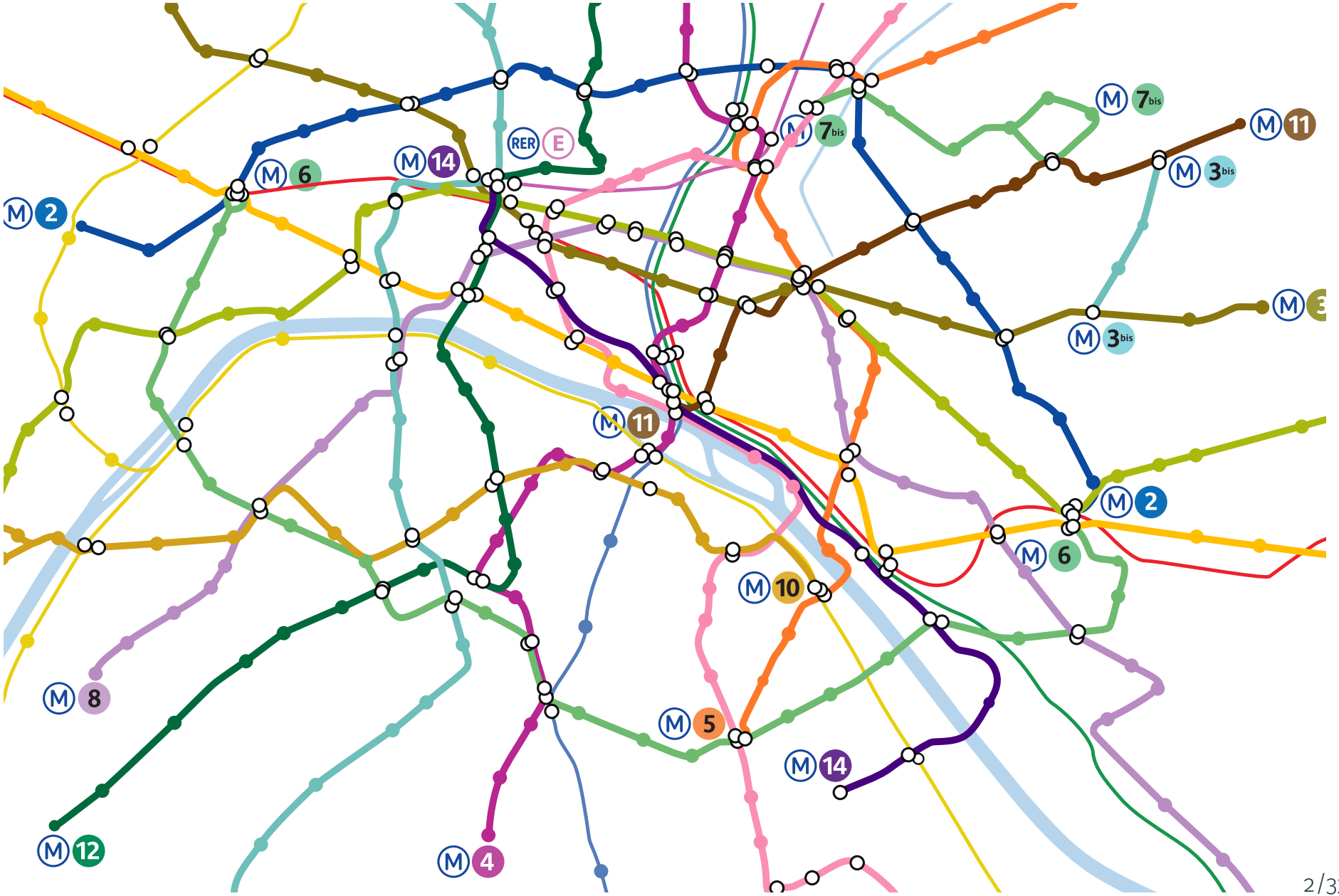
Leveraging the structure of uncertain data

Antoine Amarilli

May 16, 2018

Example application: Subway routing



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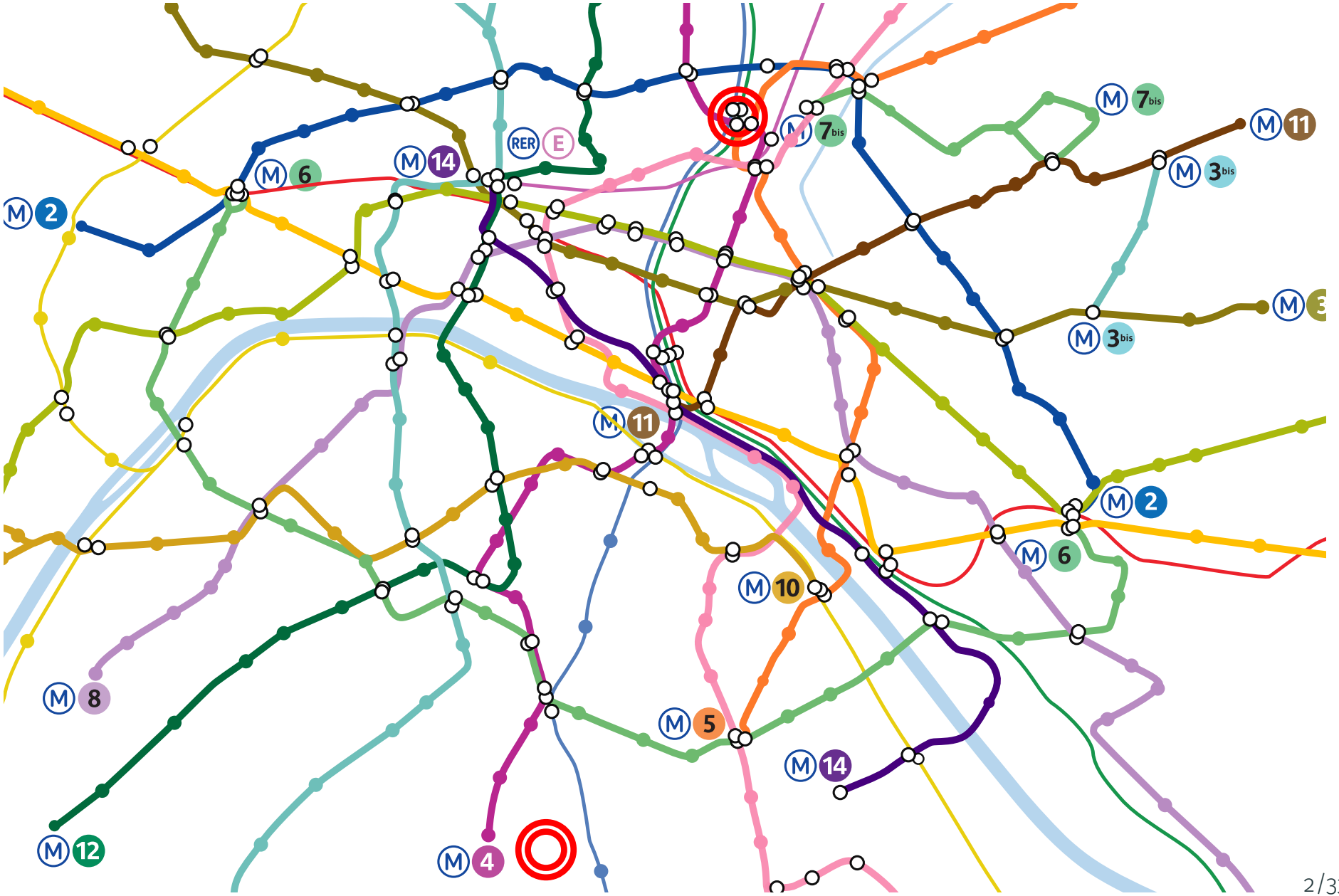

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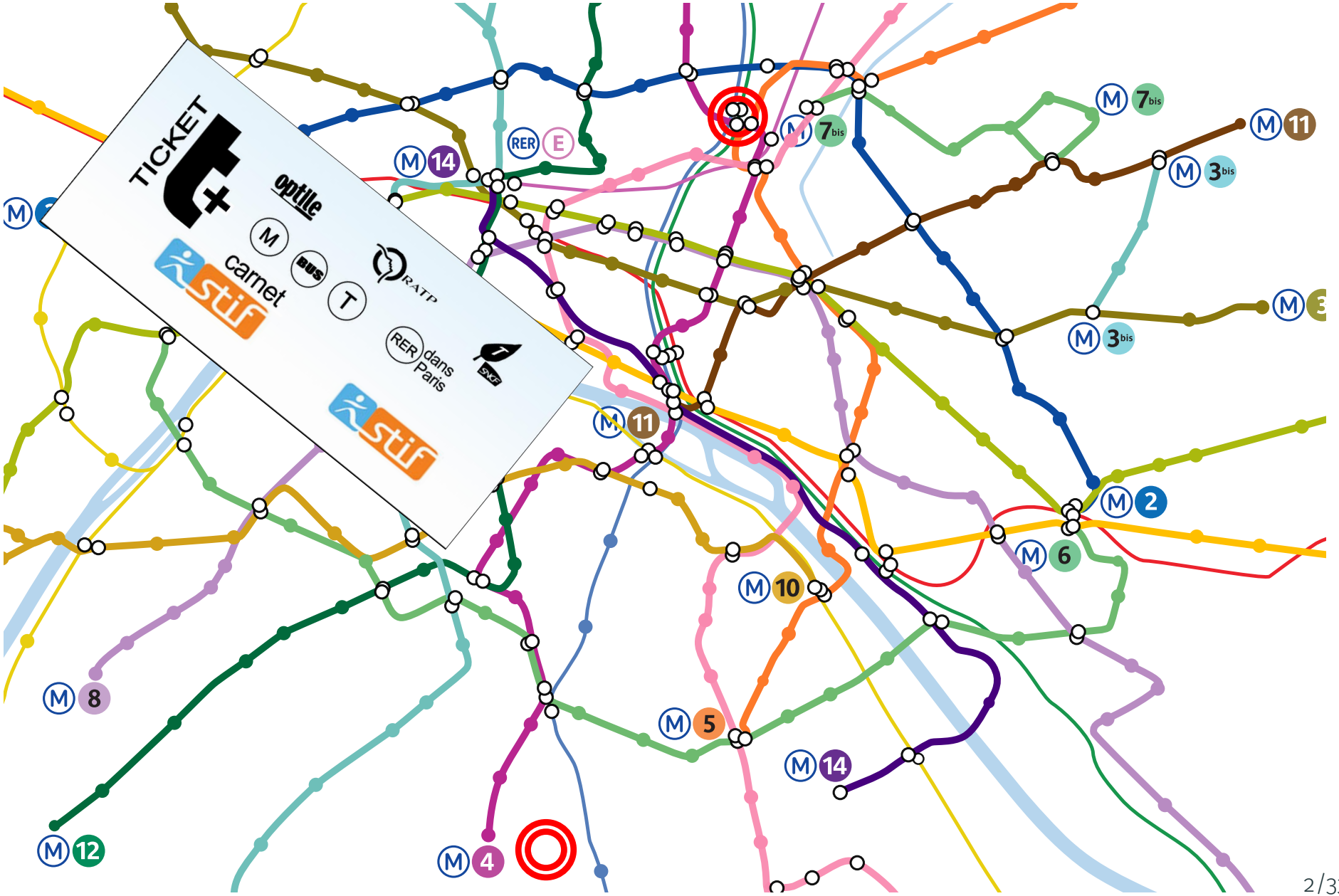

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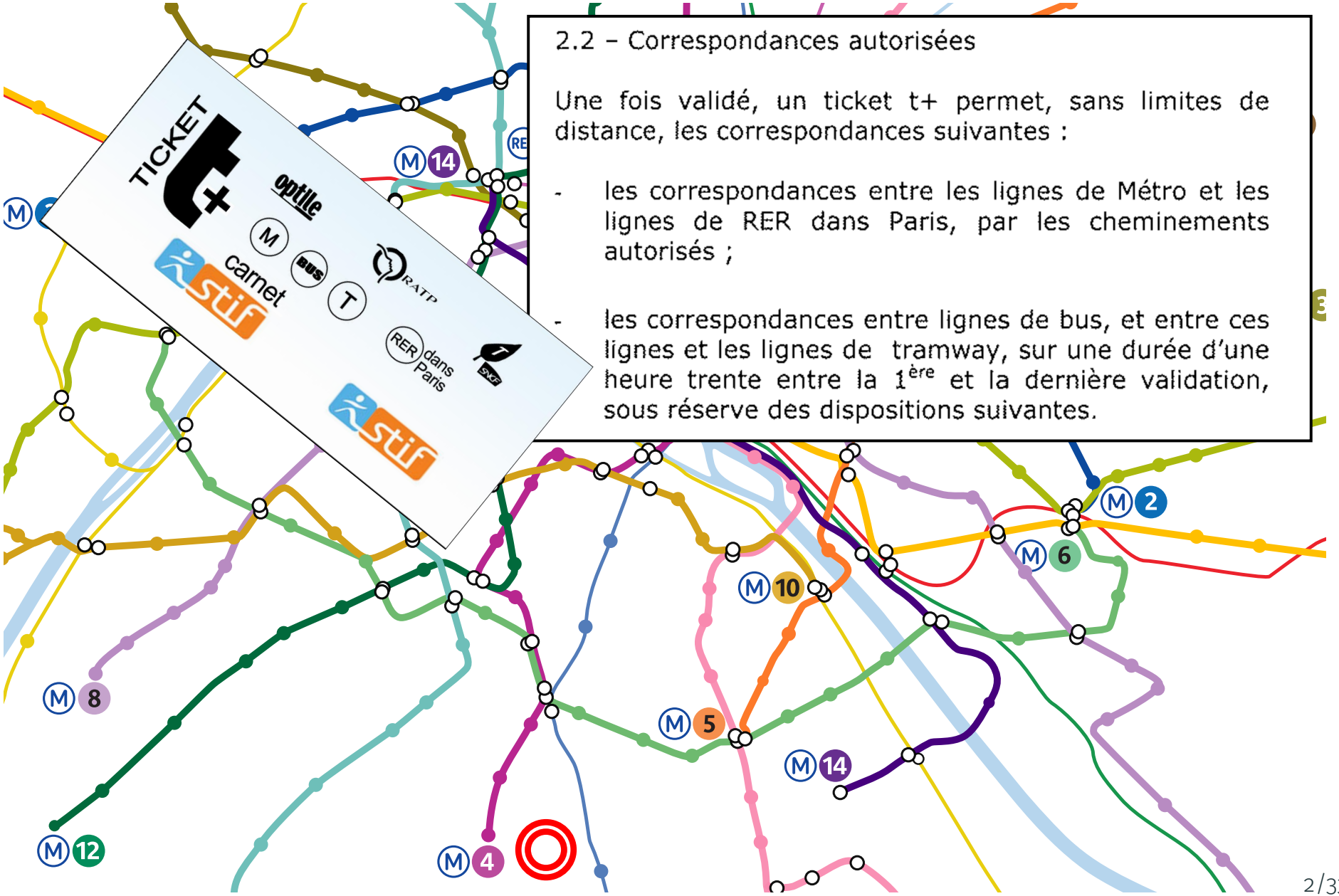

