



# A Circuit-Based Approach to Efficient Enumeration

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**Antoine Amarilli**<sup>1</sup>, Pierre Bourhis<sup>2</sup>, Louis Jachiet<sup>3</sup>, Stefan Mengel<sup>4</sup>

September 20th, 2017

<sup>1</sup>Télécom ParisTech

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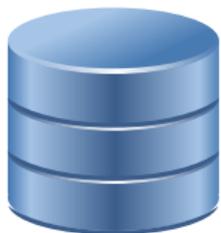
<sup>3</sup>Université Grenoble-Alpes

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## **Problem statement**

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## Problem: Enumerating large result sets



Input

## Problem: Enumerating large result sets

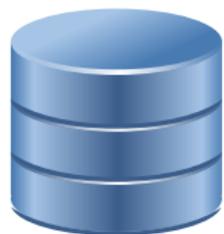


Input



Algorithm

## Problem: Enumerating large result sets



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A	B	C
a	b	c
a'	b	c
a	b'	c
a'	b'	c

Output

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- **Problem:** The output may be **too large** to compute efficiently

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Q knowledge compilation



Search

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Results **1 - 20** of **10,514**

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View (previous 20 | [next 20](#)) (20 | 50 | 100 | 250 | 500)

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→ **Solution:** Enumerate solutions **one after the other**

