ICDT.

References ii

- Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M. (2019b).
 Enumeration on Trees with Tractable Combined Complexity and Efficient Updates. In PODS.
- Amarilli, A., Bourhis, P., and Senellart, P. (2015).
 Provenance Circuits for Trees and Treelike Instances.
 In ICALP.
- Amarilli, A., Jachiet, L., Muñoz, M., and Riveros, C. (2022). Efficient Enumeration for Annotated Grammars. In PODS.
- Amarilli, A., Jachiet, L., and Paperman, C. (2021).
 Dynamic Membership for Regular Languages.
 In ICALP.

References iii

🔋 Bagan, G. (2006).

MSO queries on tree decomposable structures are computable with linear delay. In *CSL*.

- Florenzano, F., Riveros, C., Ugarte, M., Vansummeren, S., and Vrgoc, D. (2018).
 Constant delay algorithms for regular document spanners.
 In PODS.
 - Kazana, W. and Segoufin, L. (2013).

Enumeration of monadic second-order queries on trees. *TOCL*, 14(4).

📔 Niewerth, M. (2018).

MSO queries on trees: Enumerating answers under updates using forest algebras. In *LICS.*



Peterfreund, L. (2021). **Grammars for document spanners.** In *ICDT*.