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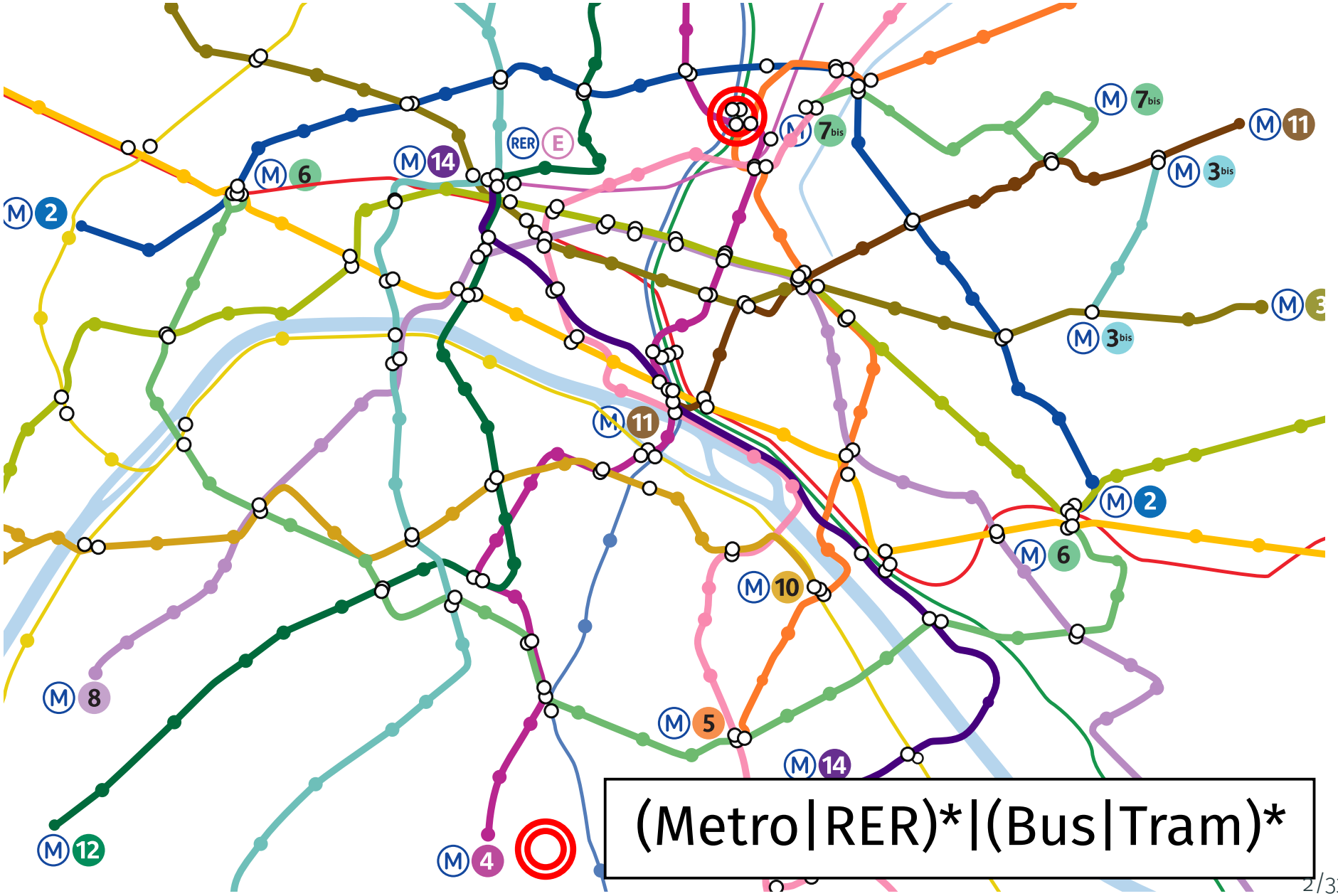
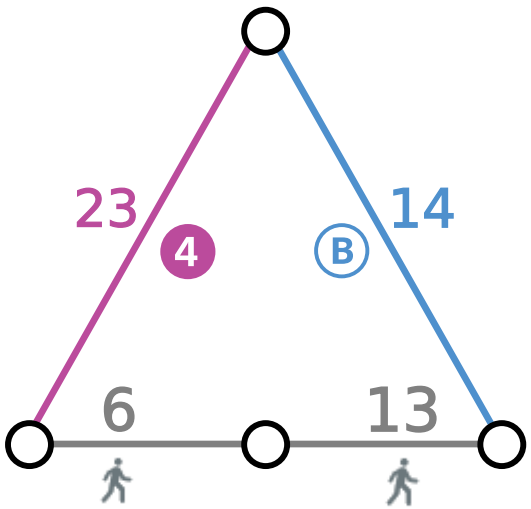


Database

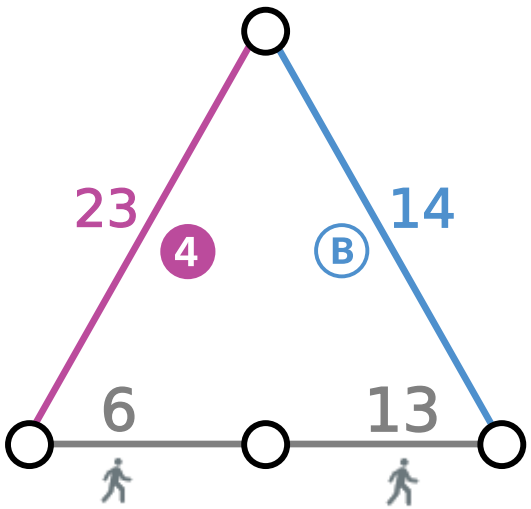


- (Hyper)graph
- Collection of ground facts

$G(aa_1, ab_2)$, $G(ab_2, ac_3)$,
$S(aa_1, m_4)$, $S(ab_2, r_B)$, ...

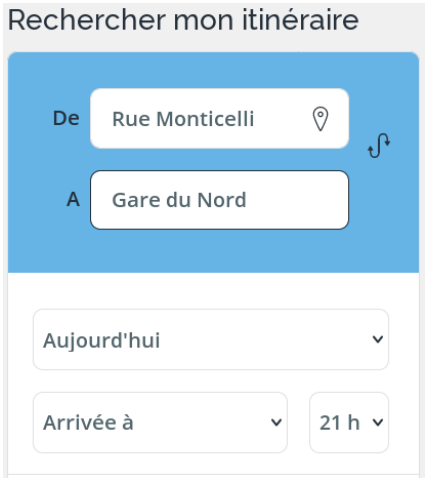
Database theory and query evaluation

Database



+

Query



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  (Metro|RER)*
  |(Bus|Tram)*

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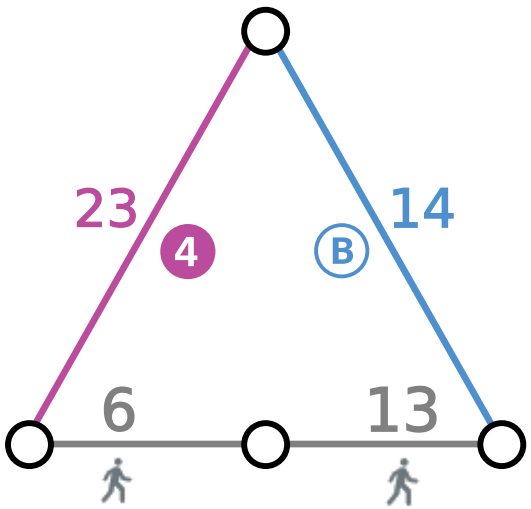

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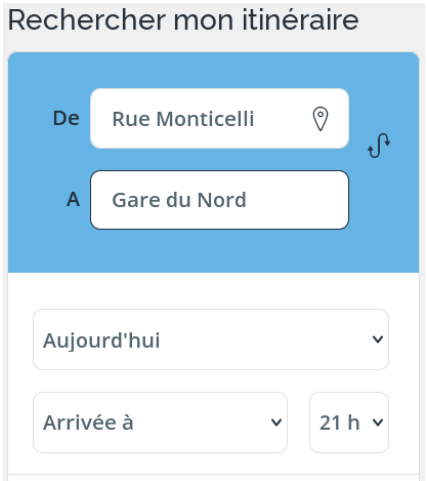

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$\forall X(rm \in X \wedge \forall xy$
$(x \in X \wedge G(x, y) \rightarrow$
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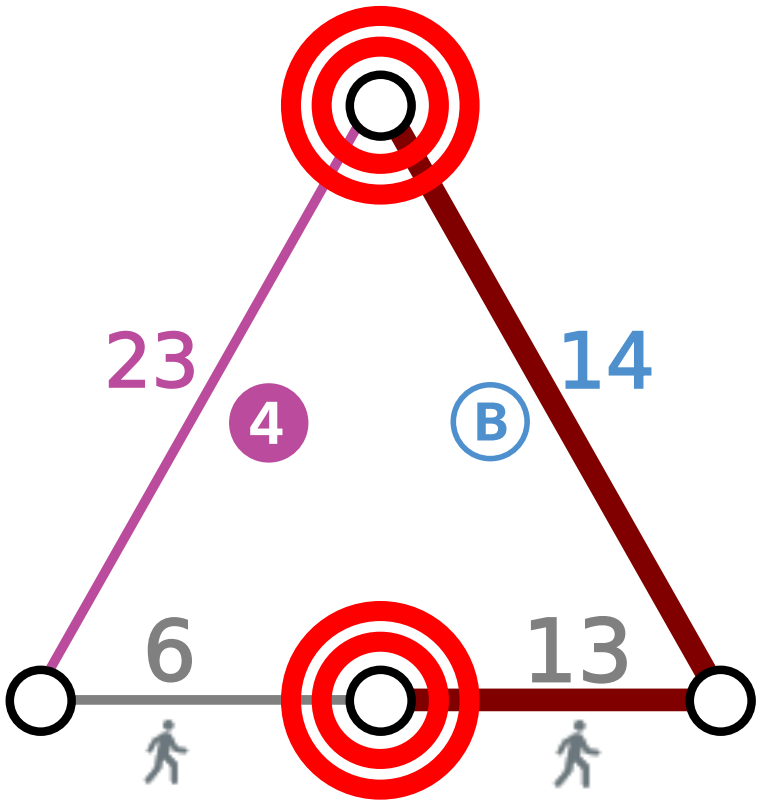
Result

- TRUE/FALSE

↔ Model checking

Probabilistic query evaluation



Panne du RER B : trafic interrompu entre Paris et Roissy, des TGV en renfort

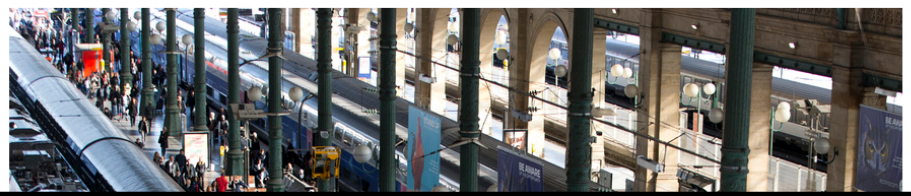
> Transports | 06 décembre 2016, 9h56 | MAJ : 06 décembre 2016, 17h03 |

Probabilistic query evaluation

Panne du RER B : trafic interrompu entre

Paris : pourquoi il y a autant de perturbations sur le RER B et à Gare du Nord

La circulation de l'ensemble des trains au départ de gare du Nord est totalement interrompue à la suite d'une panne électrique.



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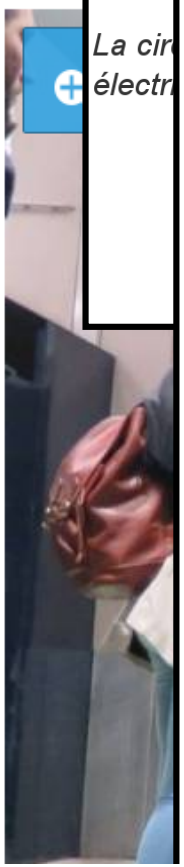

INCIDENT SUR LE RER B : QUE S'EST-IL PASSÉ CE MATIN ?

Malaise voyageur et application des mesures de sécurité : pour quelles raisons le trafic a-t-il été perturbé ce matin sur la ligne B ?

Pour beaucoup, le voyage a été difficile ce matin. Au fil de vos réactions sur Twitter notamment, je constate que les raisons de ces perturbations ne paraissent pas cohérentes. Je tiens donc à vous apporter des premiers

Probabilistic query evaluation

Panne du RER B : trafic interrompu entre

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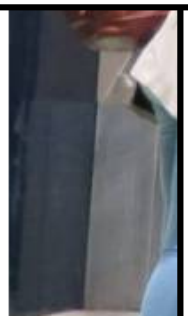
INCIDENT SUR LE RER B · QUE

ACTUALITÉS

Le RER B en panne, les voyageurs n'ont pas eu d'autre choix que de descendre sur les voies

Alors que la circulation alternée a augmenté le nombre de voyageurs dans les transports en commun, le RER B s'est retrouvé à l'arrêt.

06/12/2016 11:57 CET | **Actualisé** 06/12/2016 20:14 CET



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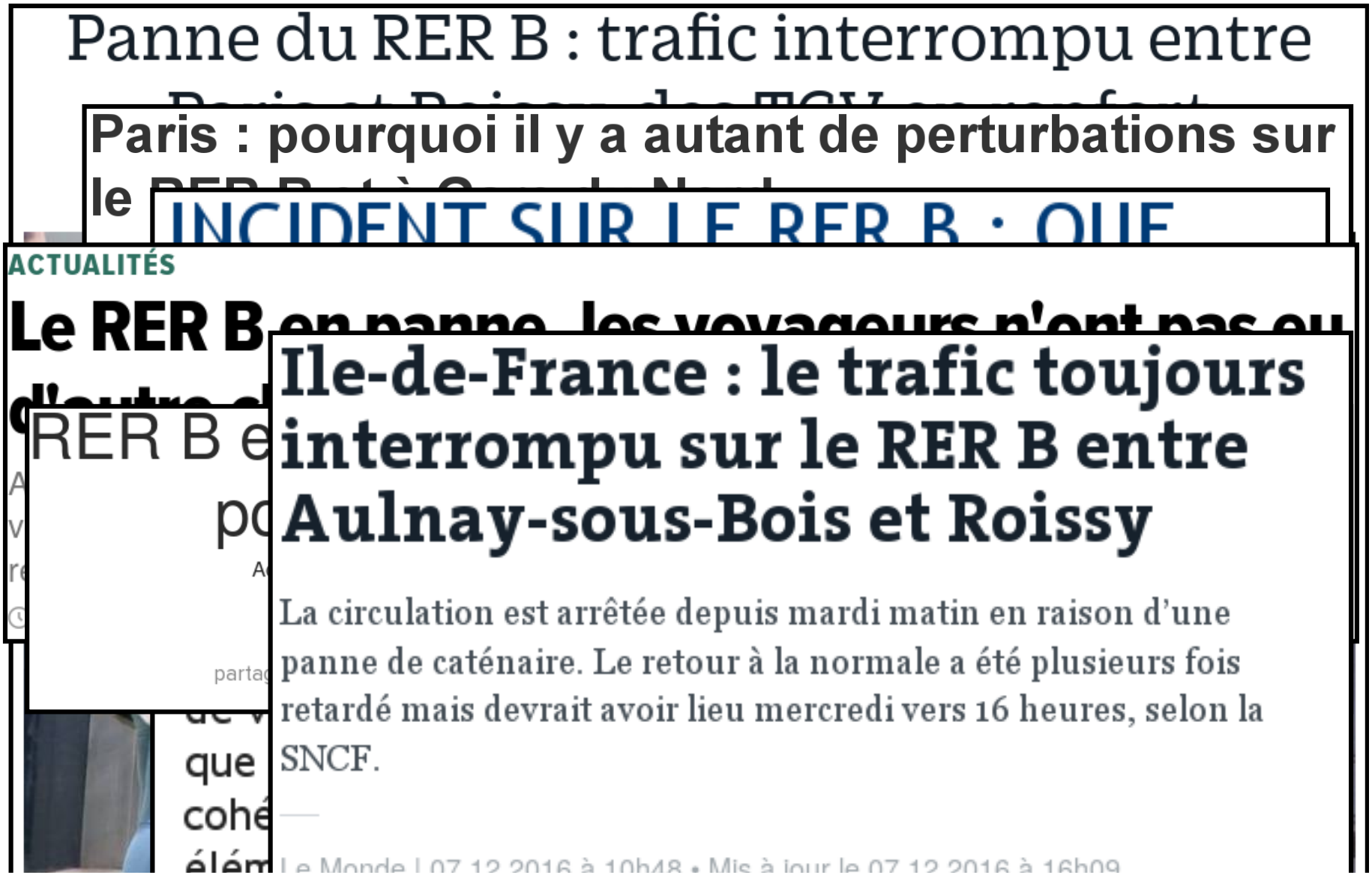

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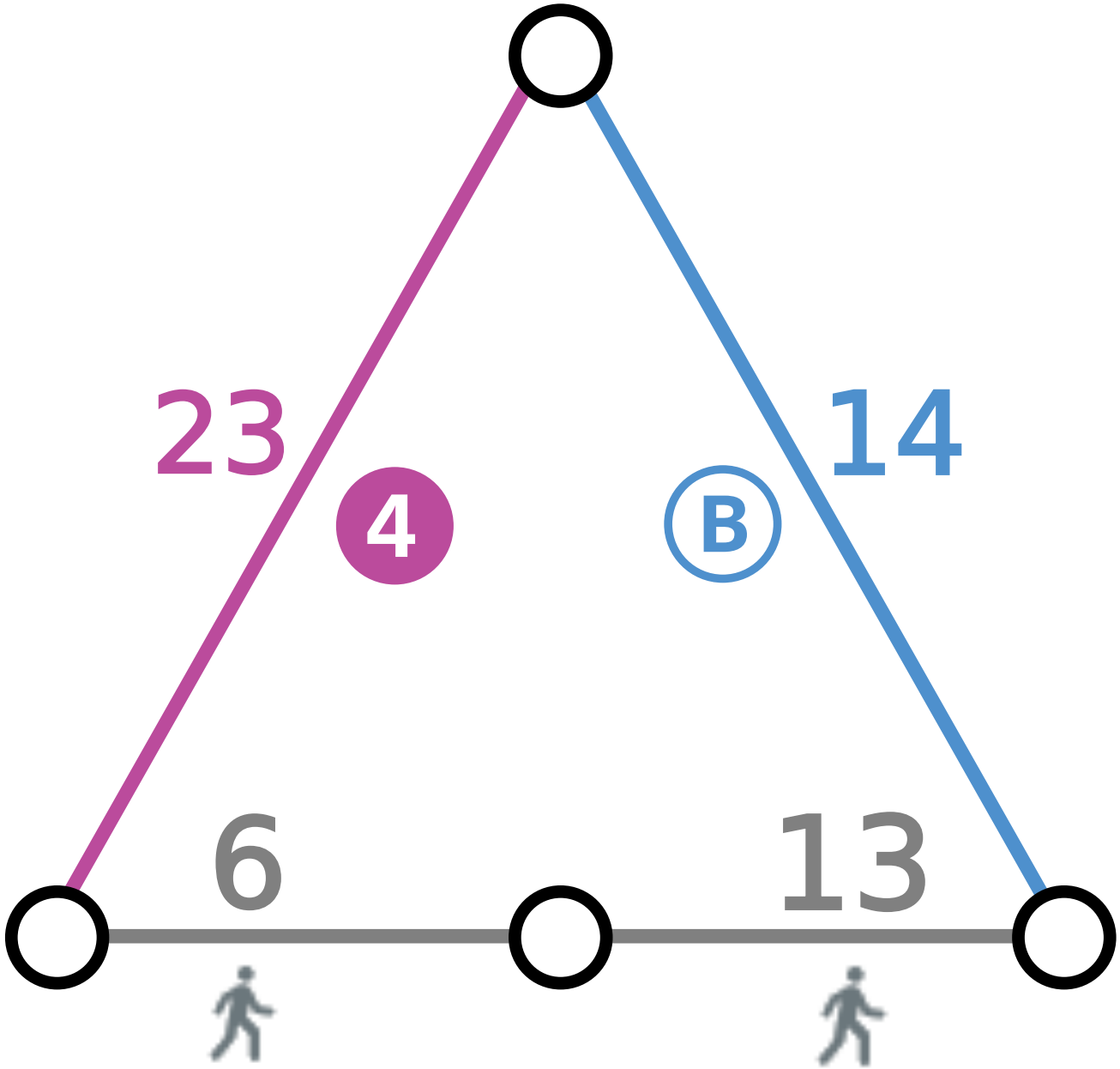