# Enumeration algorithm



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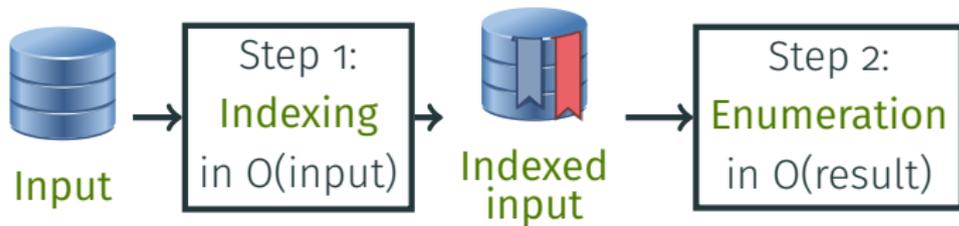
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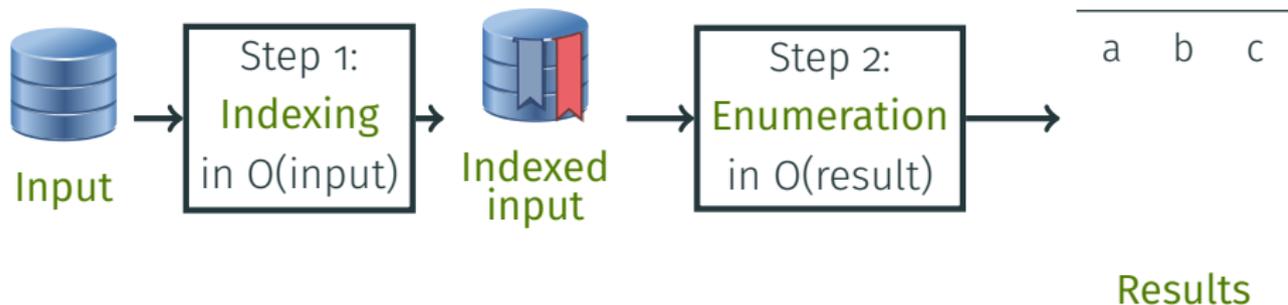
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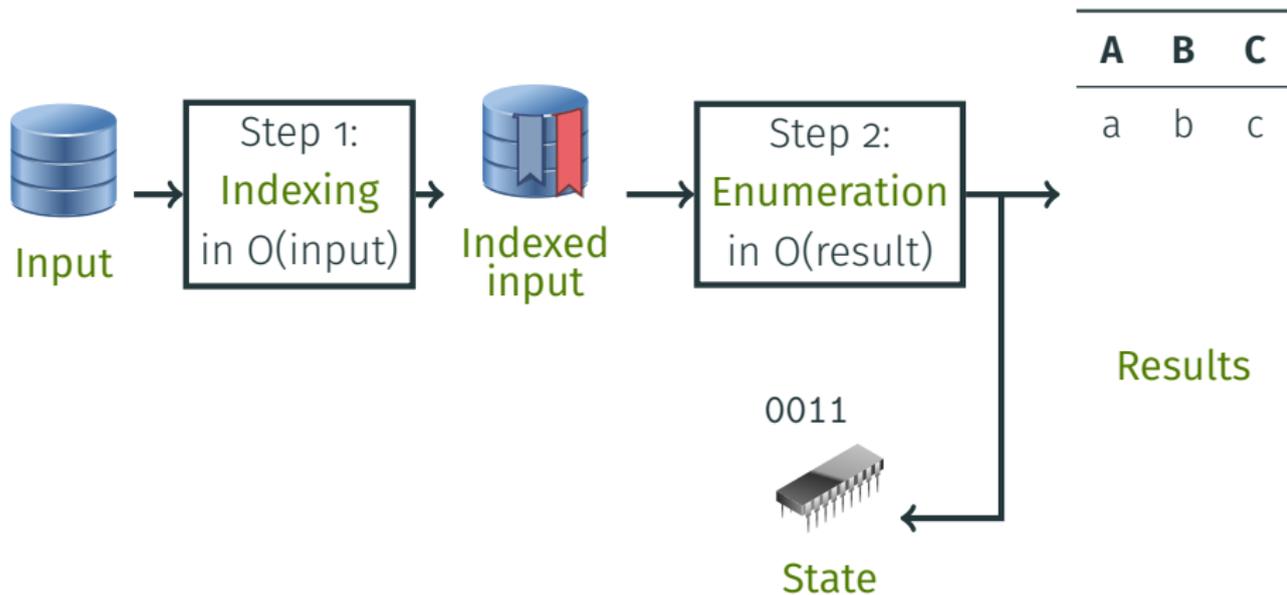
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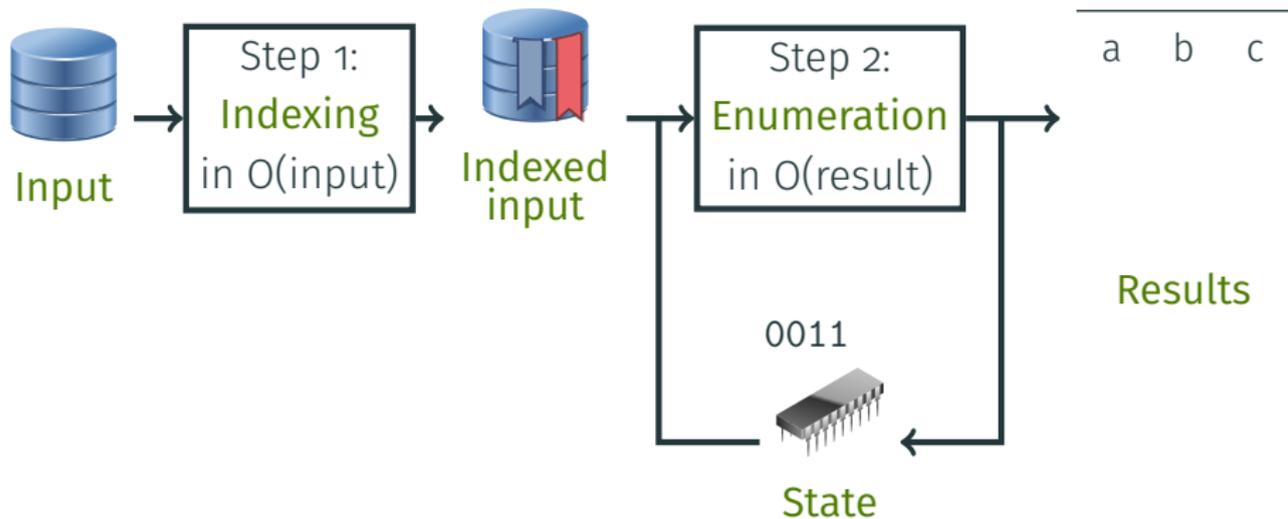
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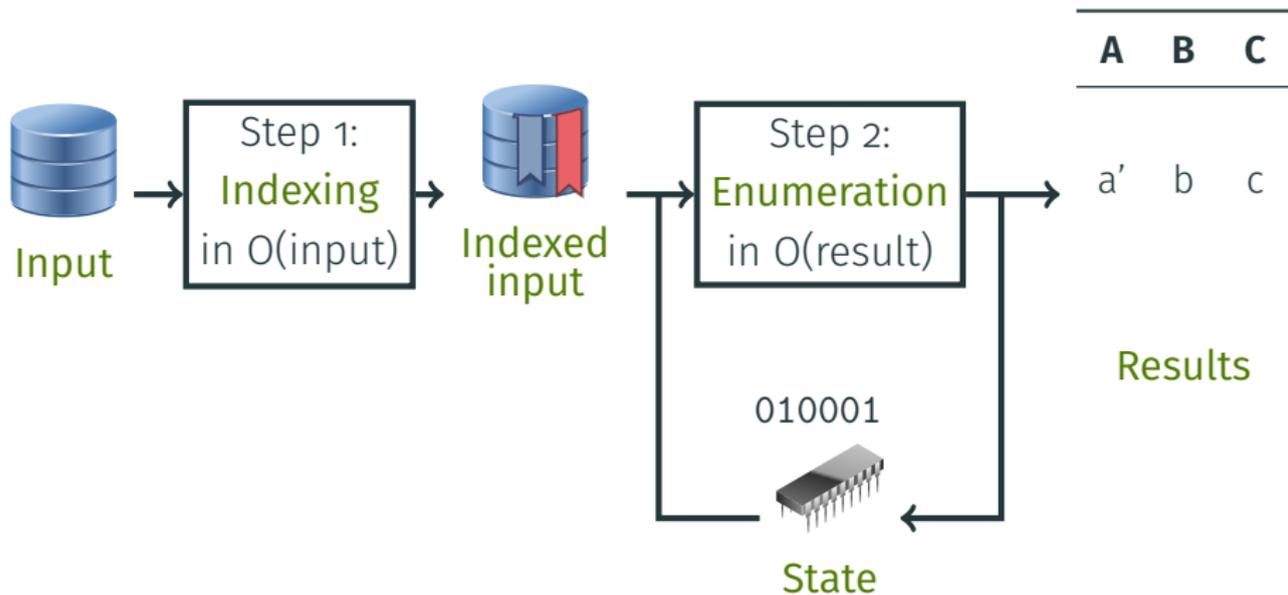
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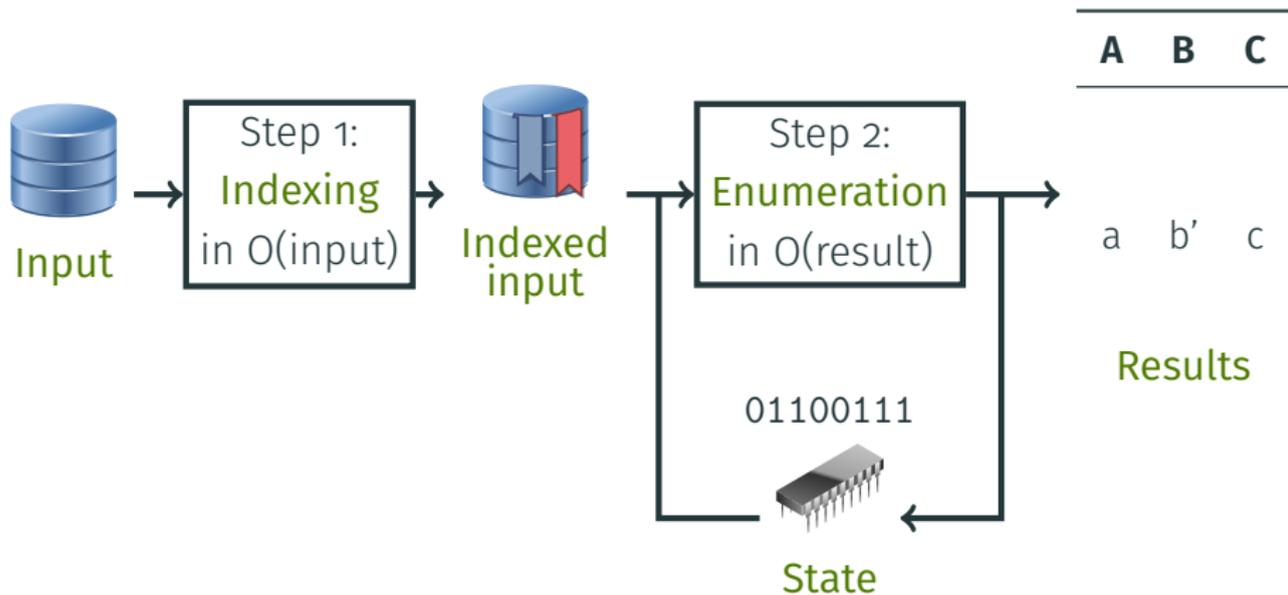
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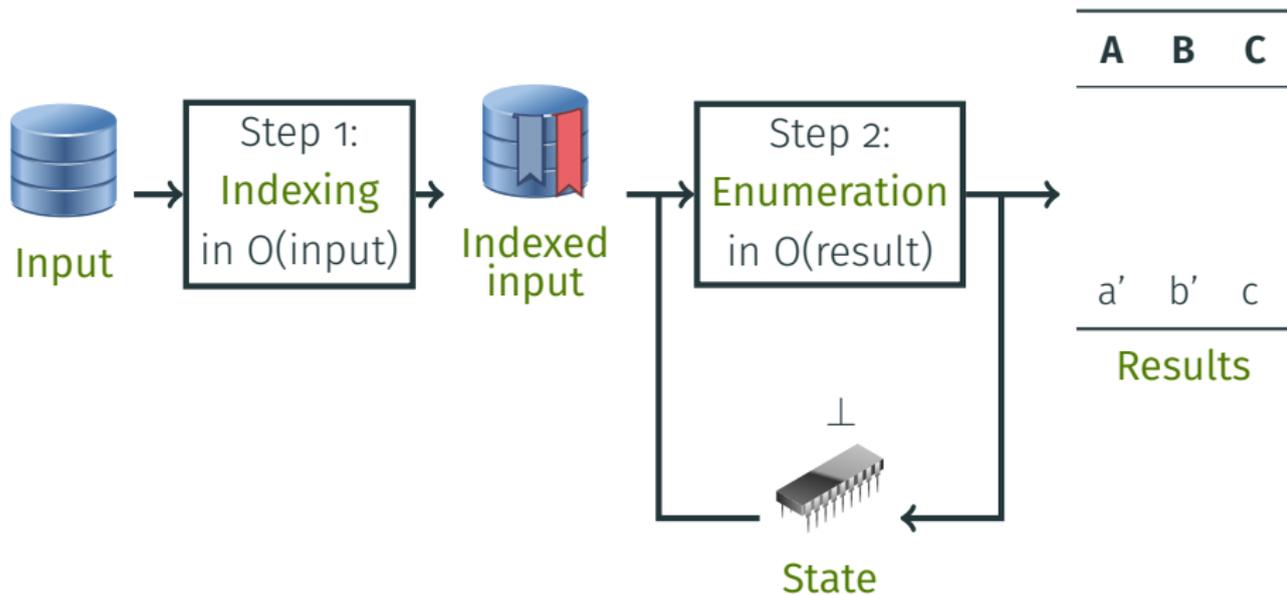
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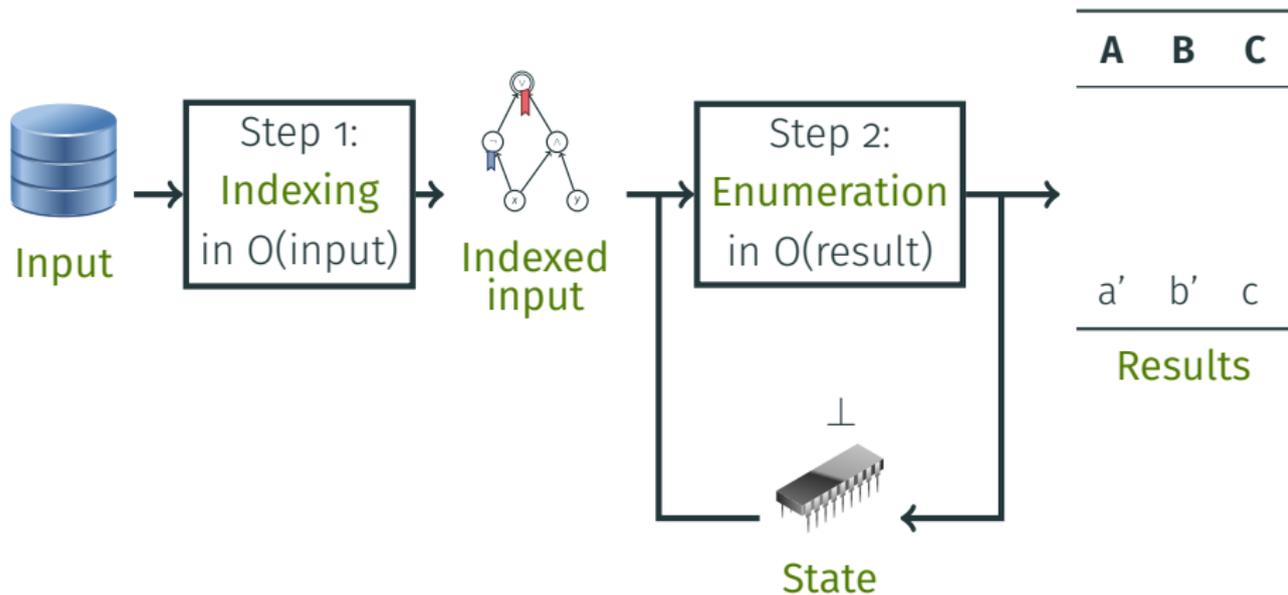
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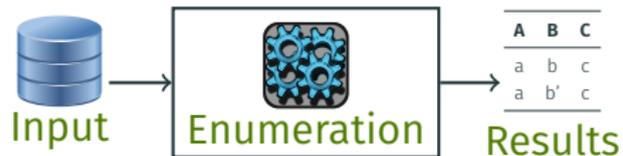


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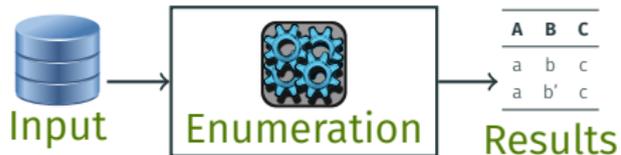
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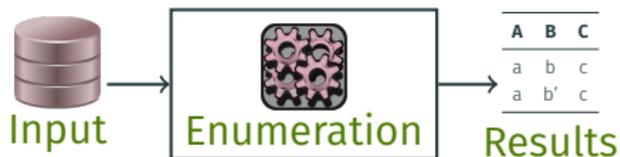
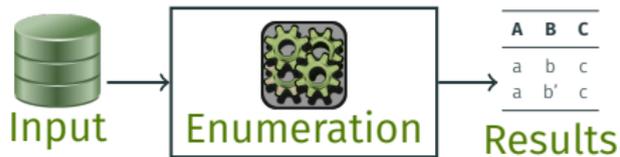
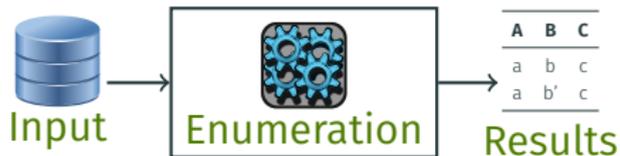
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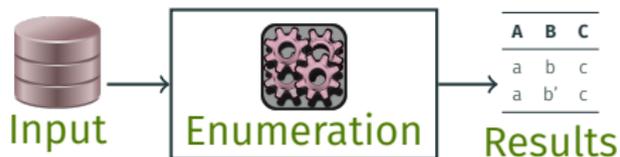
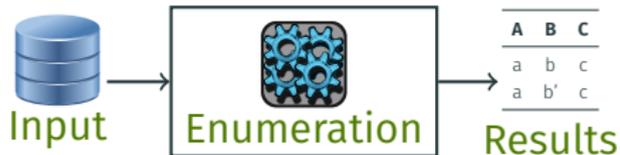
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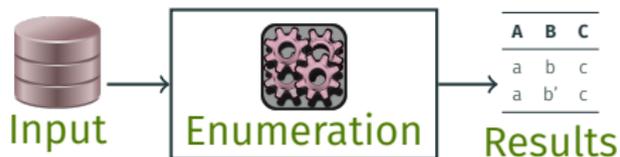


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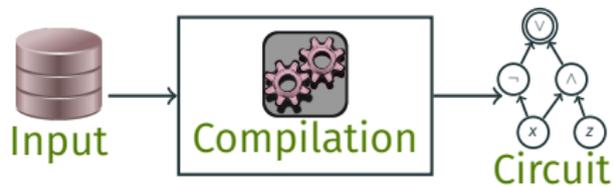


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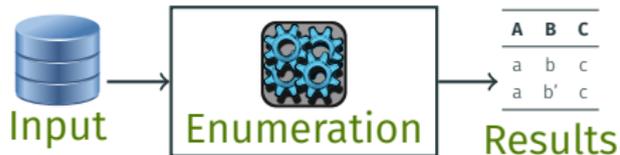


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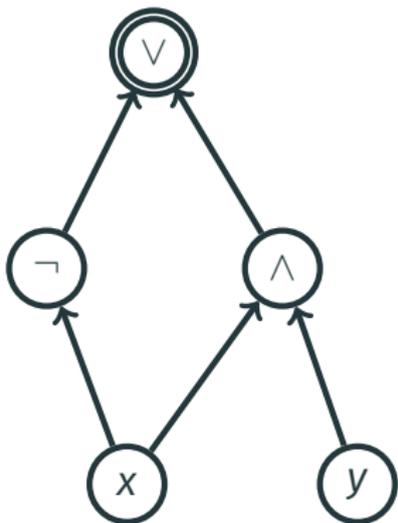
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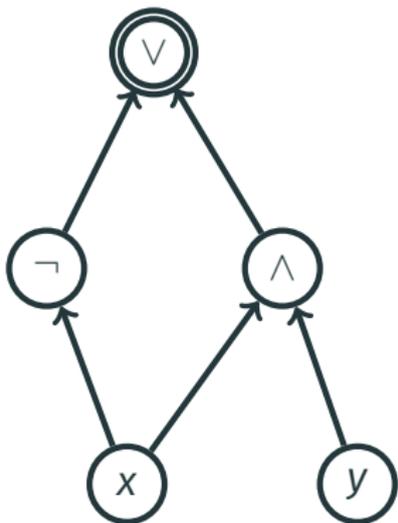
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- **Variable** gates:

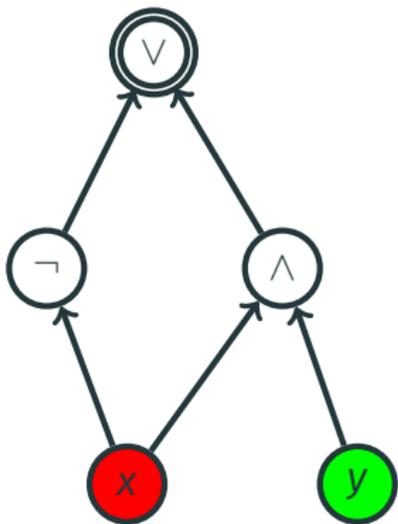
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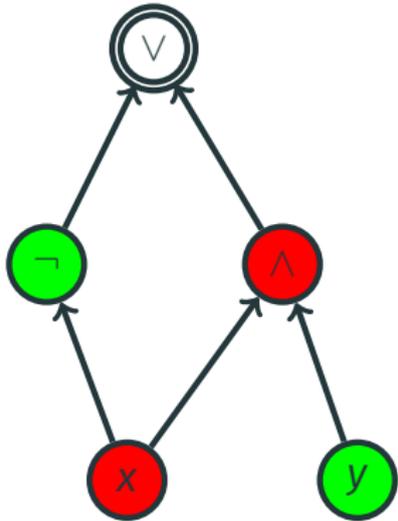
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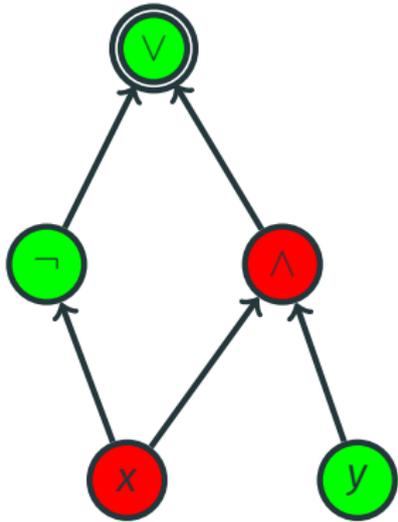
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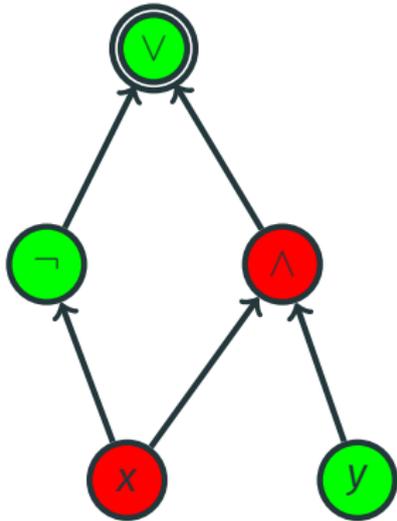
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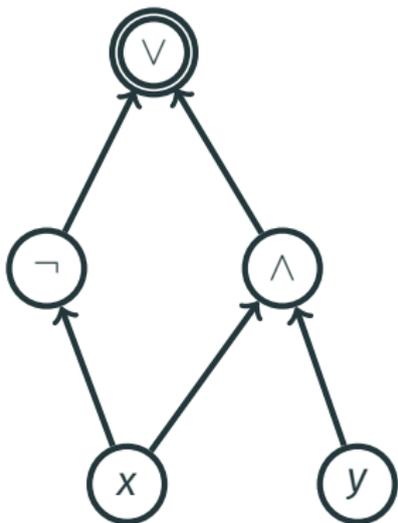
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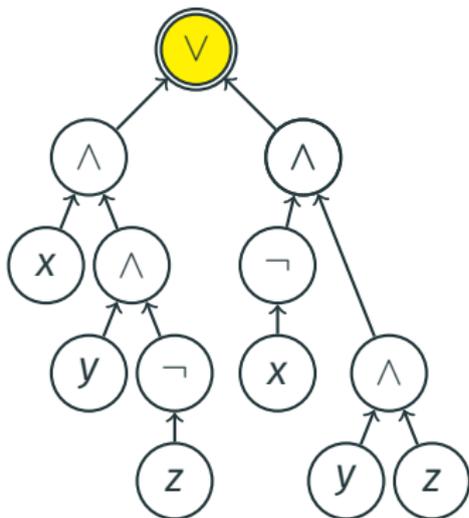
**Our task:** Enumerate all **satisfying assignments** of an input circuit