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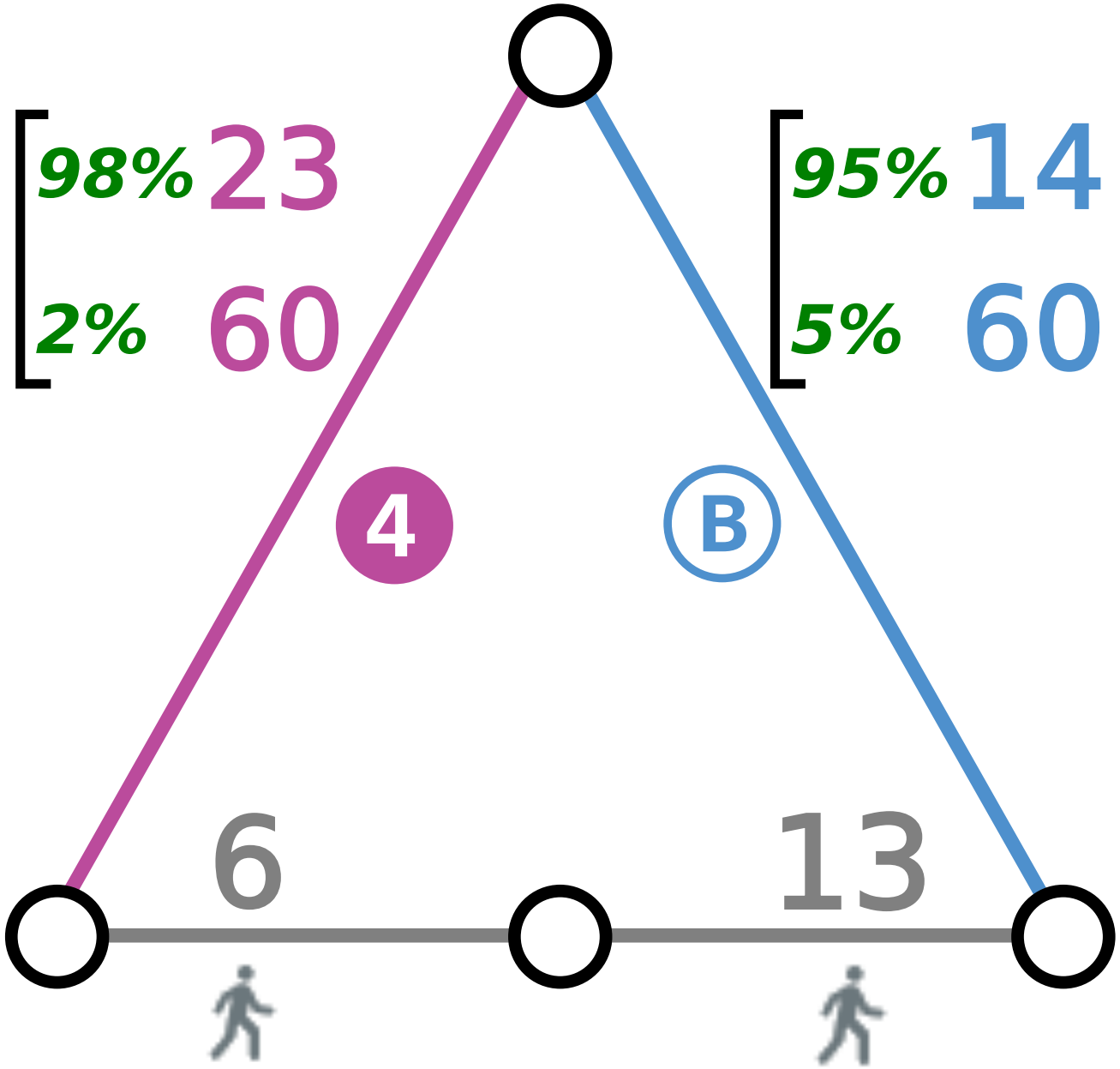
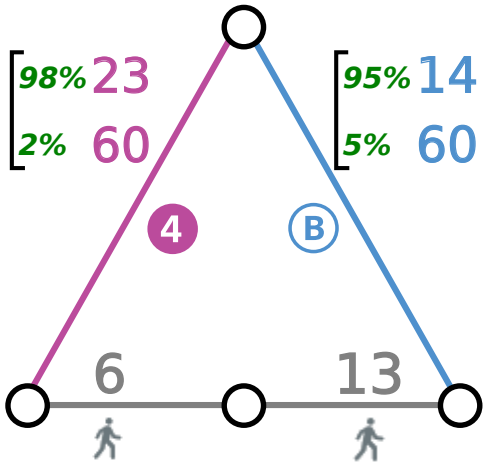


Probabilistic query evaluation

Probabilistic database



- (Hyper)graph
- Collection of ground facts

+ independent probabilities

Probabilistic query evaluation

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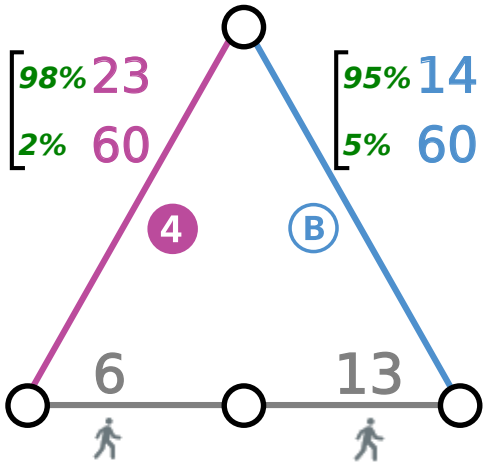

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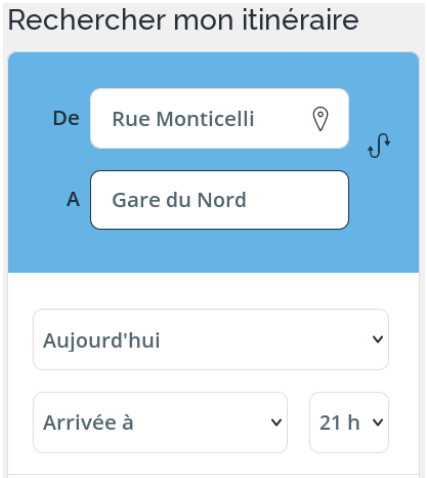
Query



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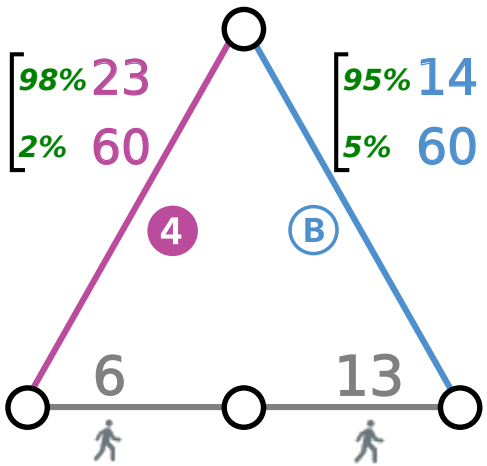

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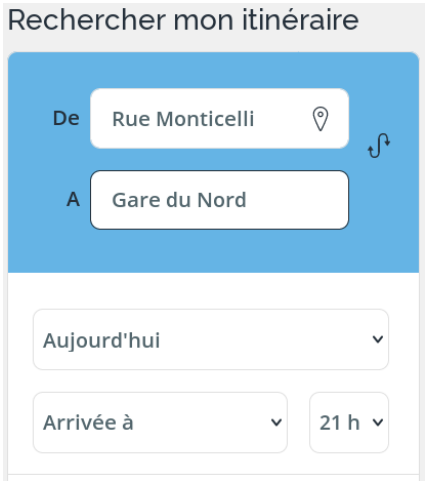

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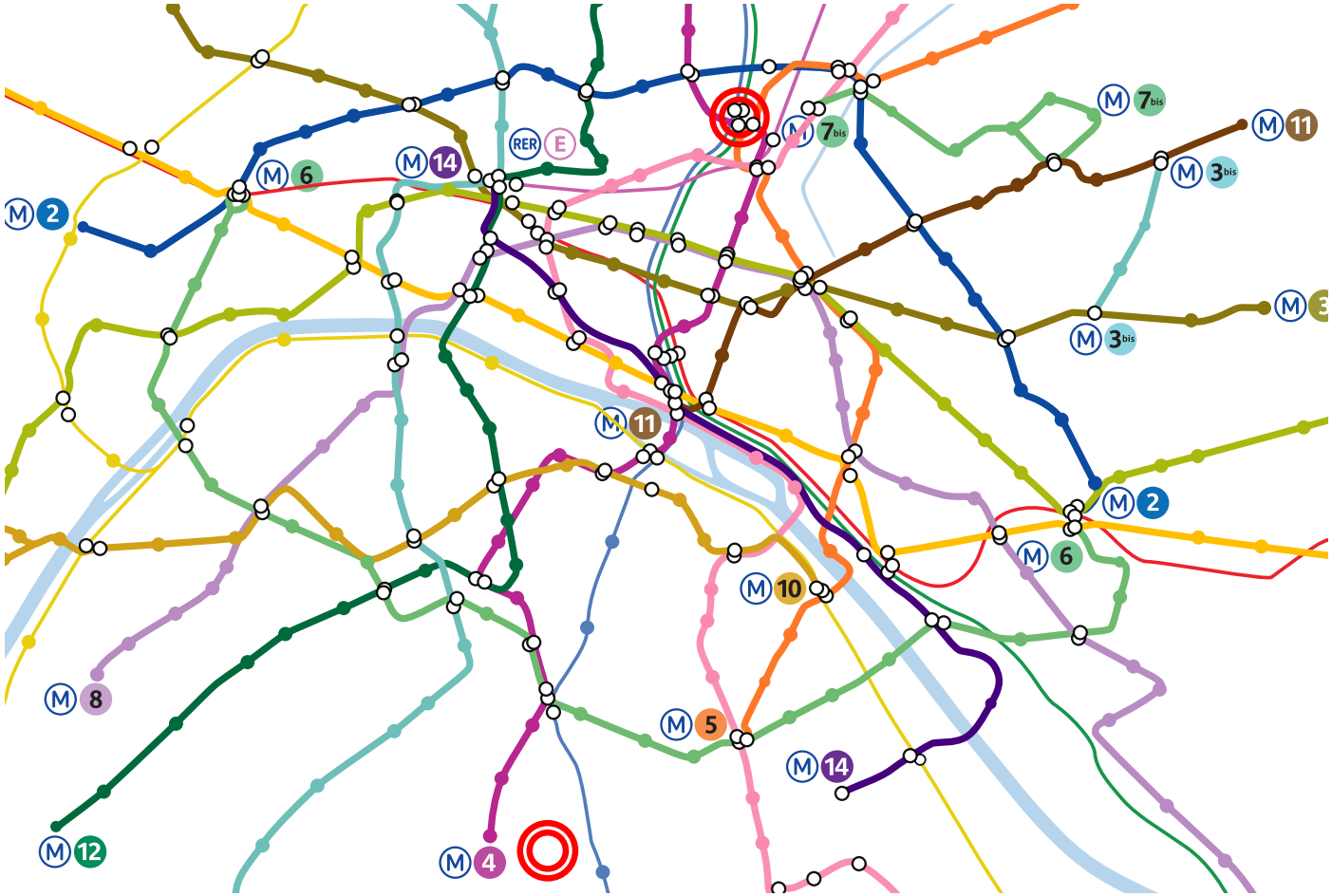
Probabilistic Result

proba to be on time: **98%**

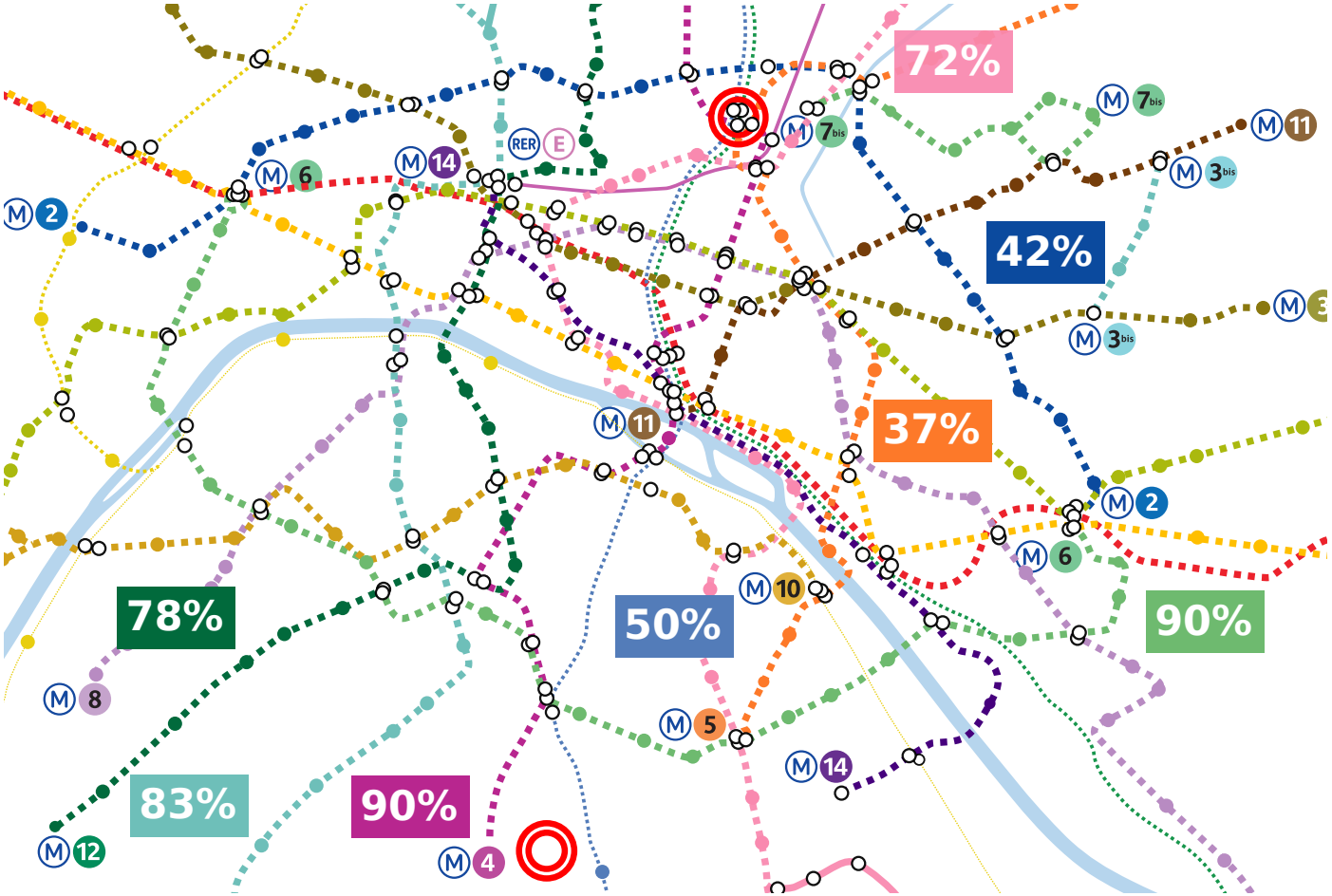
- Probability according to the input distribution