# Circuit restrictions

## d-DNNF:

- $\vee$  are all **deterministic**:

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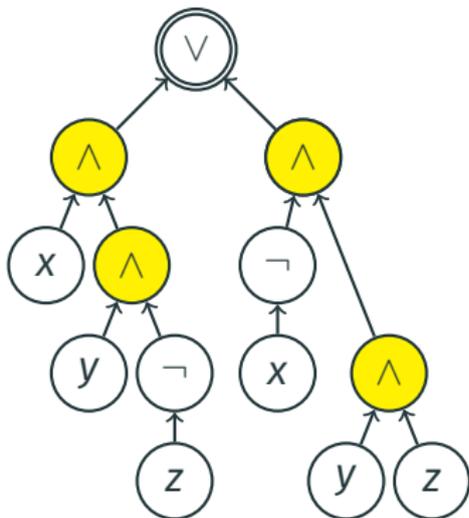
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# Circuit restrictions

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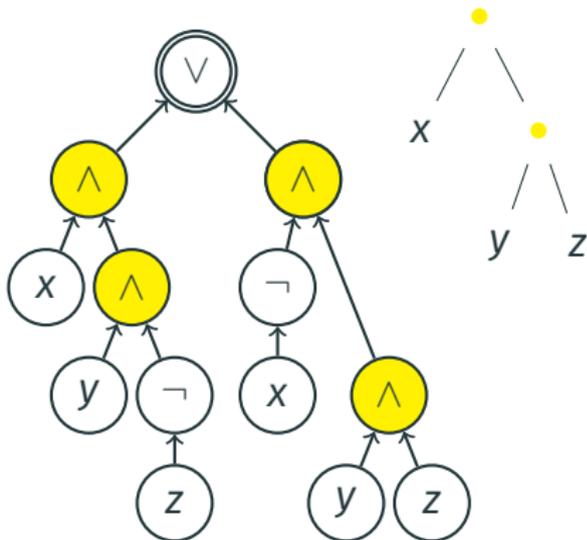
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**v-tree:**  $\bigwedge$ -gates follow a **tree**  
on the variables



# Main results

## Theorem

Given a *d-DNNF circuit*  $C$  with a *v-tree*  $T$ , we can enumerate its *satisfying assignments* with preprocessing *linear in  $|C| + |T|$*  and delay *linear in each assignment*

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Also: restrict to assignments of *constant size*  $k \in \mathbb{N}$   
(at most  $k$  variables are set to 1):

## Theorem

Given a *d-DNNF circuit*  $C$  with a *v-tree*  $T$ , we can enumerate its *satisfying assignments of size  $\leq k$*  with preprocessing *linear in  $|C| + |T|$*  and *constant delay*

# Application 1: Factorized databases

Orders (O for short)			Dish (D for short)		Items (I for short)	
customer	day	dish	dish	item	item	price
Elise	Monday	burger	burger	patty	patty	6
Elise	Friday	burger	burger	onion	onion	2
Steve	Friday	hotdog	burger	bun	bun	2
Joe	Friday	hotdog	hotdog	bun	sausage	4
			hotdog	onion		
			hotdog	sausage		

Consider the join of the above relations:

O(customer, day, **dish**), D(**dish**, **item**), I(**item**, price)

customer	day	dish	item	price
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(Slides courtesy of Dan Olteanu)

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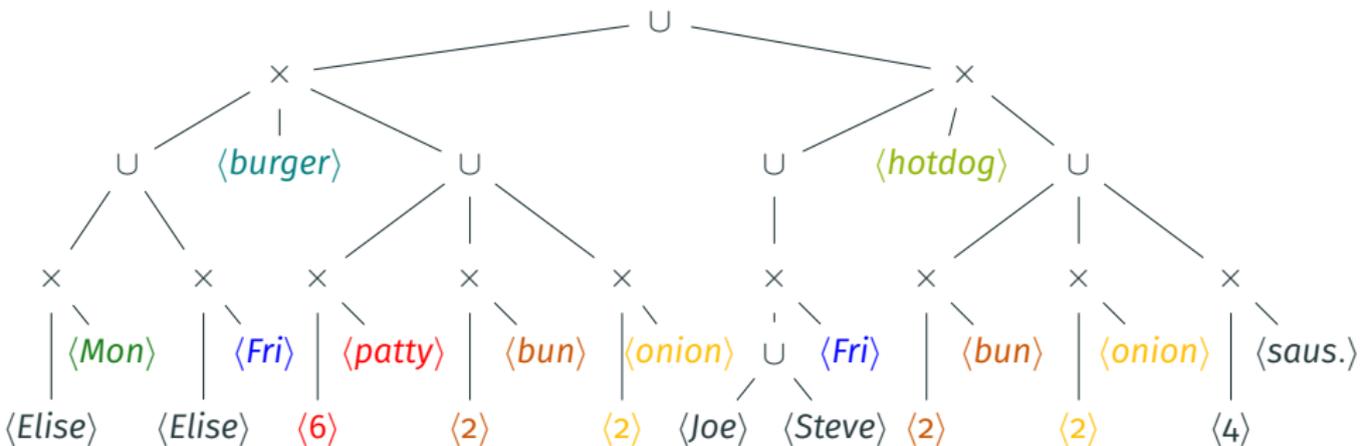
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A relational algebra expression encoding the above query result is:

$$\begin{aligned} \langle \text{Elise} \rangle &\times \langle \text{Monday} \rangle \times \langle \text{burger} \rangle \times \langle \text{patty} \rangle \times \langle 6 \rangle \cup \\ \langle \text{Elise} \rangle &\times \langle \text{Monday} \rangle \times \langle \text{burger} \rangle \times \langle \text{onion} \rangle \times \langle 2 \rangle \cup \\ \langle \text{Elise} \rangle &\times \langle \text{Monday} \rangle \times \langle \text{burger} \rangle \times \langle \text{bun} \rangle \times \langle 2 \rangle \cup \\ \langle \text{Elise} \rangle &\times \langle \text{Friday} \rangle \times \langle \text{burger} \rangle \times \langle \text{patty} \rangle \times \langle 6 \rangle \cup \\ \langle \text{Elise} \rangle &\times \langle \text{Friday} \rangle \times \langle \text{burger} \rangle \times \langle \text{onion} \rangle \times \langle 2 \rangle \cup \\ \langle \text{Elise} \rangle &\times \langle \text{Friday} \rangle \times \langle \text{burger} \rangle \times \langle \text{bun} \rangle \times \langle 2 \rangle \cup \dots \end{aligned}$$