Computational complexity



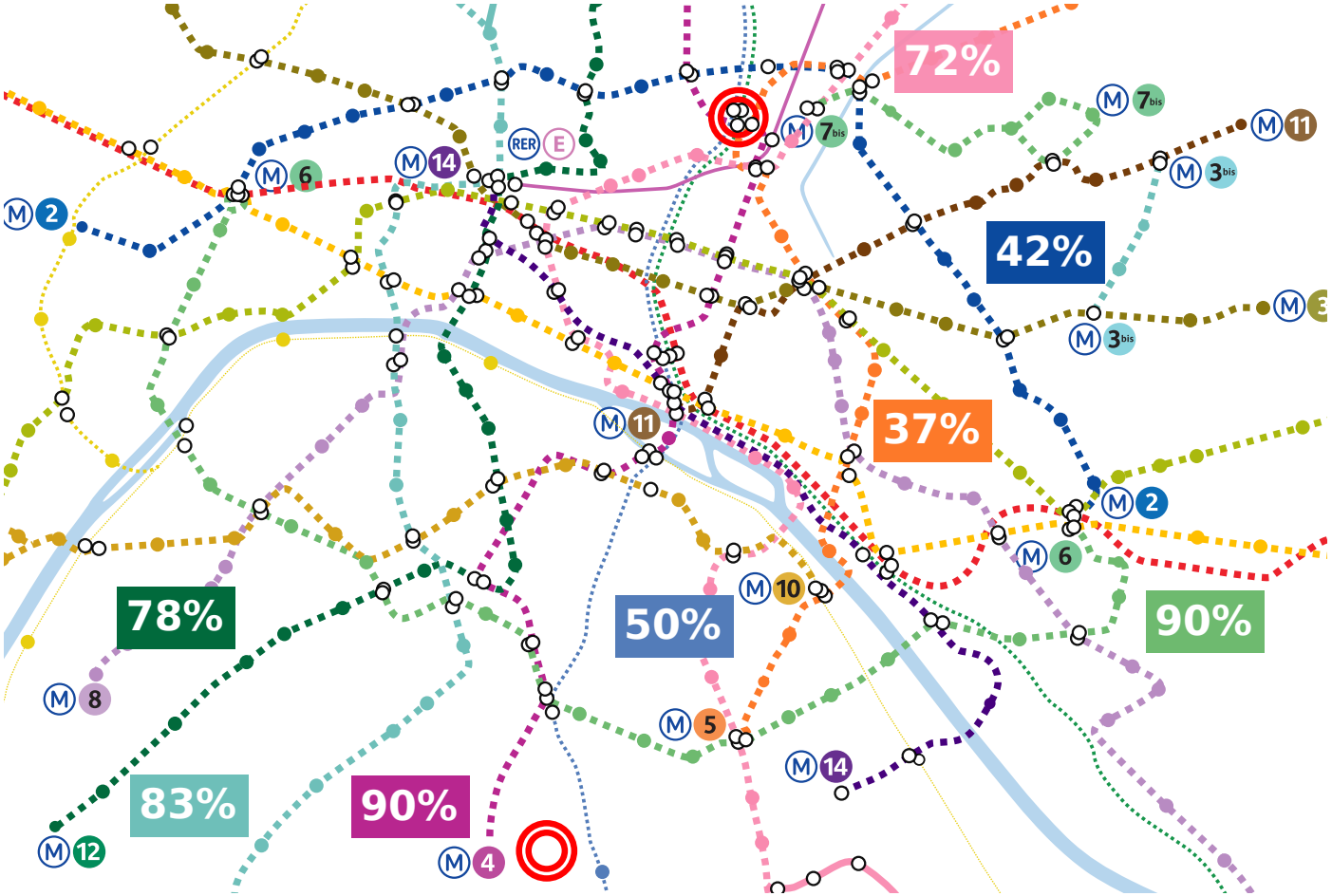
- Computing paths on a large graph:
  - → Well-studied problem, efficient algorithms

Computational complexity



- Computing paths on a large probabilistic graph:
  → ???

Computational complexity



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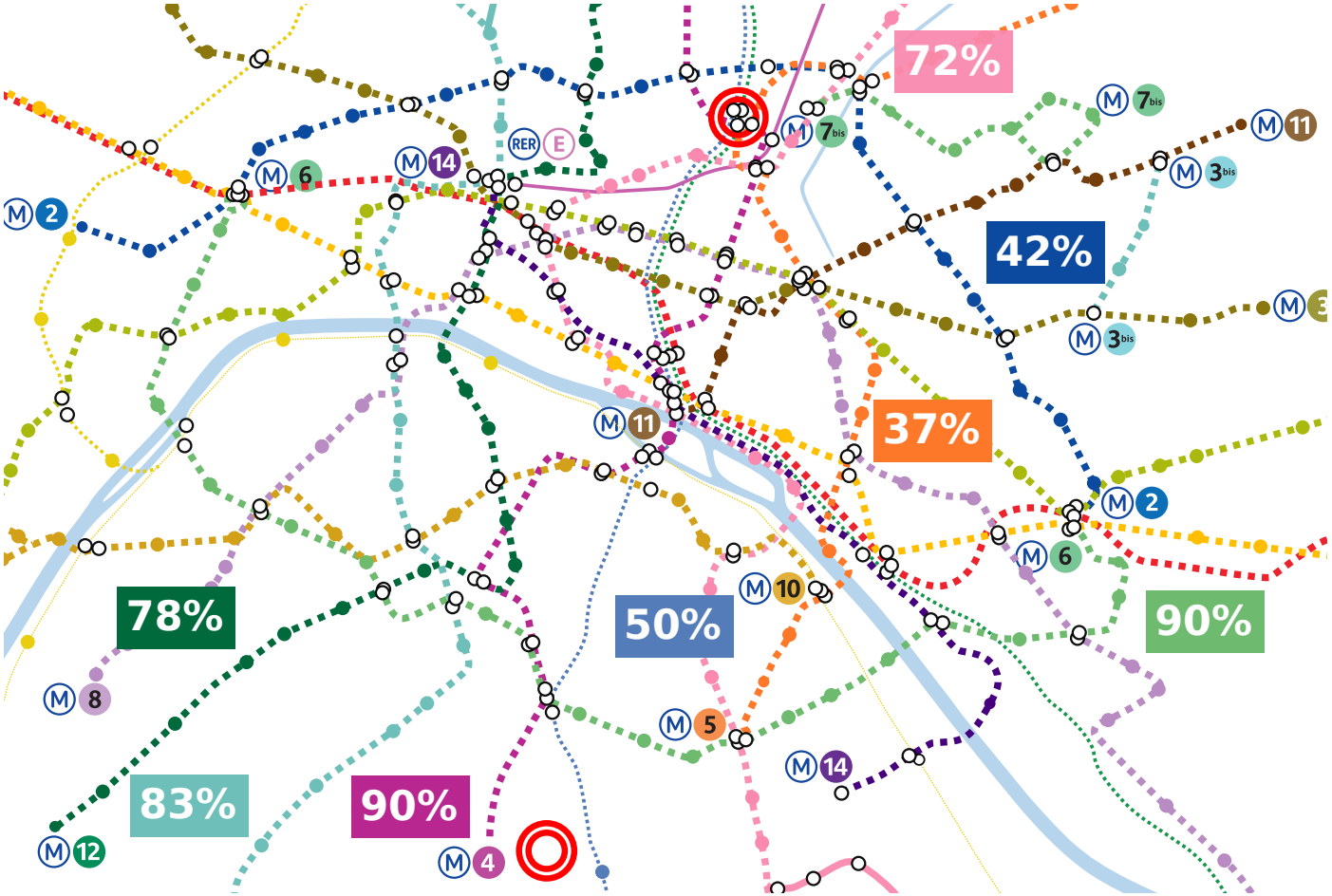
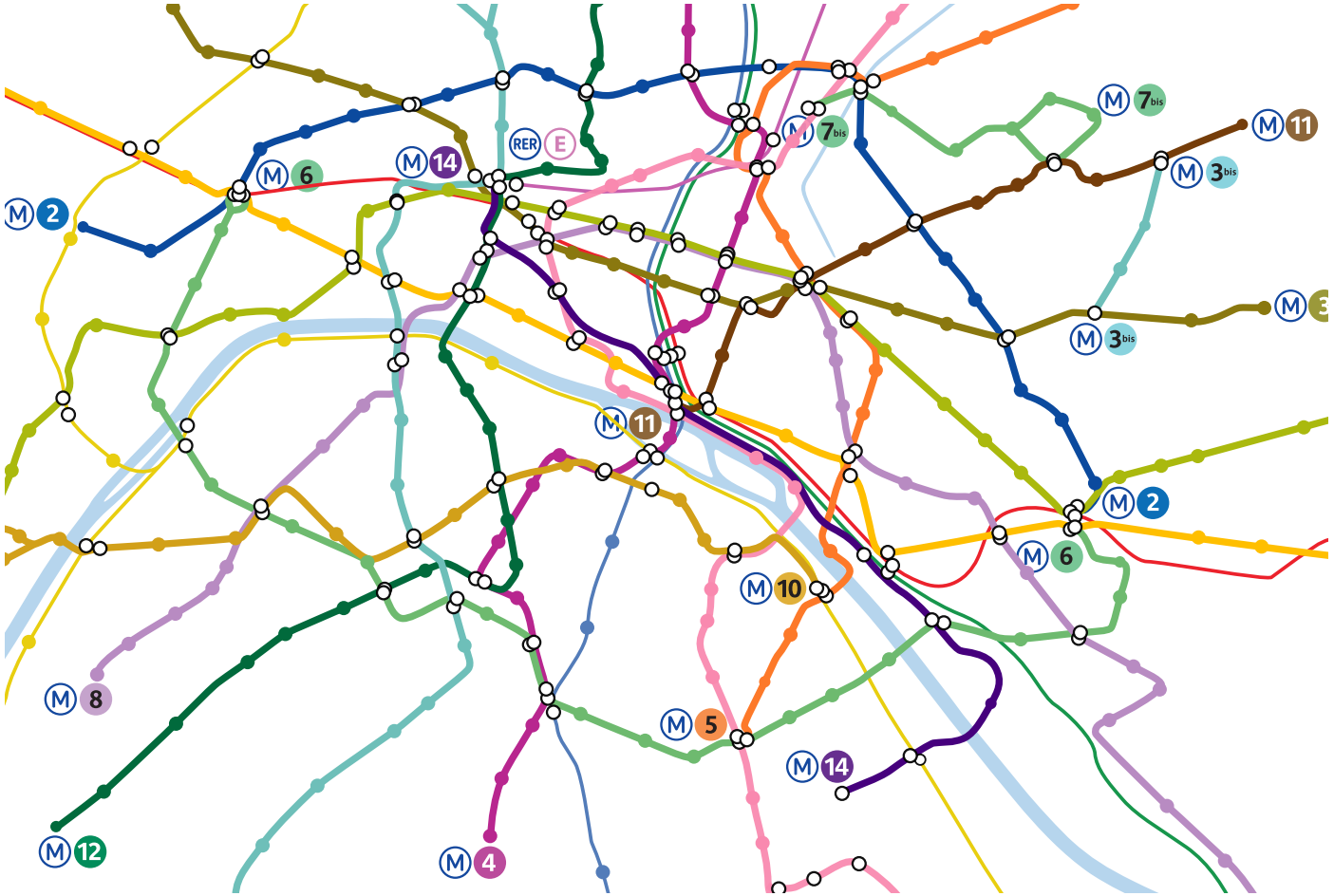


- Computing paths on a large **probabilistic** graph:
  - → **Exponential** number of possibilities
  - → **#P-hard** computational complexity in the **database**

Idea: use the structure of data



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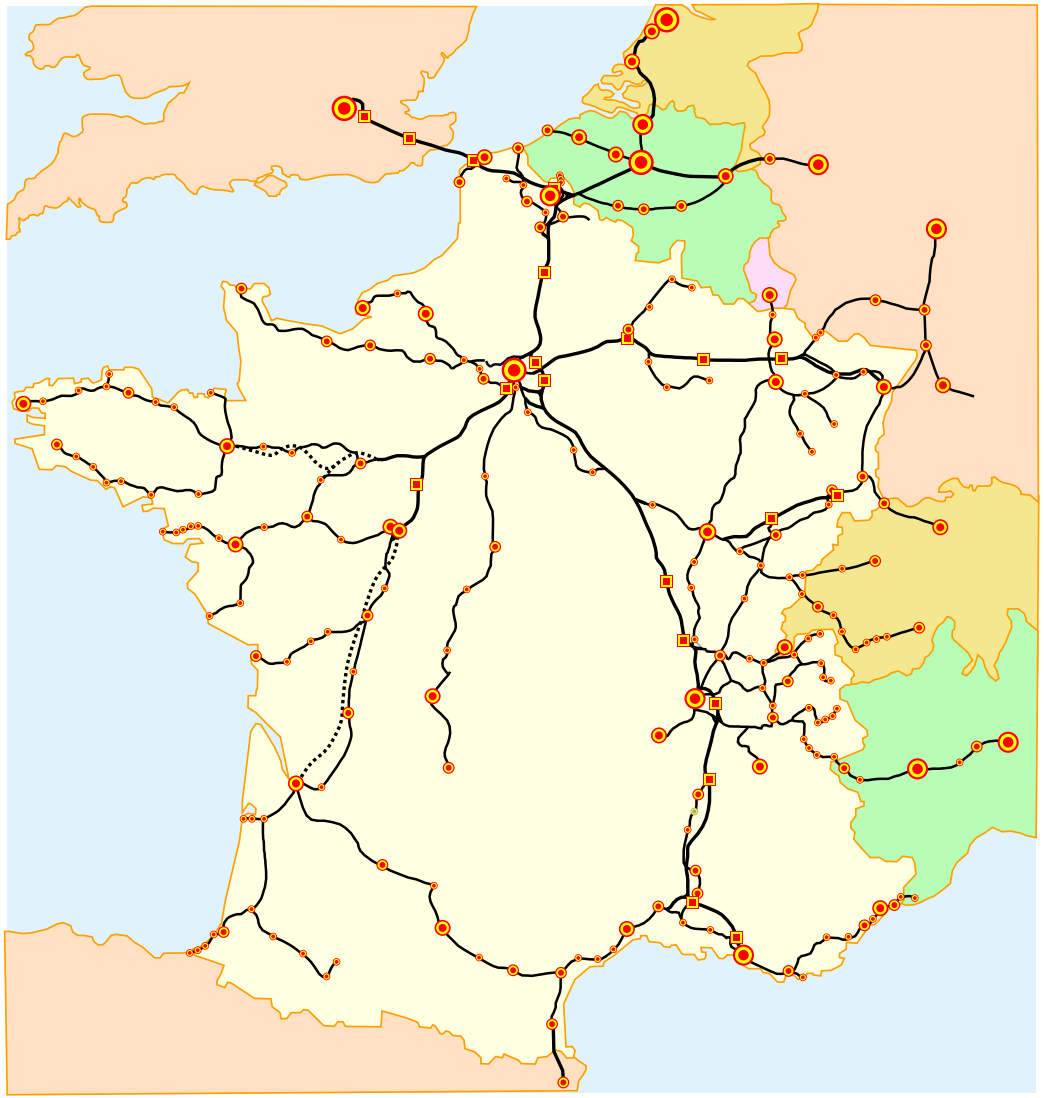

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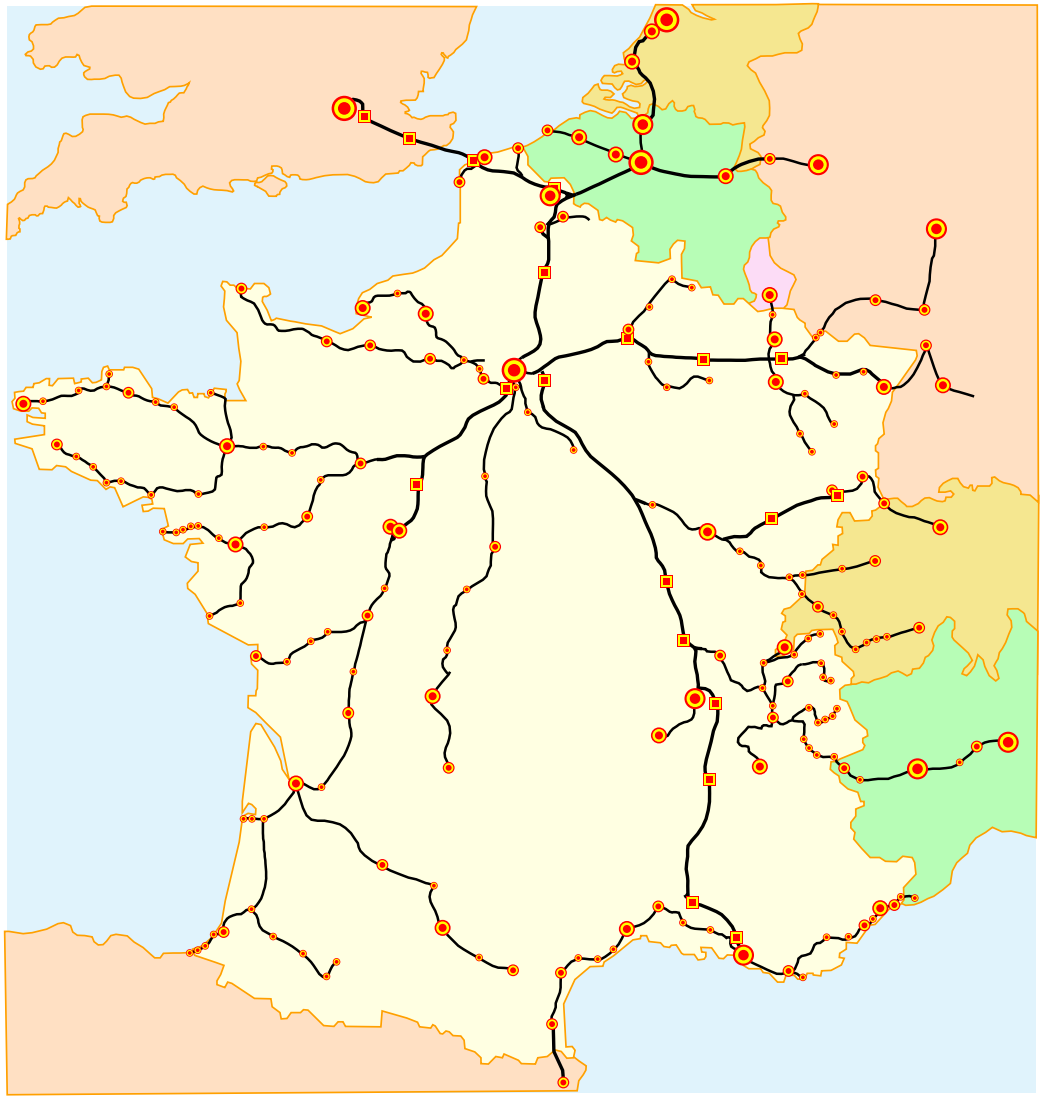

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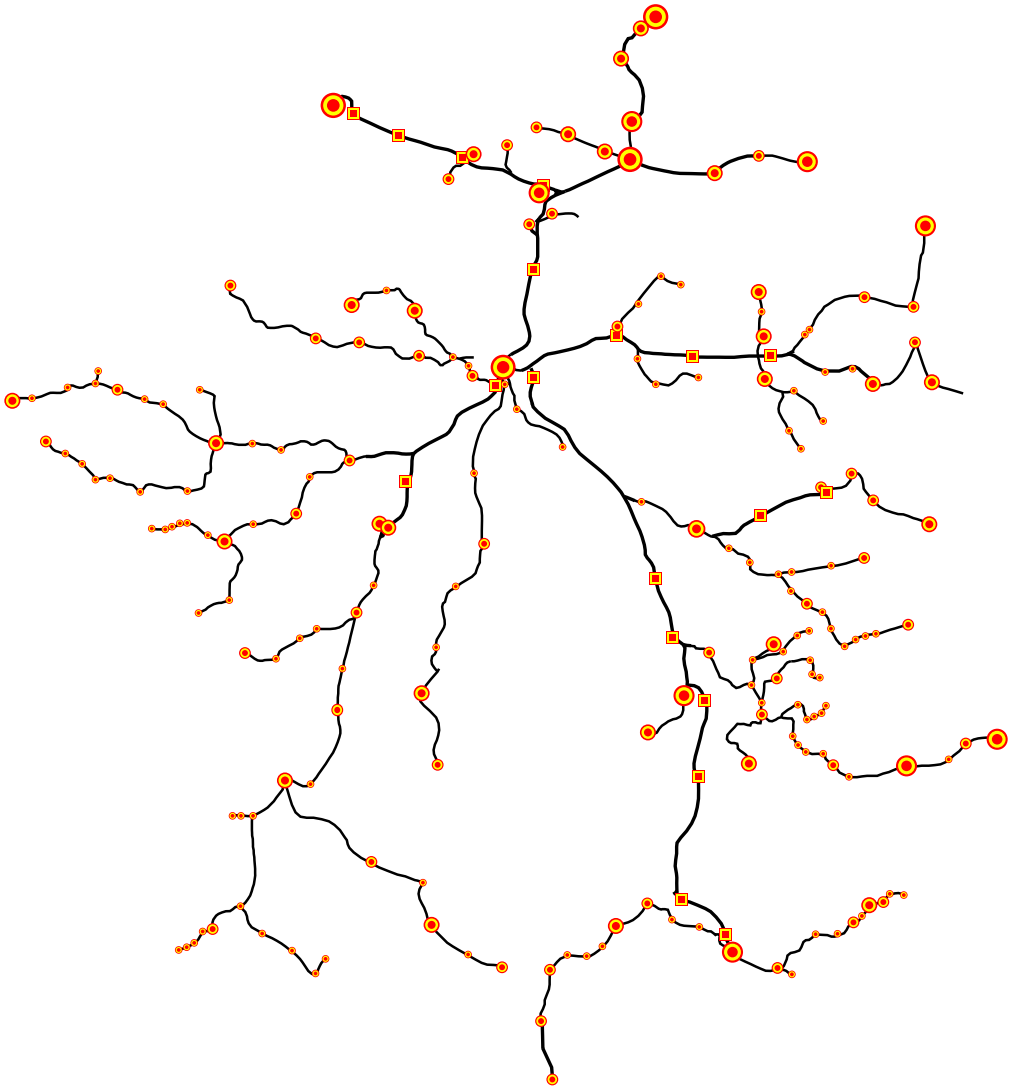

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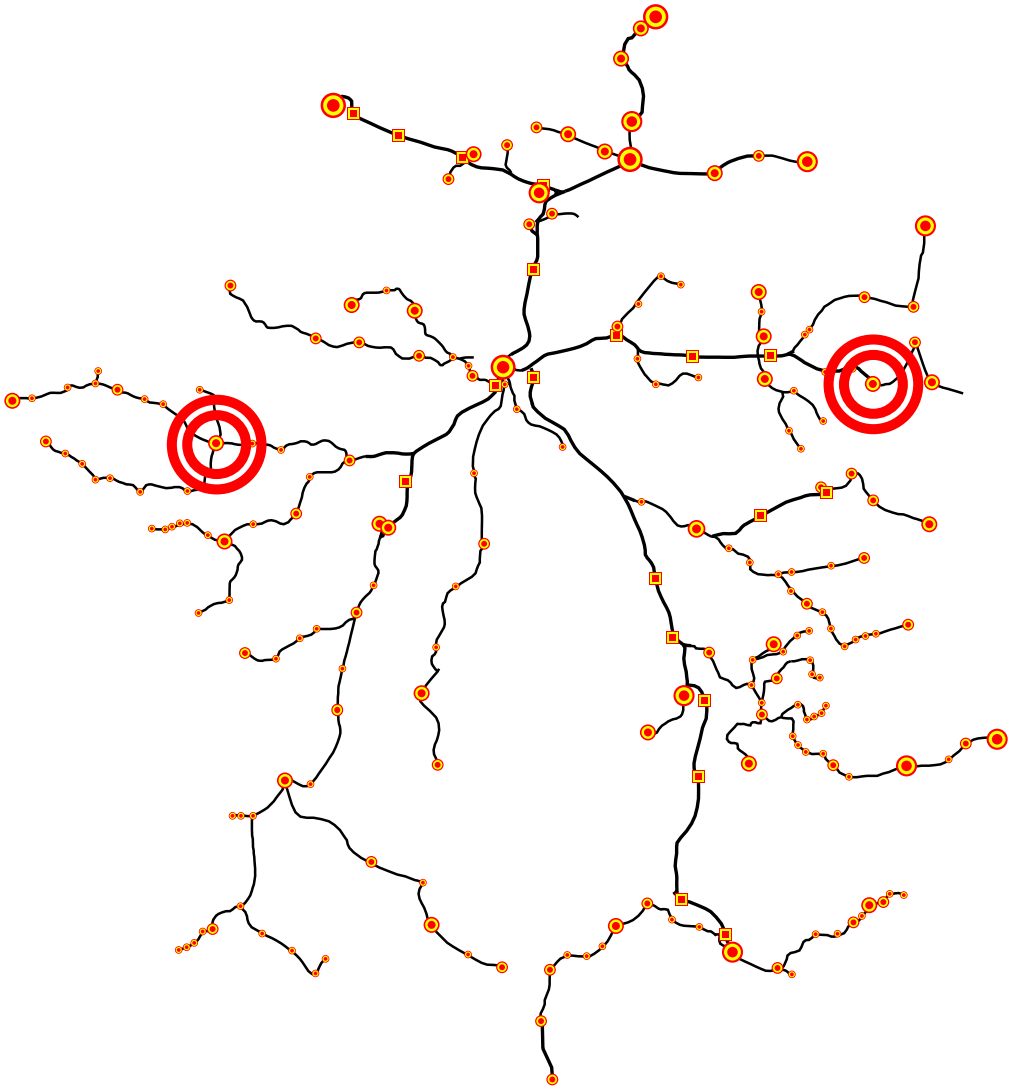

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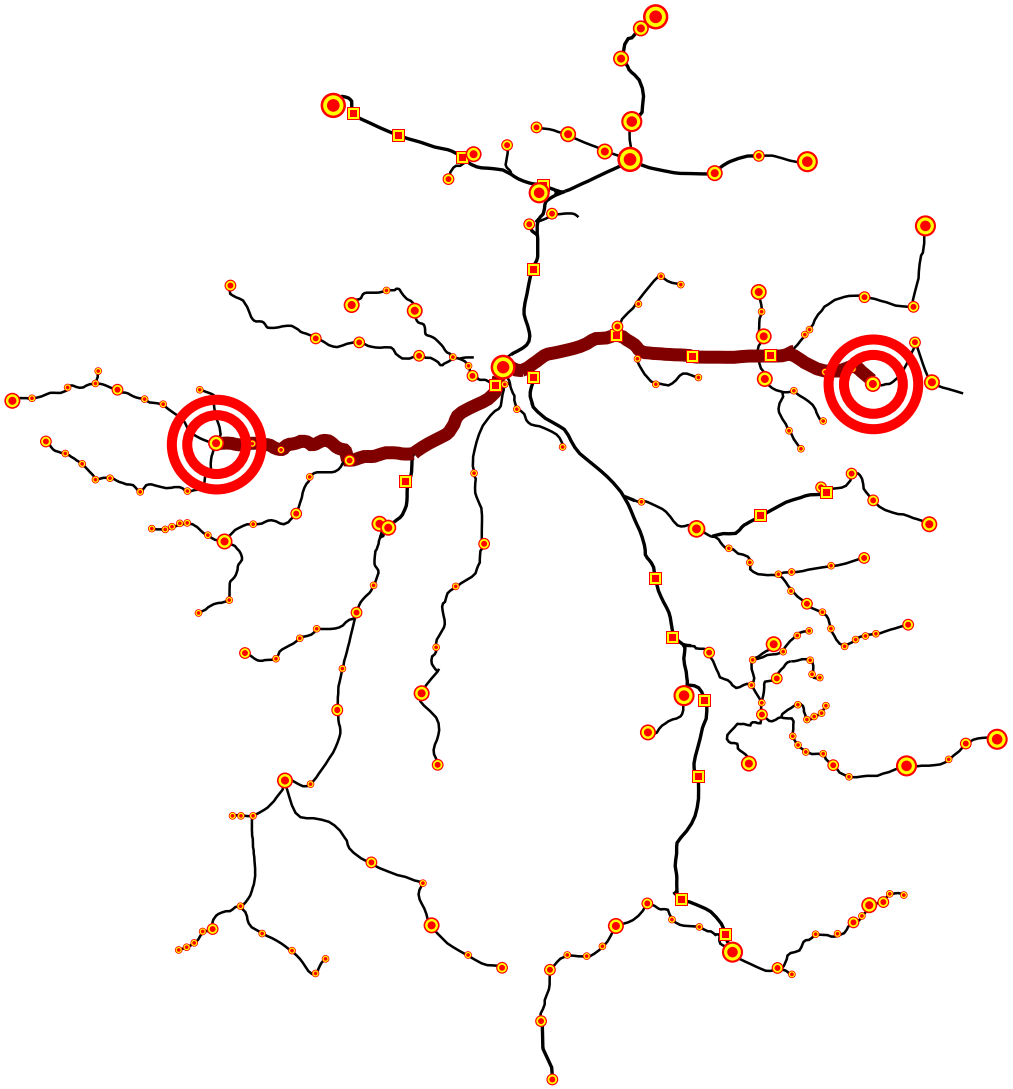

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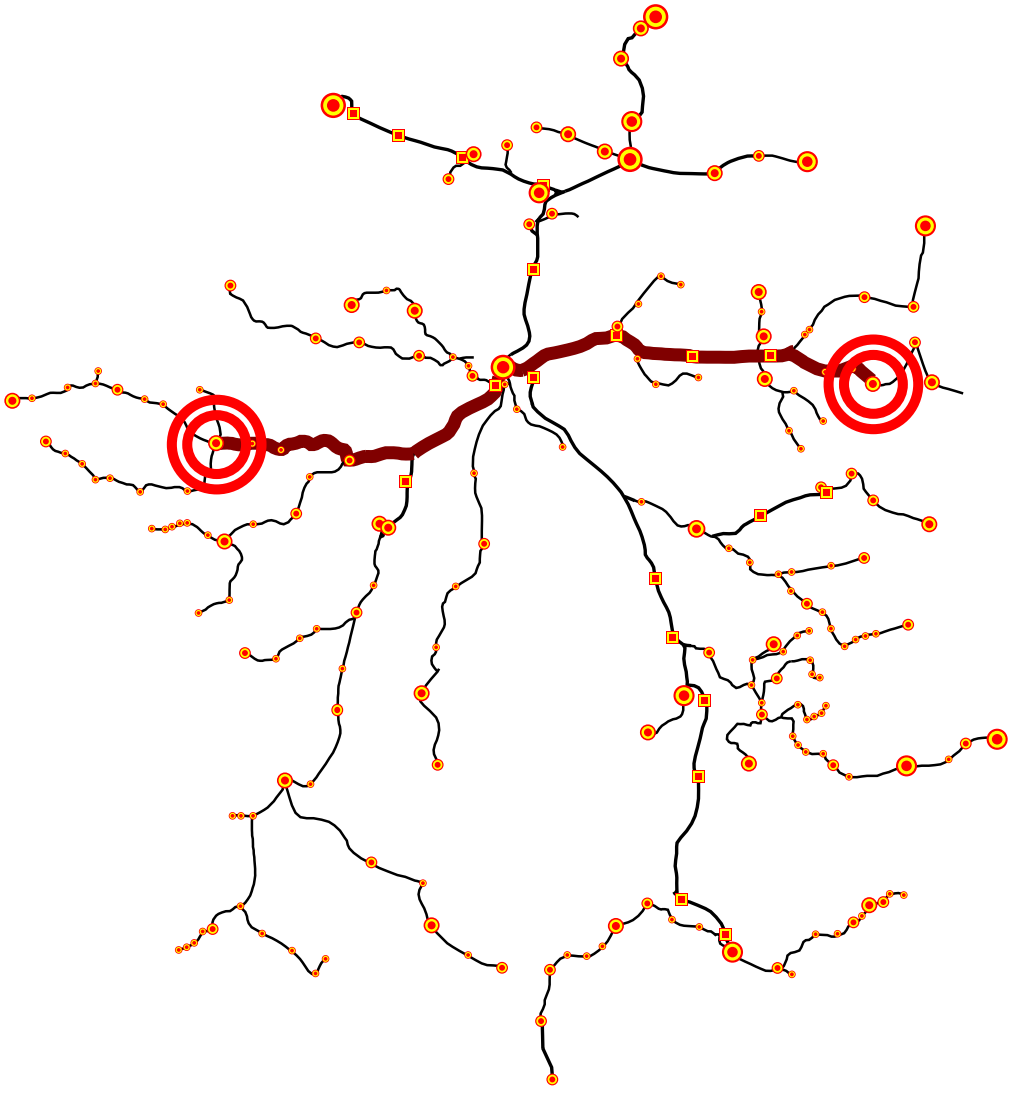

→ Shortest path: very easy on a large tree

Leveraging the structure of uncertain data

Does query evaluation on probabilistic data have ***lower complexity*** *when the* ***structure*** *of the data is* ***close to a tree?***

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- Introduce **new tools** and **results**:

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  - Applications to **probabilistic query evaluation**
- **Other applications:** Counting, enumeration, provenance...

Table of contents

Query evaluation on words

Database: a word w where nodes have a color from an alphabet 

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"Is there both a pink and a blue node?"