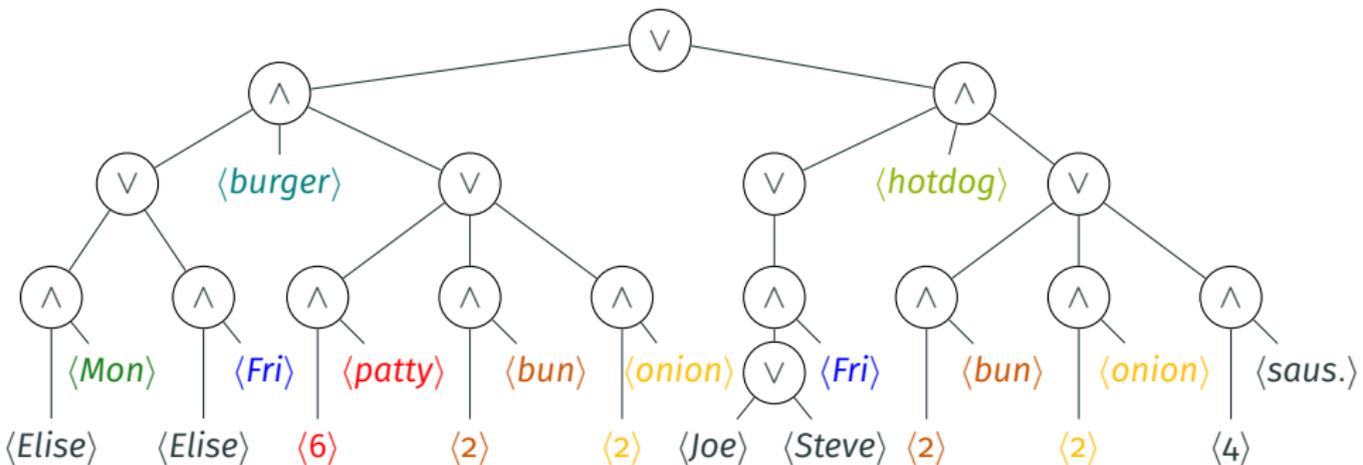
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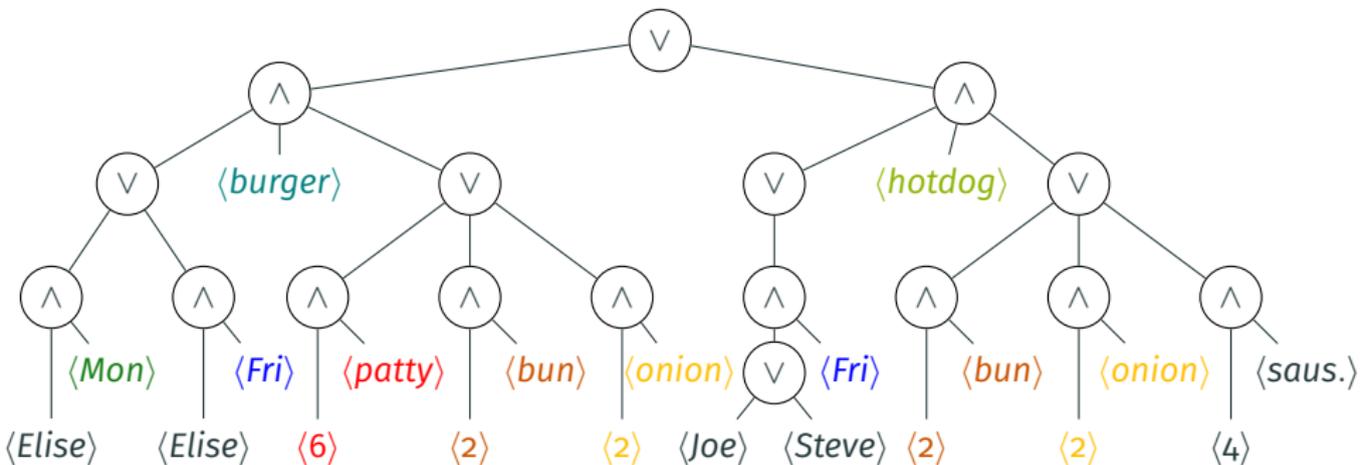
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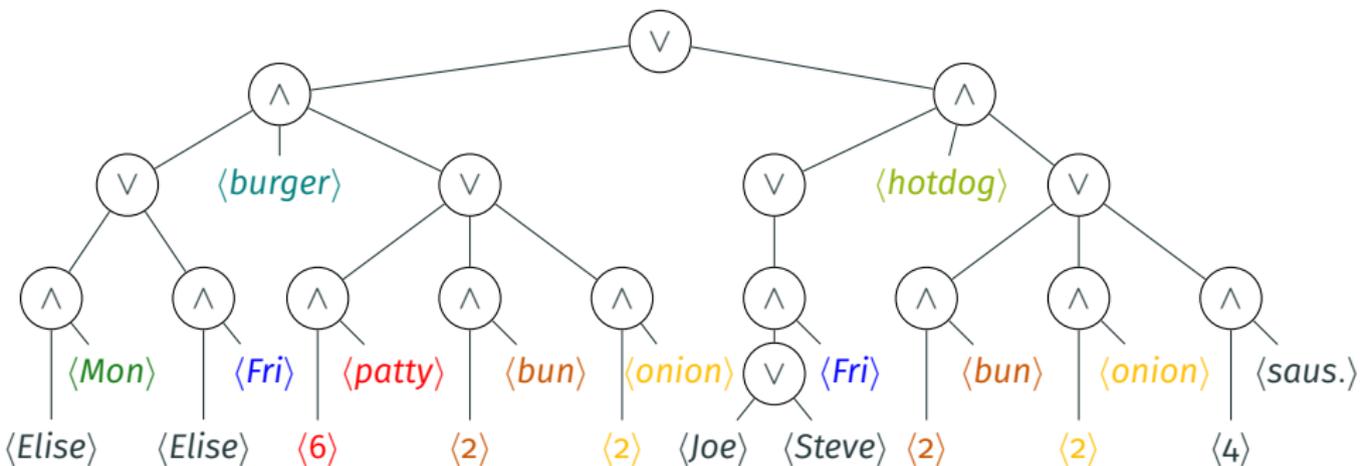
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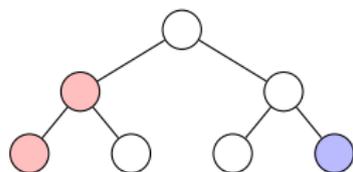
### Theorem (**Strenghtens** result of [Olteanu and Závodný, 2015])

Given a deterministic factorized representation, we can enumerate its tuples with *linear preprocessing* and *constant delay*

## Application 2: Query evaluation

### Query evaluation on trees

**Database:** a tree  $T$  where nodes have a color from an alphabet  $\{\circ, \text{pink}, \text{blue}\}$



**Query  $Q$ :** a sentence in monadic second-order logic (MSO)

- $P_{\text{blue}}(x)$  means “ $x$  is blue”
- $x \rightarrow y$  means “ $x$  is the parent of  $y$ ”

“Is there both a pink and a blue node?”

$$\exists x y P_{\text{pink}}(x) \wedge P_{\text{blue}}(y)$$

**Result:** TRUE/FALSE indicating if  $T$  satisfies the query  $Q$

**Computational complexity** as a function of the tree  $T$   
(the query  $Q$  is fixed)

(Slides courtesy of Pierre Bourhis)

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- Compute the results  $(a, b, c)$  of a query  $Q(x, y, z)$  on a tree  $T$ 
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### **Theorem (Recaptures [Bagan, 2006], [Kazana and Segoufin, 2013])**

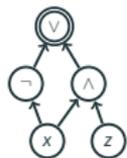
*For any constant  $k \in \mathbb{N}$  and fixed MSO query  $Q$ ,  
given a database  $D$  of treewidth  $\leq k$ , the results of  $Q$  on  $D$   
can be enumerated with **linear preprocessing** in  $D$  and **linear delay**  
in each answer (→ **constant delay** for free first-order variables)*