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Computational complexity as a function of w
(the query Q is fixed)

Monadic second-order logic (MSO)

- $P_{\bigcirc}(x)$ means "x is blue"; also $P_{\bigcirc}(x)$, $P_{\bigcirc}(x)$
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- First-order logic: adds existential quantifier $\exists$ and universal quantifier $\forall$
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  - $\exists x\, y\; P_{\bullet}(x) \wedge P_{\bullet}(y)$ means *"There is both a pink and a blue node"*

- Monadic second-order logic (MSO): adds quantifiers over sets
  - $\exists S\; \forall x\; S(x)$ means *"there is a set S containing every element x"*
  - Can express transitive closure $x \to^* y$, i.e., *"x is before y"*
  - $\forall x\, P_{\bullet}(x) \Rightarrow \exists y\, P_{\bullet}(y) \wedge x \to^* y$
    means *"There is a blue node after every pink node"*

Word automata

Translate the query Q to a deterministic word automaton

Alphabet: 

w: 

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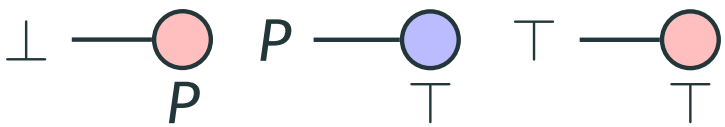
- Transitions (examples): 

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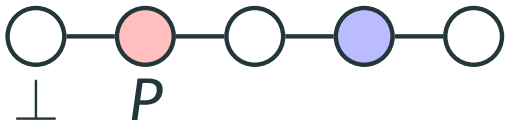
Translate the query Q to a deterministic word automaton

Alphabet:  w: 

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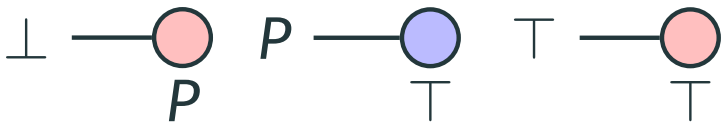
- Transitions (examples): 

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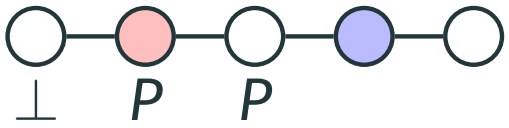
Alphabet:  w: 

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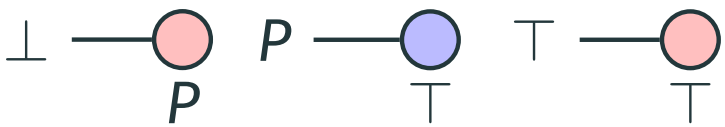
- Transitions (examples): 

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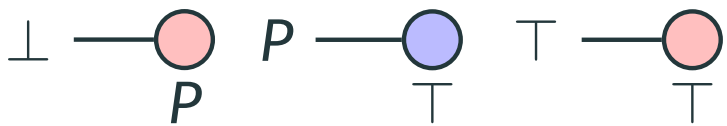
- Transitions (examples): 

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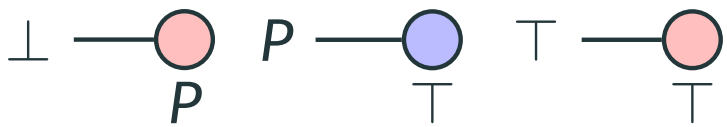
- Transitions (examples): 

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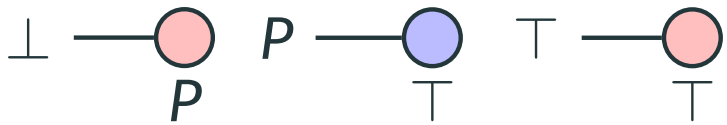
- Transitions (examples): 

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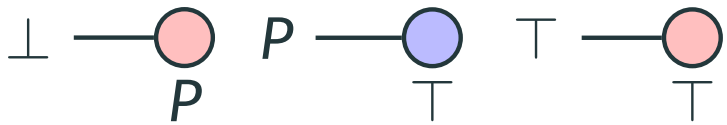
- Transitions (examples): 

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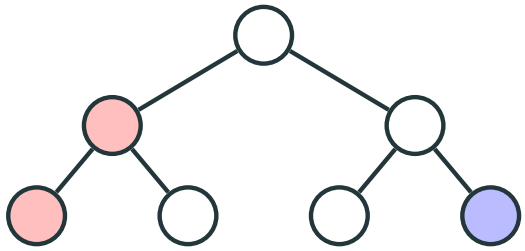
Query evaluation on trees

Database: a tree T where nodes have a color from an alphabet ○ ● ●



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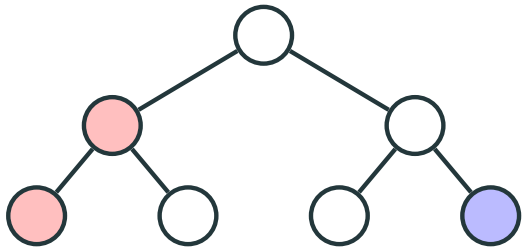

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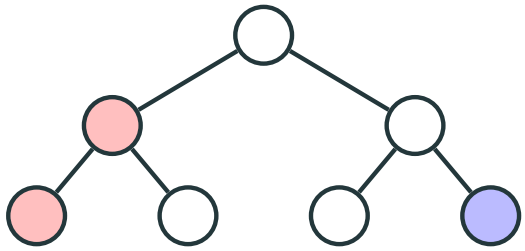

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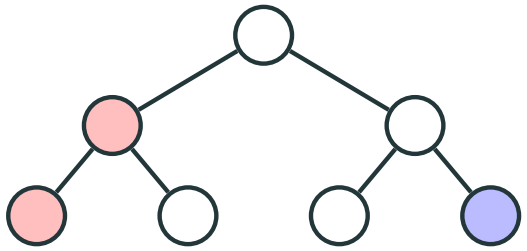

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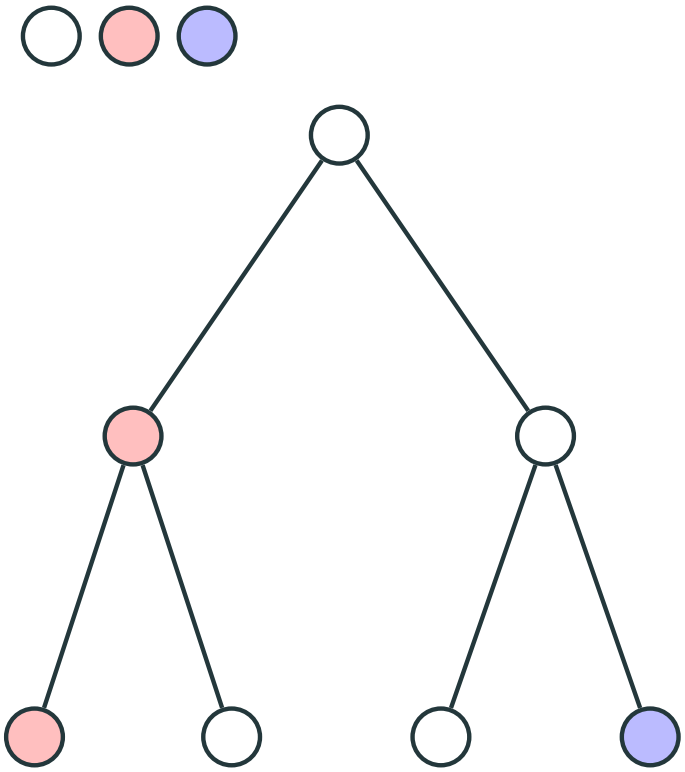

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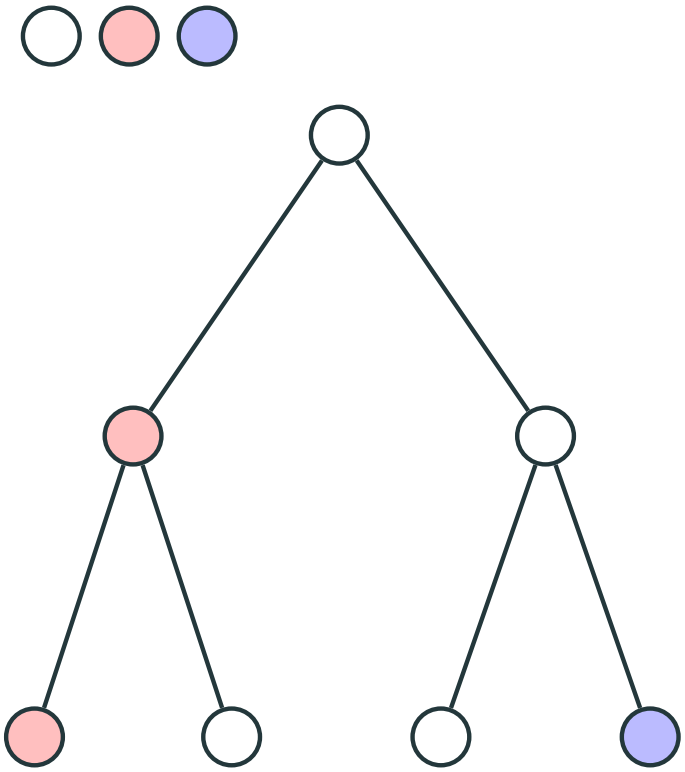

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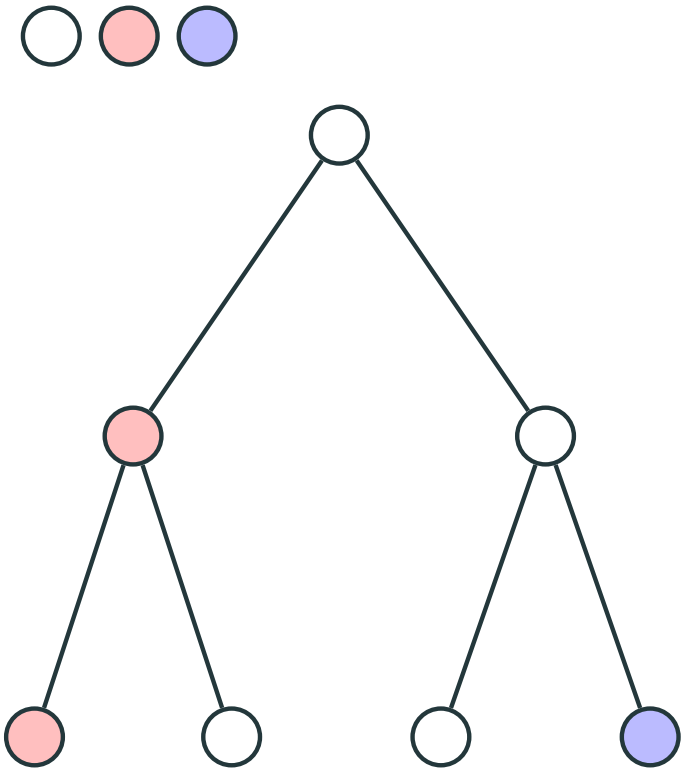

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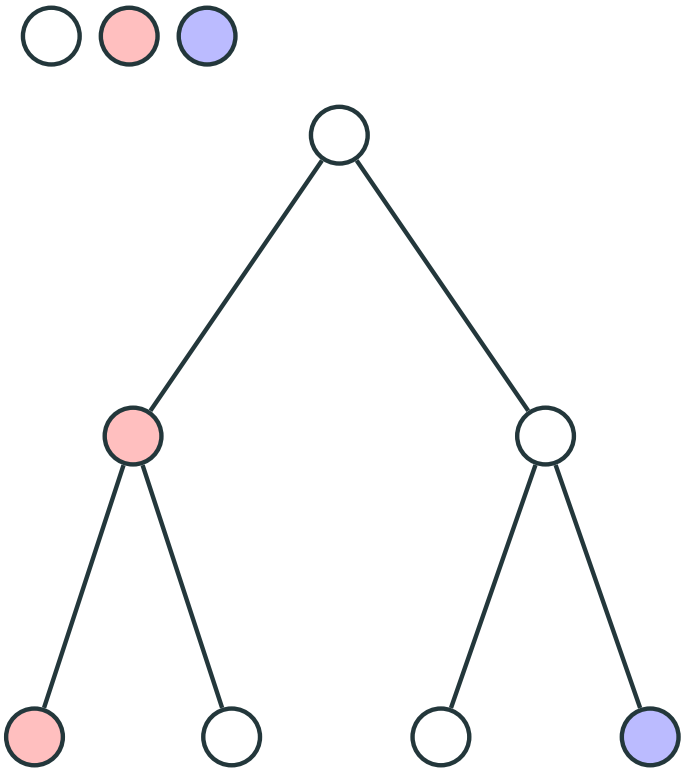

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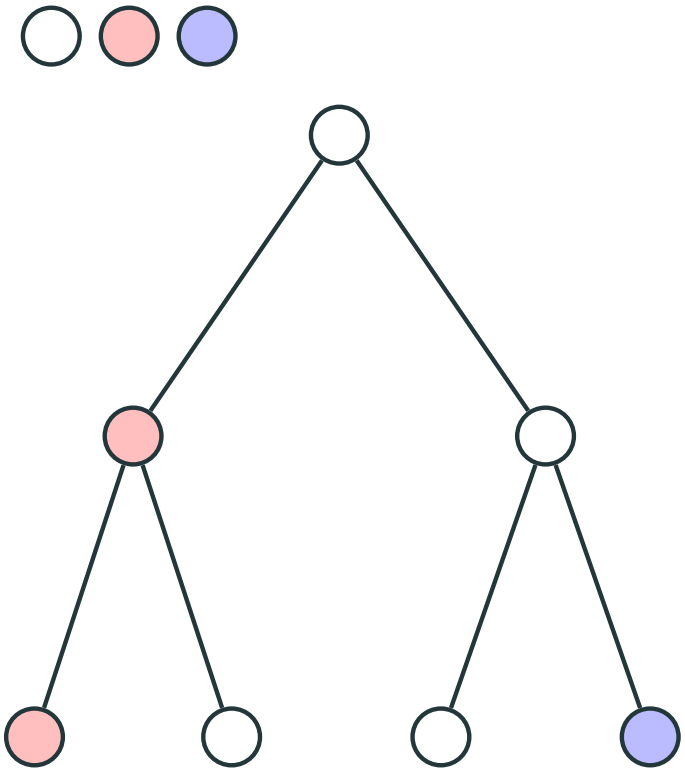

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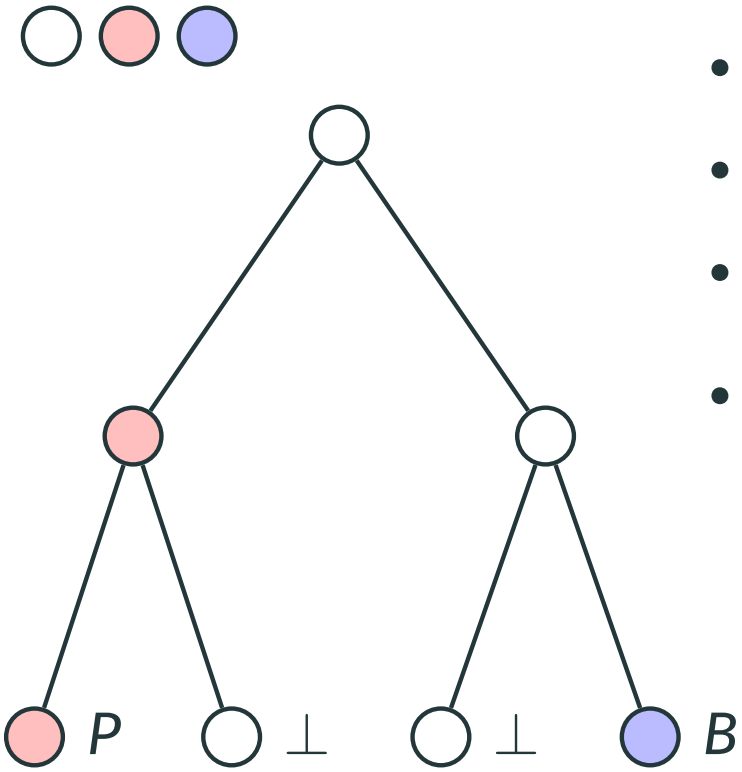

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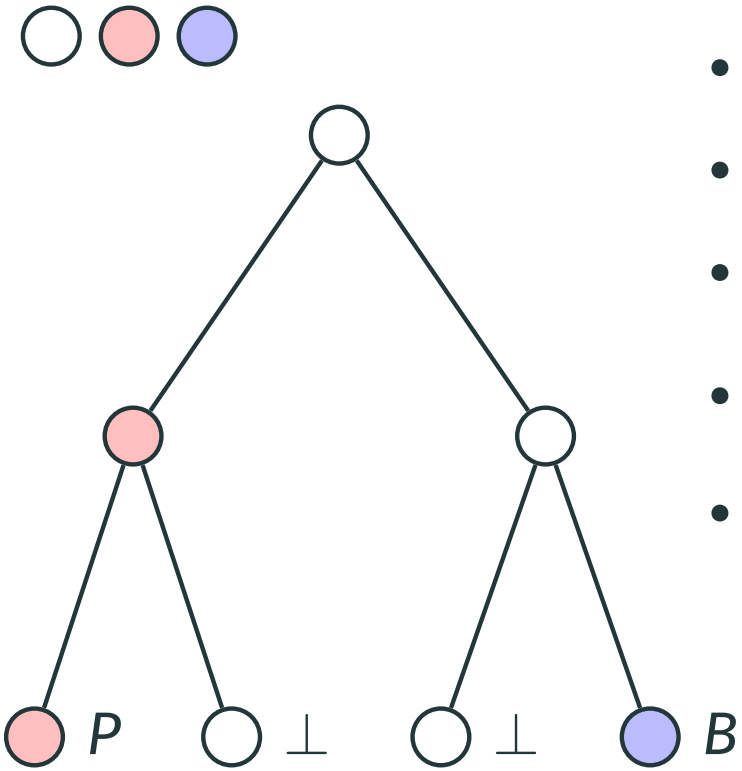

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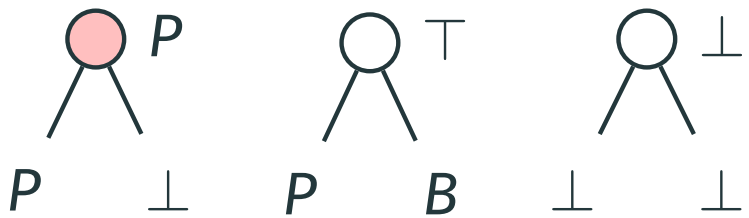

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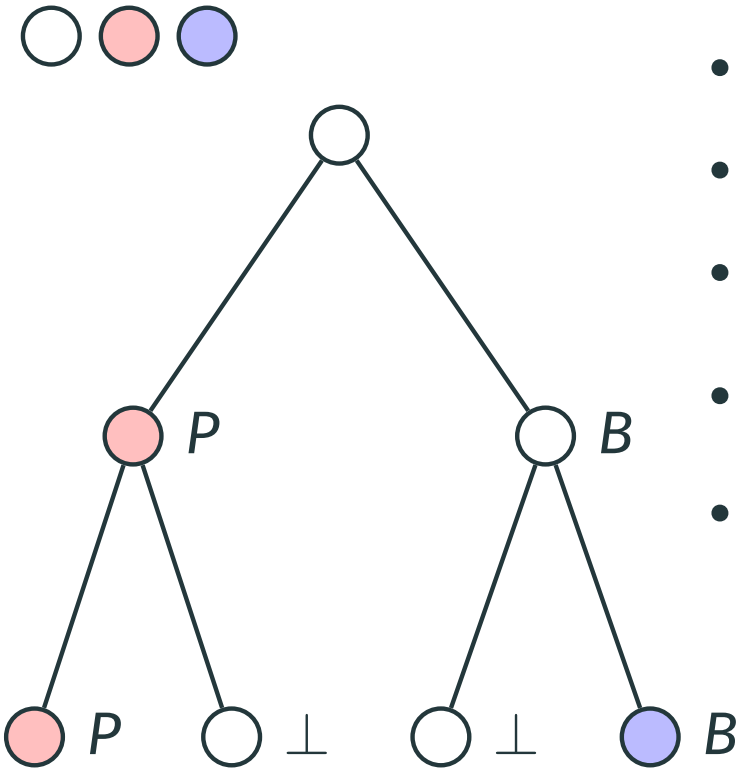

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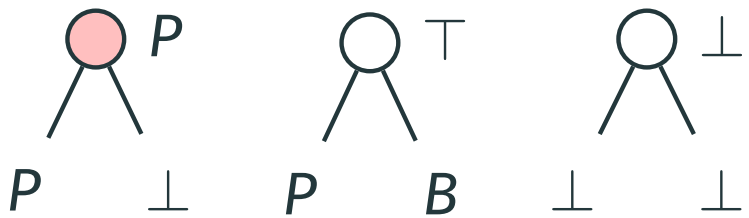

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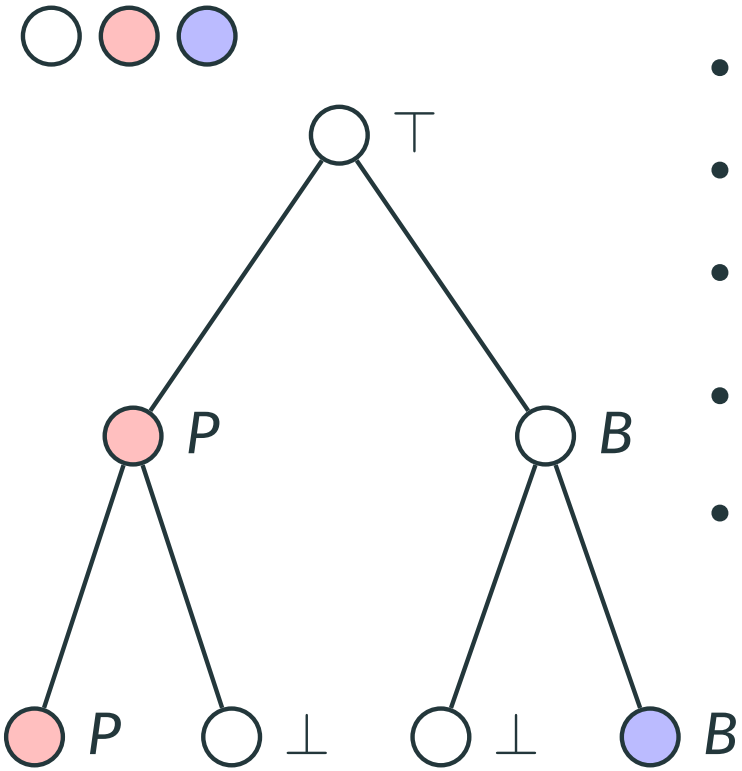

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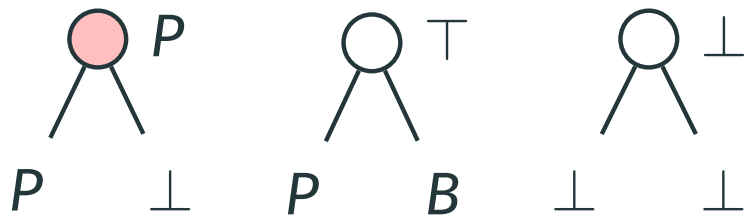

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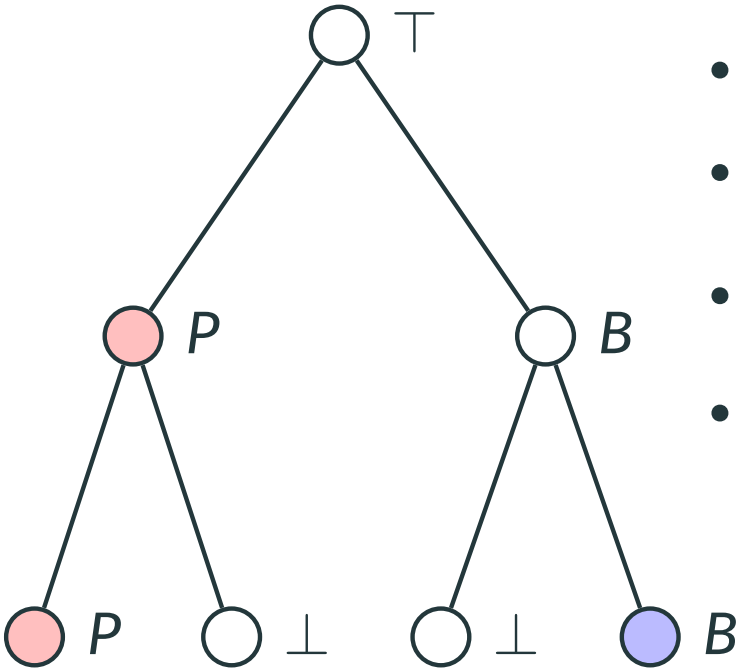

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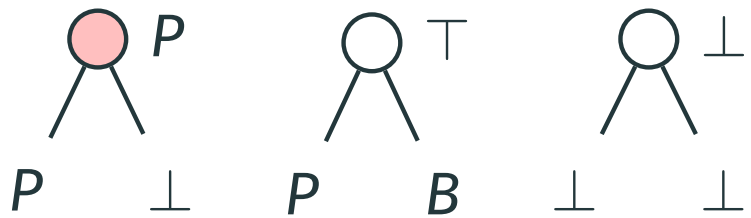

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MSO and tree automata have the same expressive power on trees

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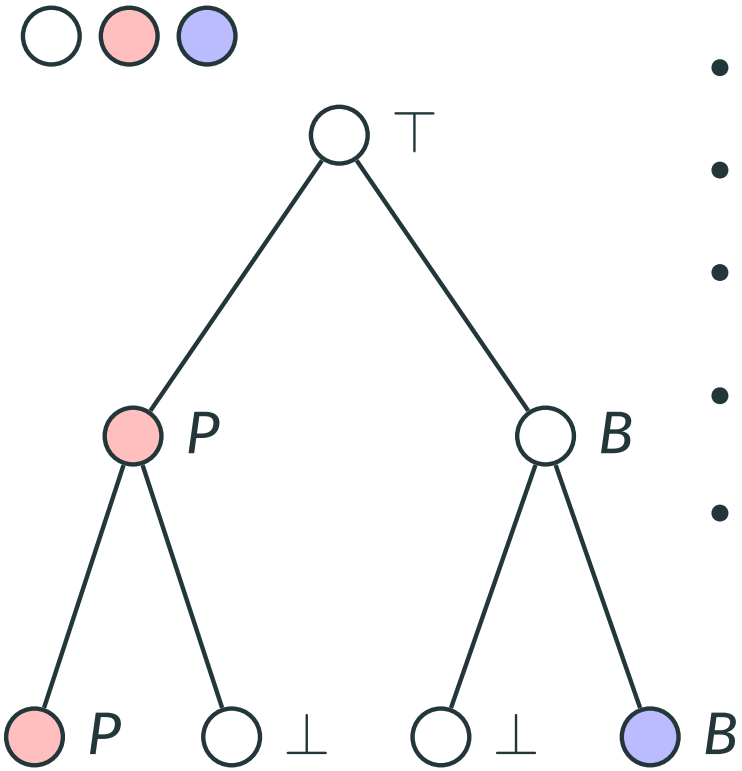

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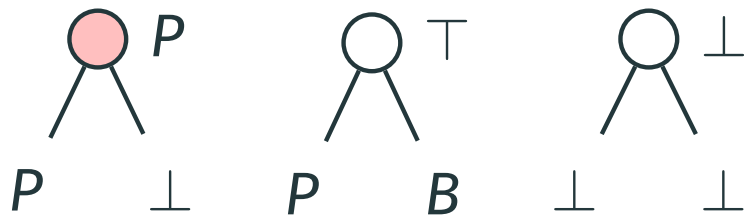

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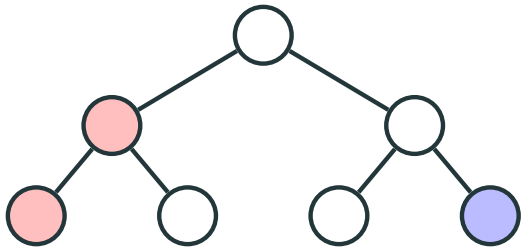

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Query evaluation on treelike data

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(Metro|RER)*
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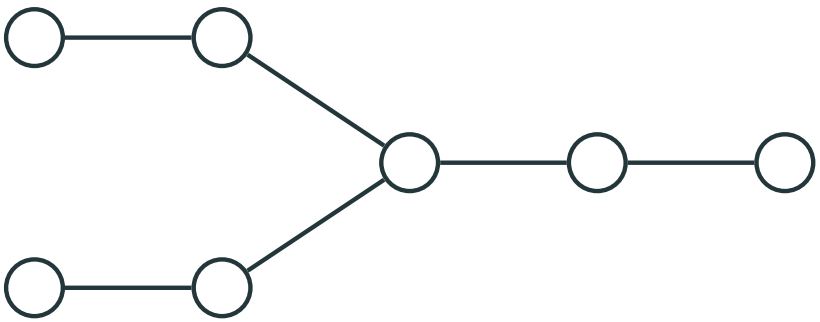

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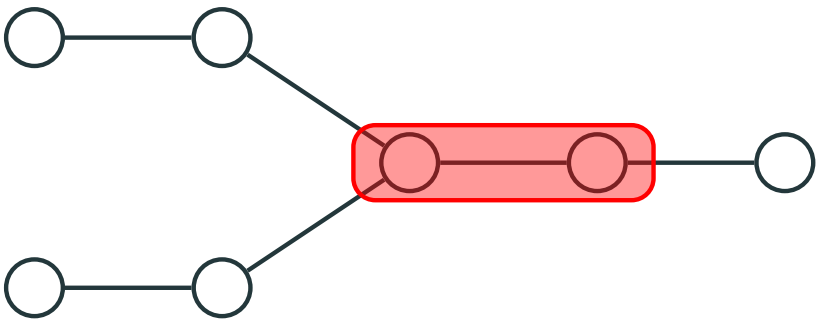

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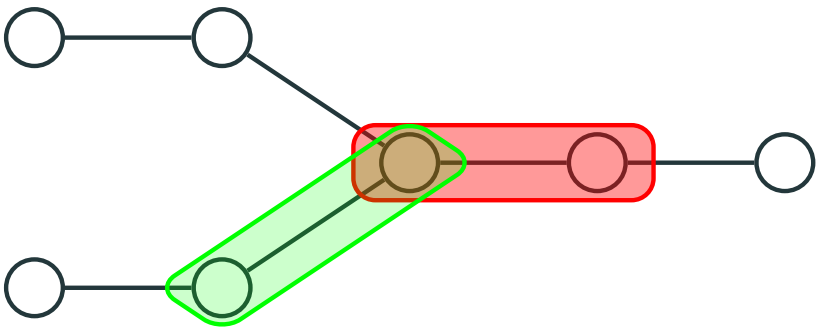

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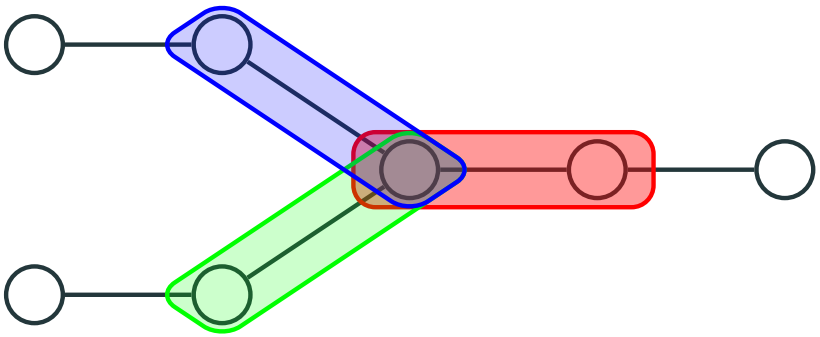

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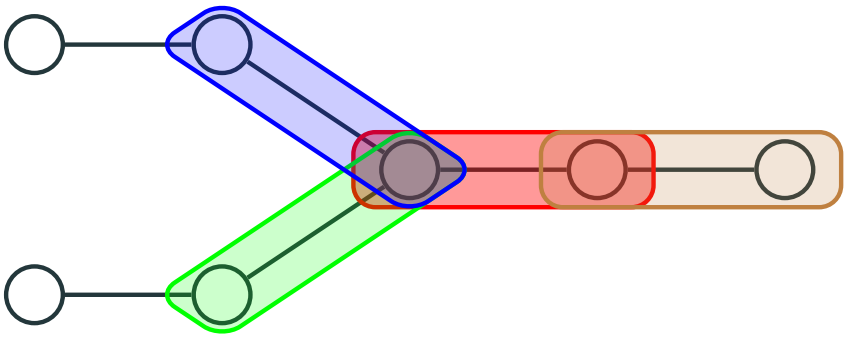

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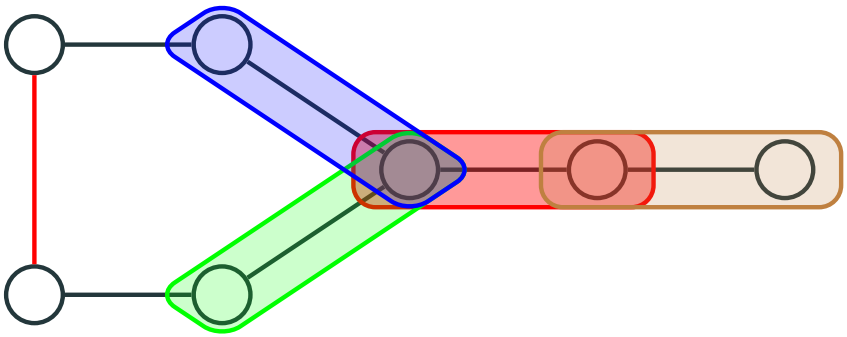

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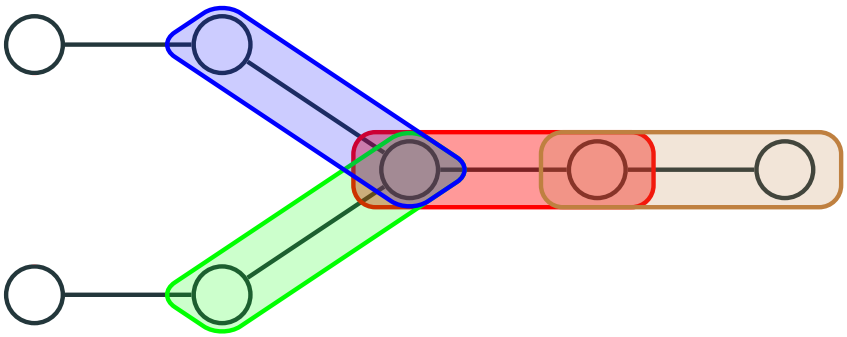

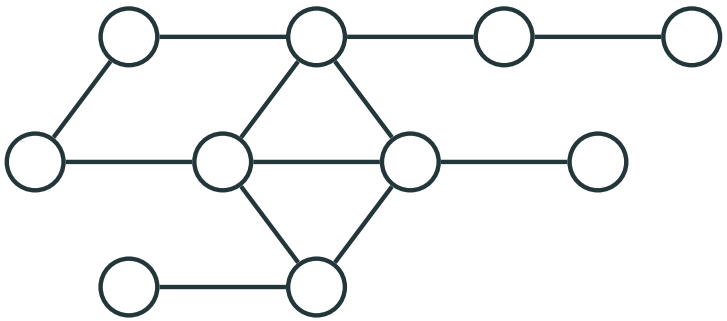

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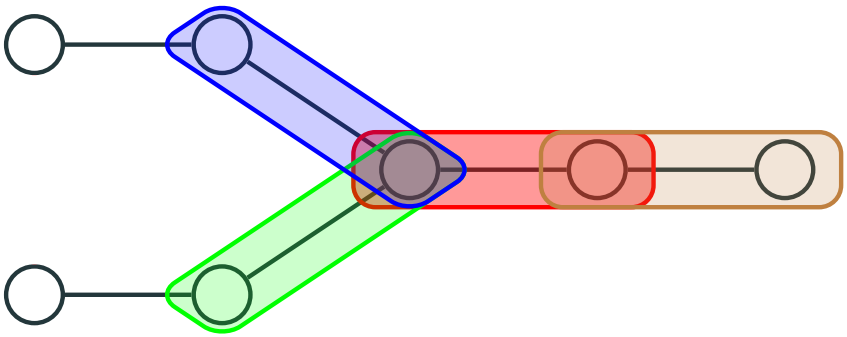