# Proof techniques

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# Proof overview

## Preprocessing phase:



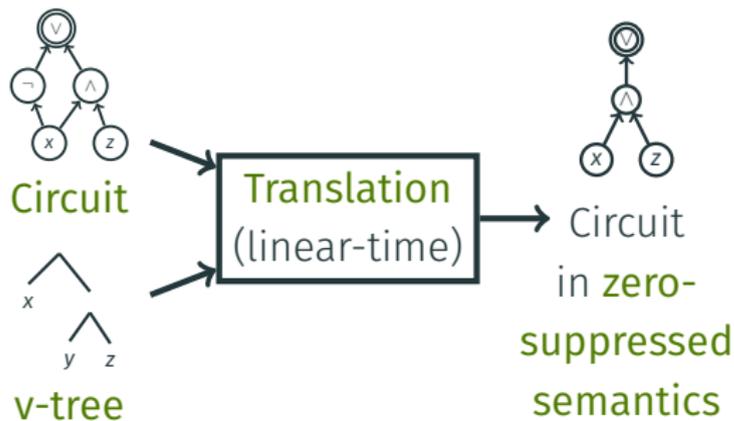
Circuit



v-tree

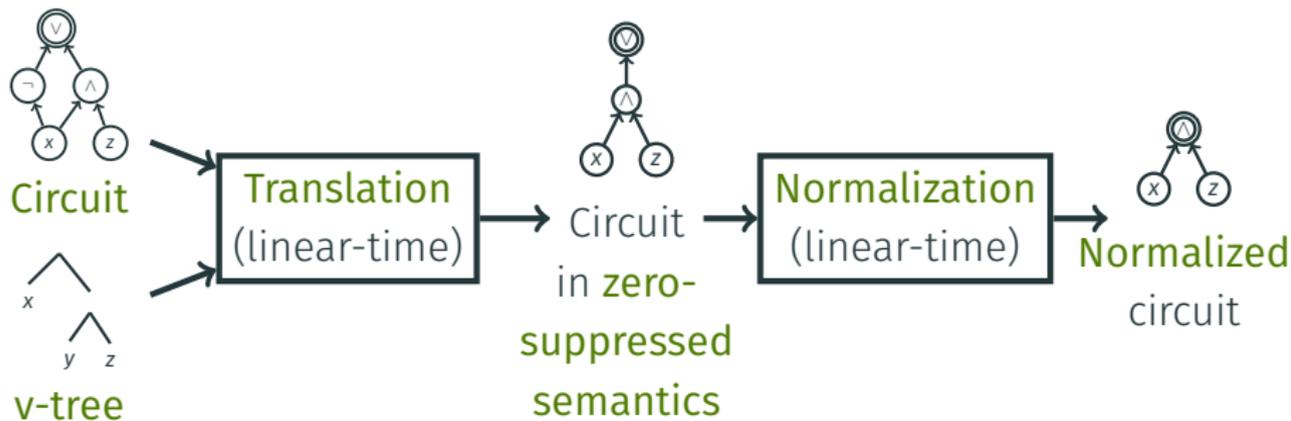
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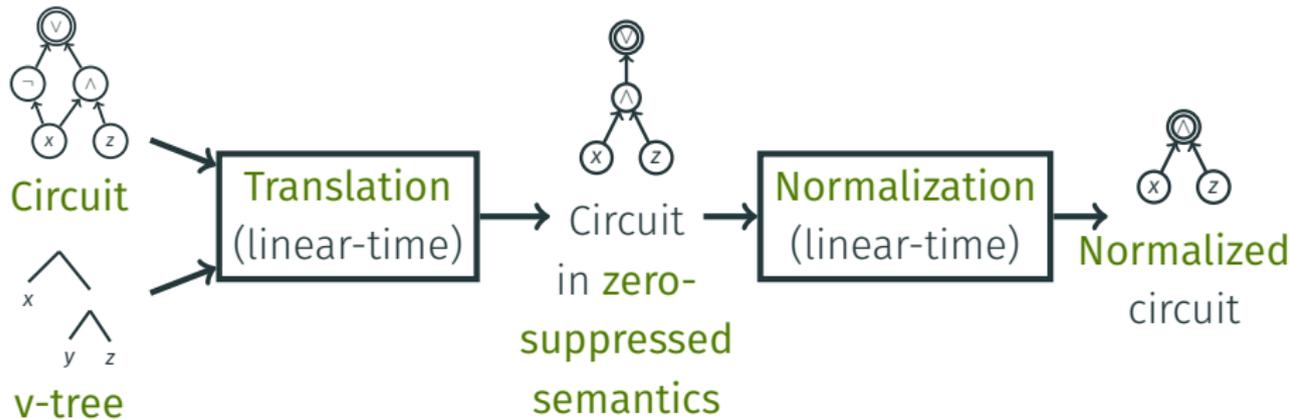
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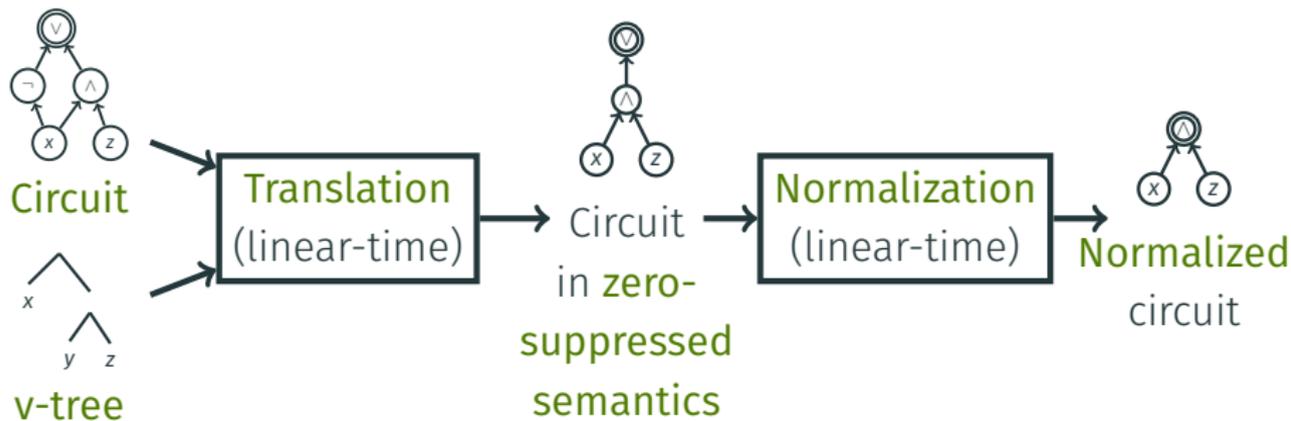
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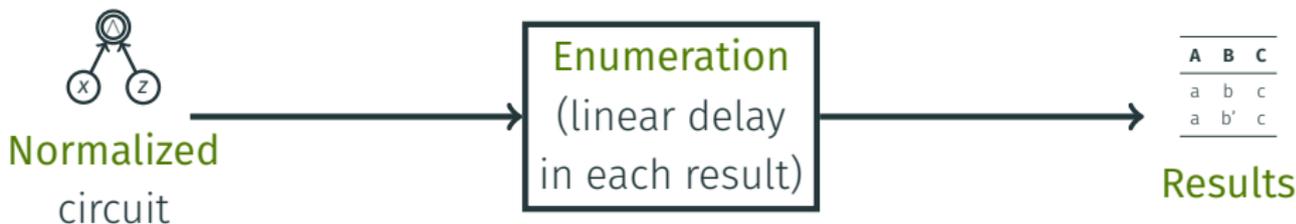
Normalized  
circuit

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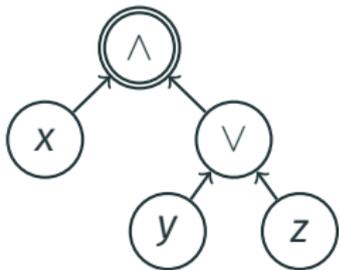
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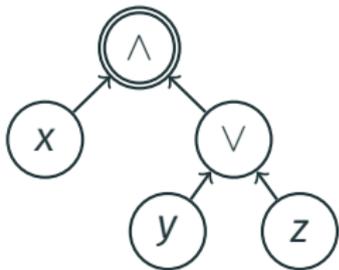


# Zero-suppressed semantics



Special **zero-suppressed semantics** for circuits:

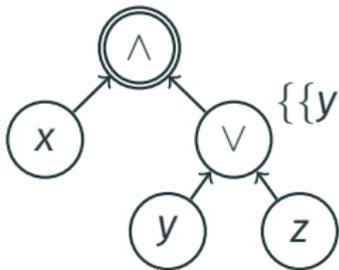
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- No **NOT**-gate
- Each gate **captures** a set of assignments
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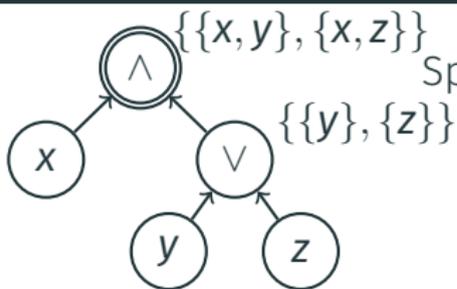


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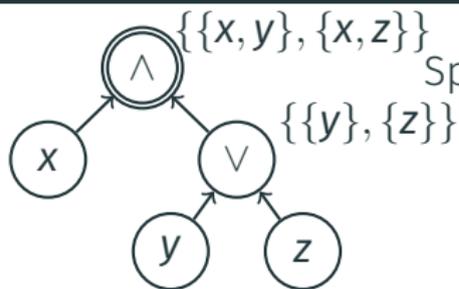
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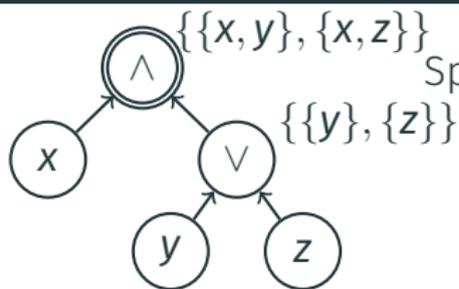
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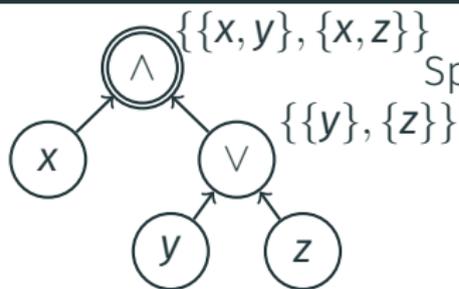
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Many **equivalent ways** to understand this:

- Generalization of **factorized representations**
- Analogue of **zero-suppressed OBDDs** (implicit negation)
- **Arithmetic circuits**:  $\times$  and  $+$  on polynomials

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- No **NOT**-gate
- Each gate **captures** a set of assignments
- **Bottom-up** definition with  $\times$  and  $\cup$

- **d-DNNF**:  $\cup$  are disjoint,  $\times$  are on disjoint sets

Many **equivalent ways** to understand this:

- Generalization of **factorized representations**
- Analogue of **zero-suppressed OBDDs** (implicit negation)
- **Arithmetic circuits**:  $\times$  and  $+$  on polynomials

**Simplification**: rewrite circuits to arity-two (fan-in  $\leq 2$ )

## Enumerating assignments in the zero-suppressed semantics

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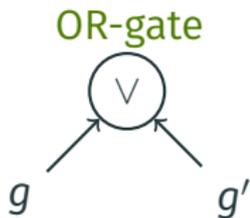
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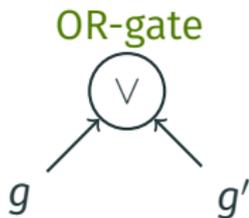
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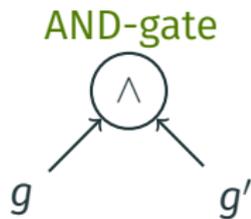
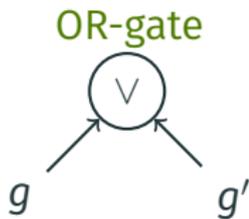
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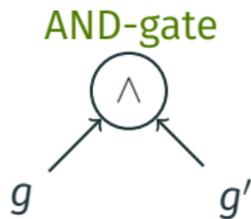
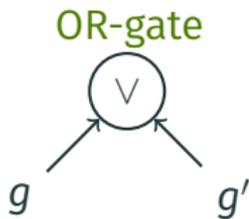
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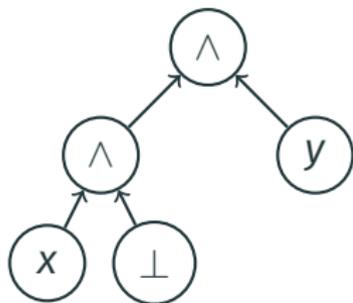
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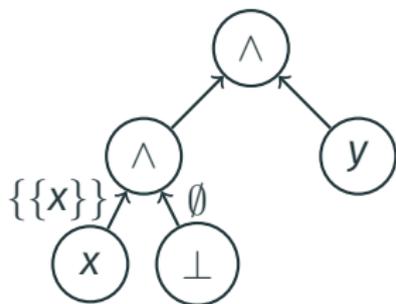
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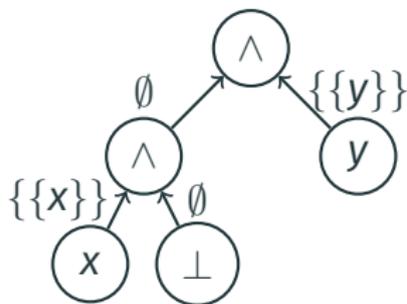
## Normalization: handling $\emptyset$



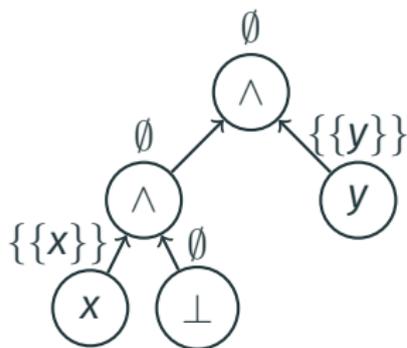
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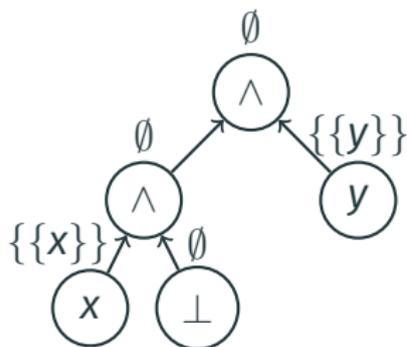
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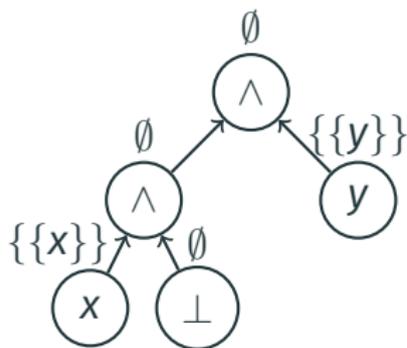


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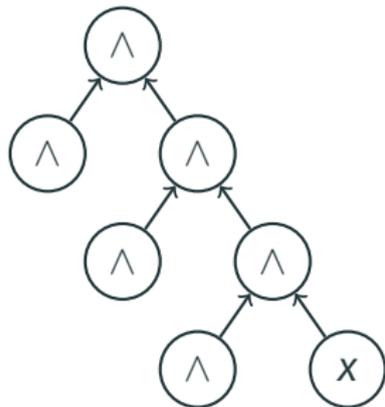
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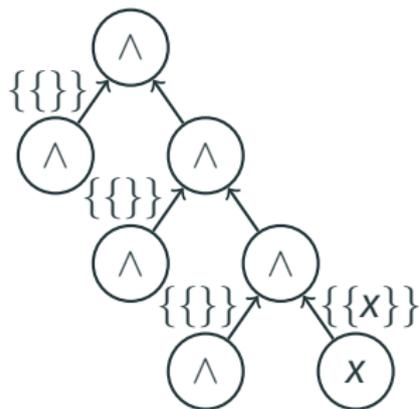


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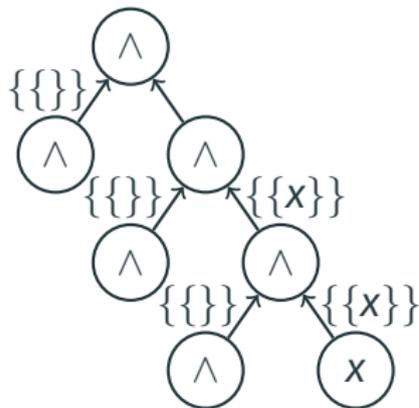
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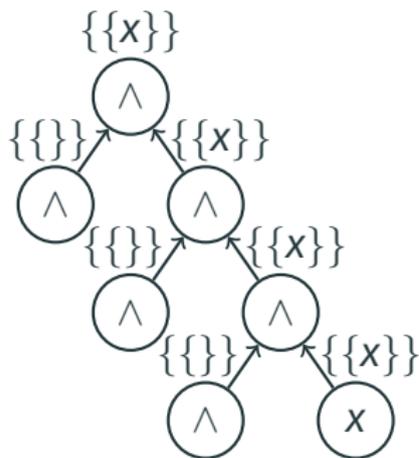


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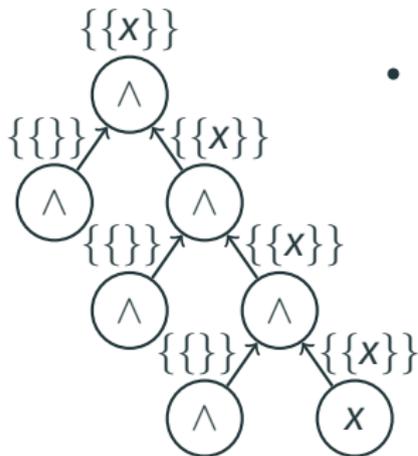




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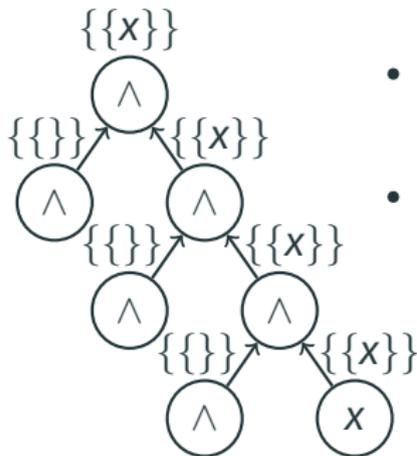


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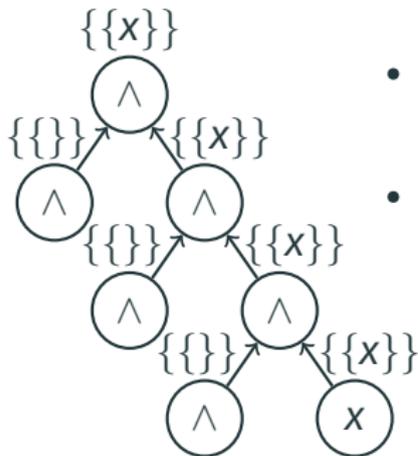
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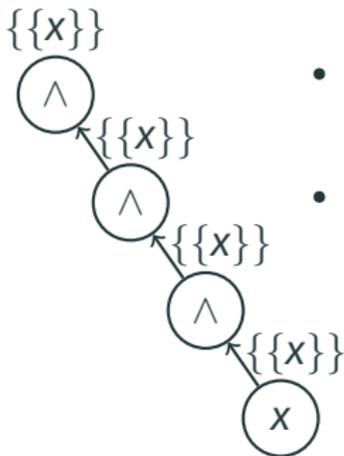


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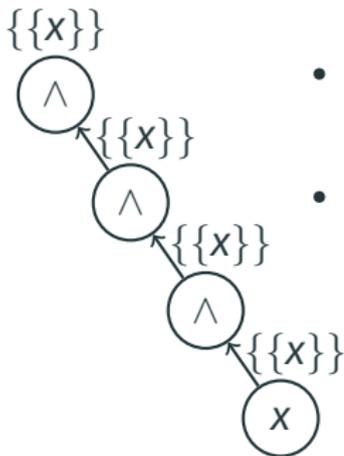
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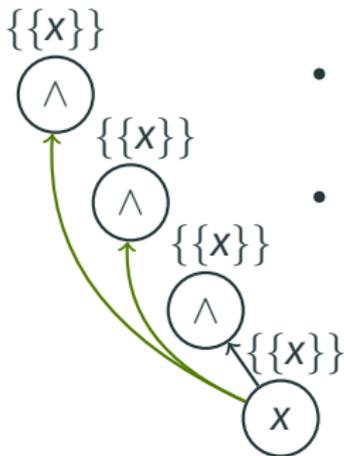
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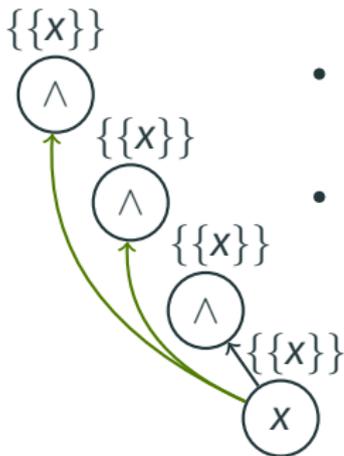
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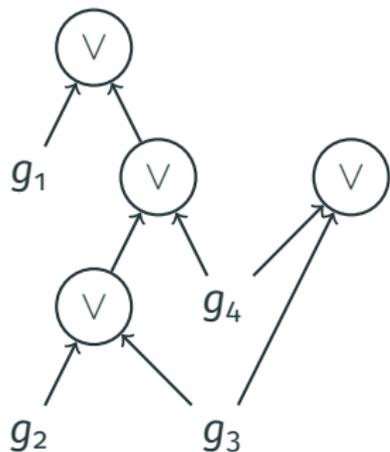
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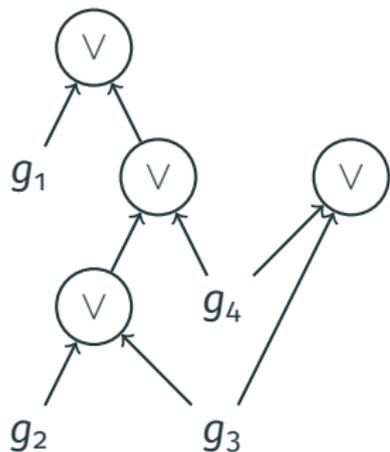
→ Now, traversing an **AND-gate** ensures that we make progress: it **splits** the assignments non-trivially

## Normalization: handling OR-hierarchies



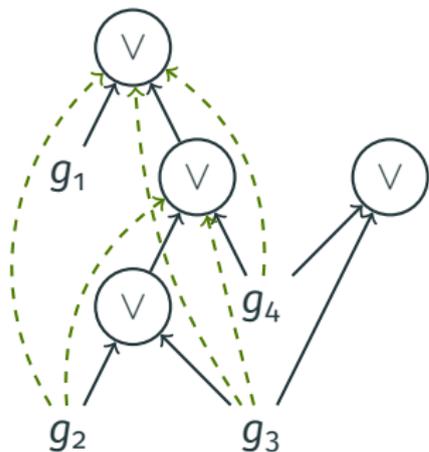
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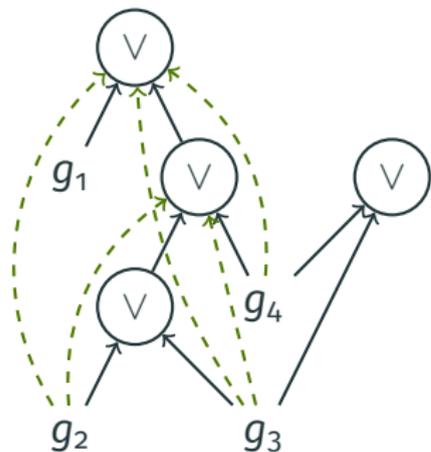
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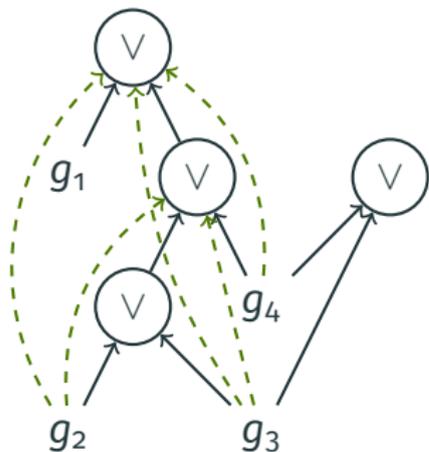
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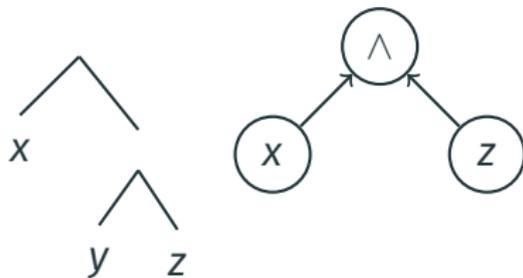
## Solution:

- **Determinism** ensures we have a **multitree** (we cannot have the pattern at the right)
- **Custom** constant-delay reachability index for multitrees



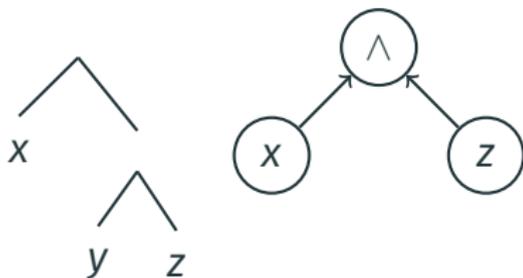
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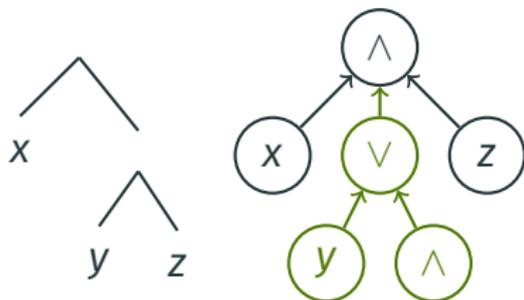
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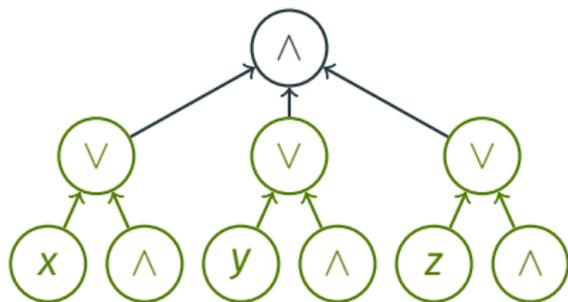
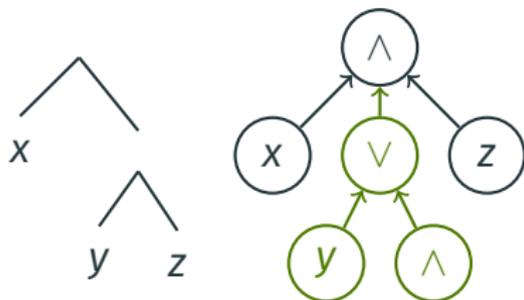
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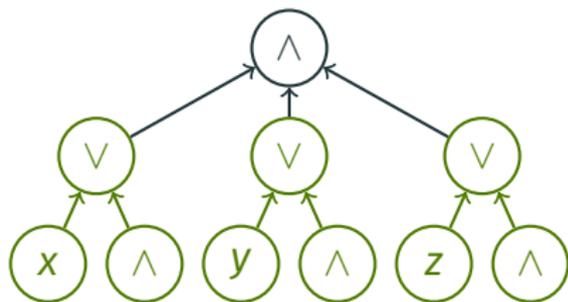
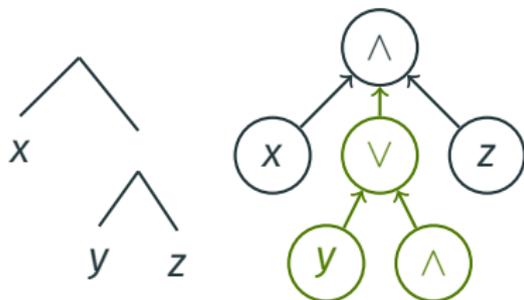
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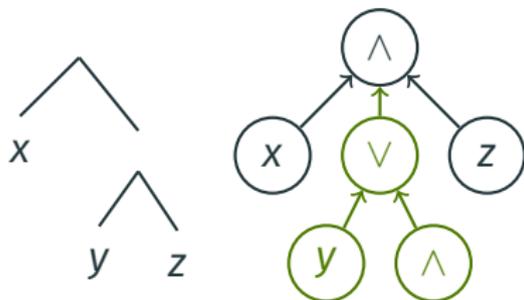
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Computer Science > Databases

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(Submitted on 18 Sep 2017)

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Thanks for your attention!

## References

-  Bagan, G. (2006).  
**MSO queries on tree decomposable structures are computable with linear delay.**  
In *CSL*.
-  Kazana, W. and Segoufin, L. (2013).  
**Enumeration of monadic second-order queries on trees.**  
*TOCL*, 14(4).
-  Olteanu, D. and Závodný, J. (2015).  
**Size bounds for factorised representations of query results.**  
*TODS*, 40(1).