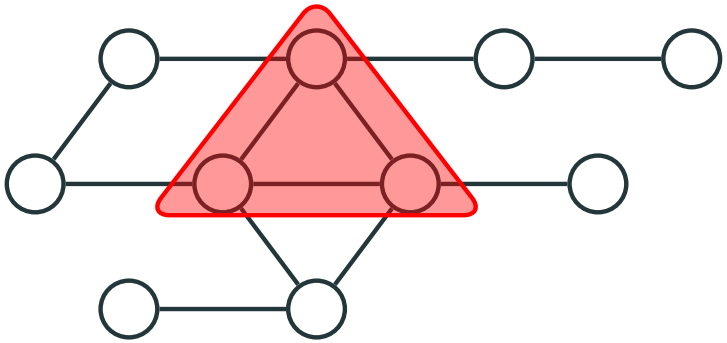

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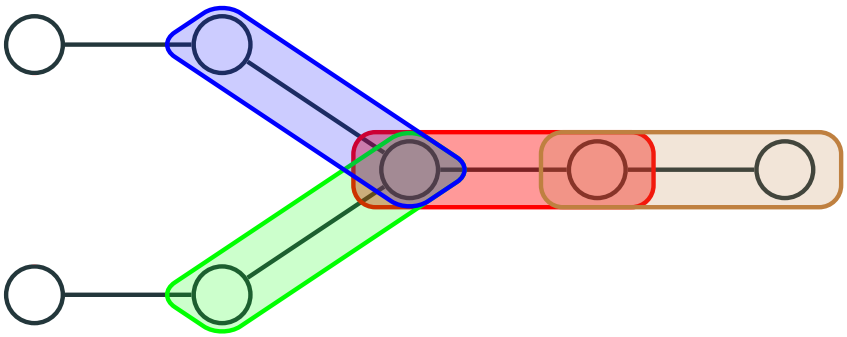

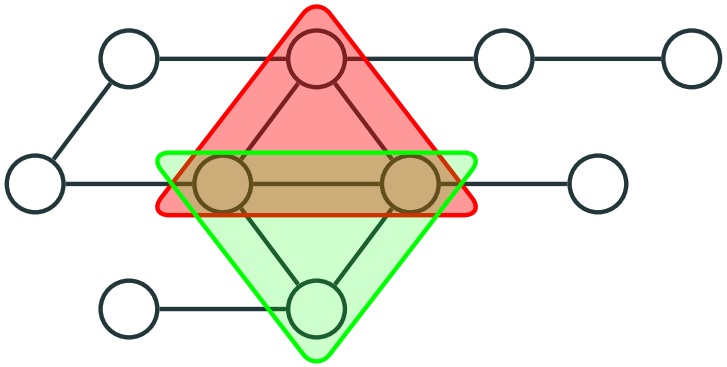

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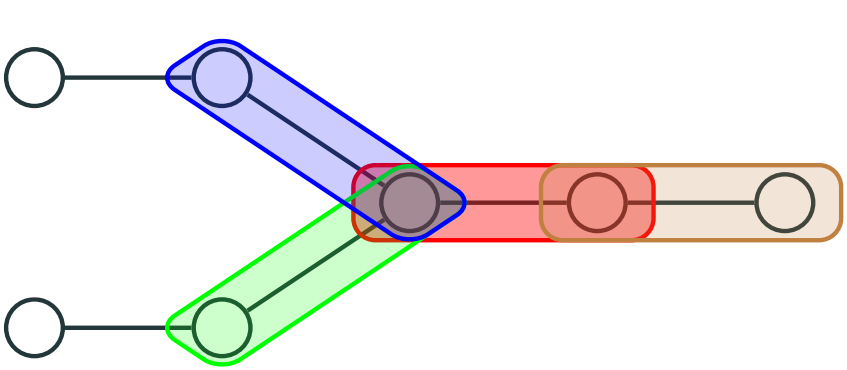

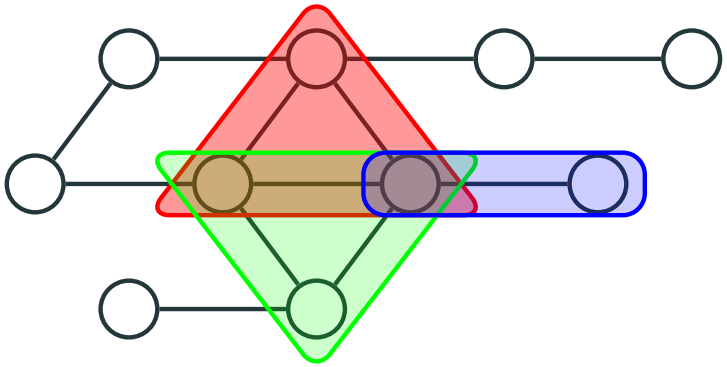

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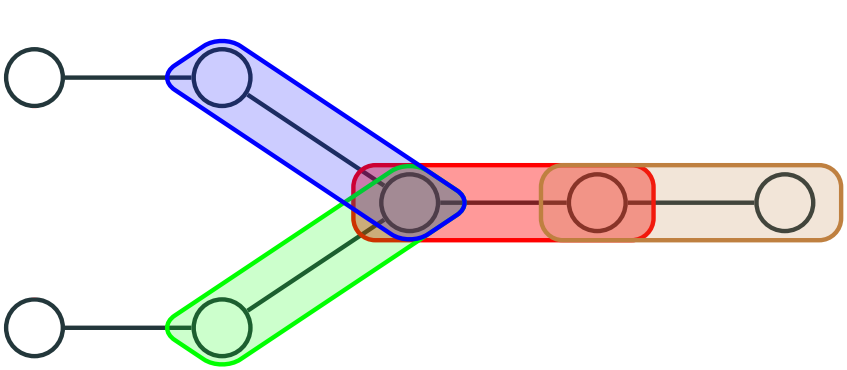

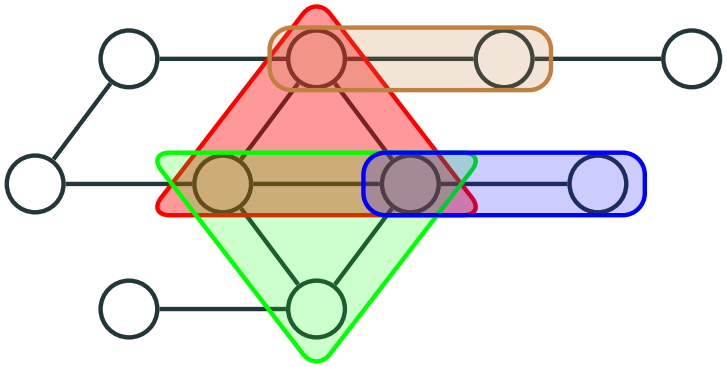

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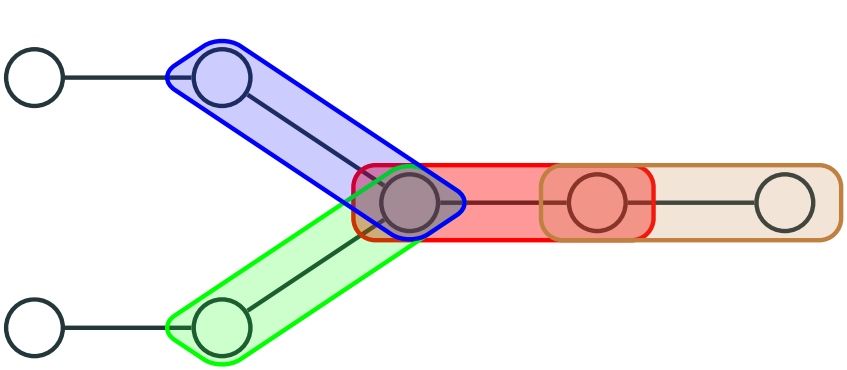

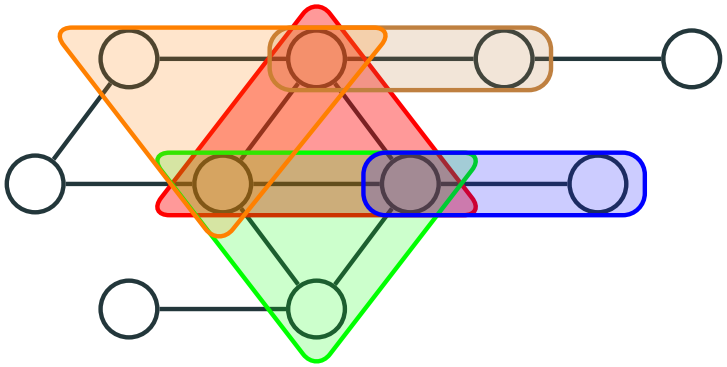

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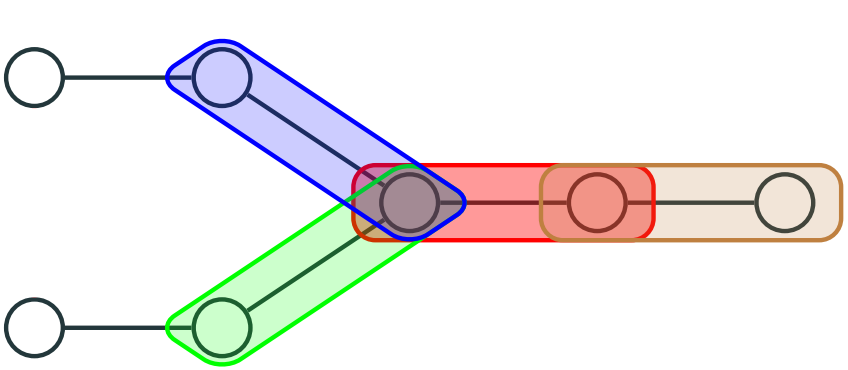

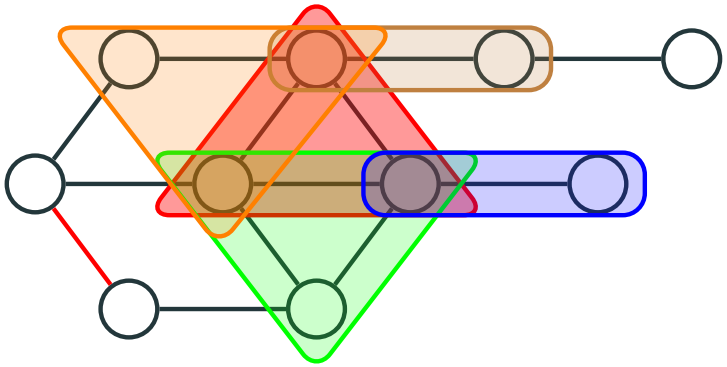

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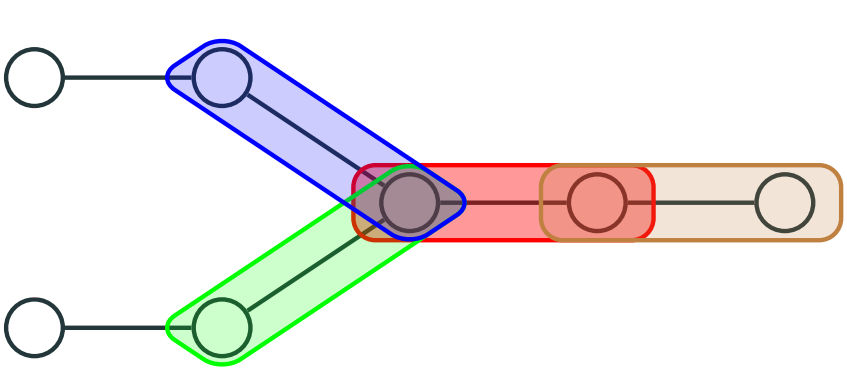

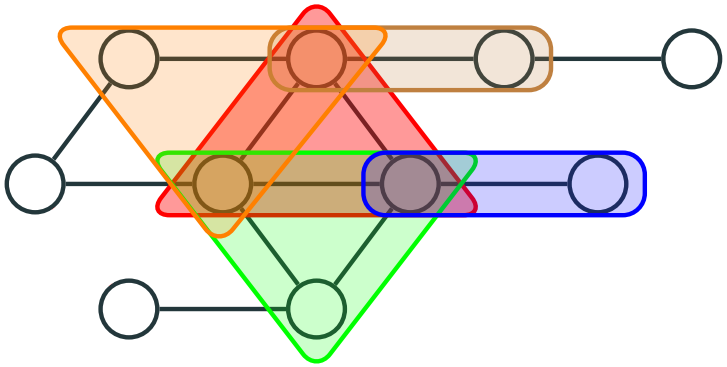

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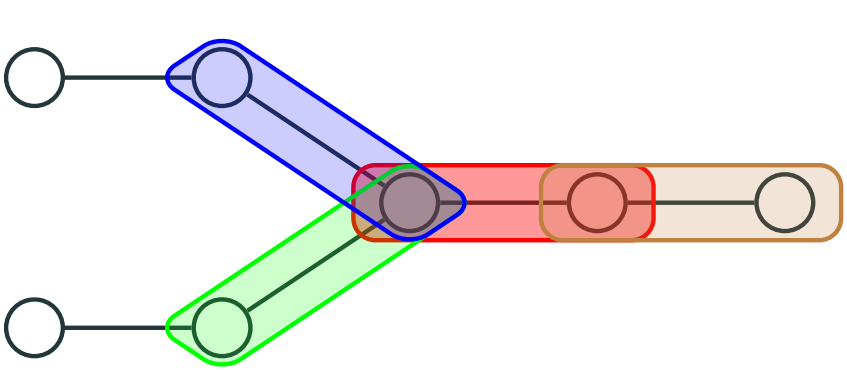

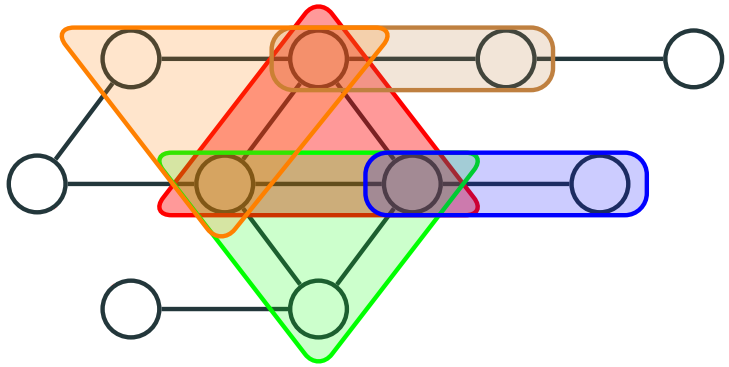

- **Trees** have treewidth **1**
- **Cycles** have treewidth **2**
- ***k*-cliques** and **$(k-1)$-grids** have treewidth $k-1$

Treewidth

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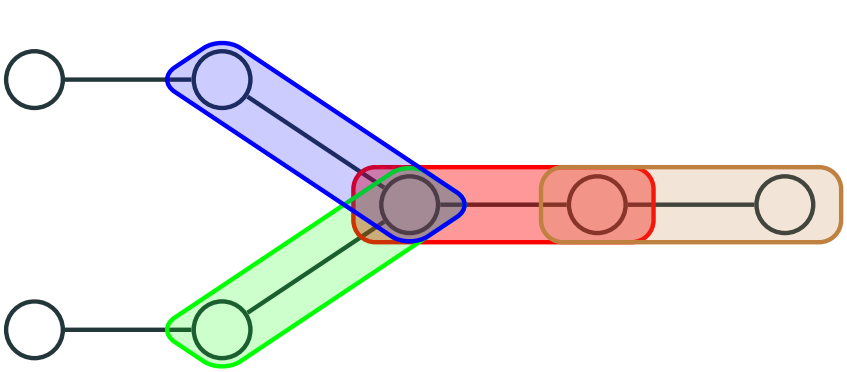

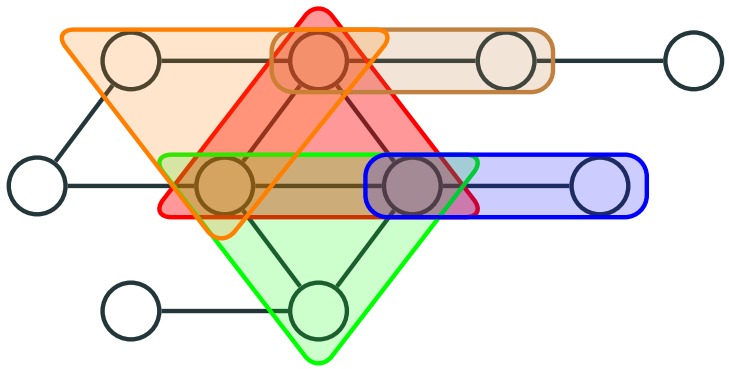

- Trees have treewidth 1
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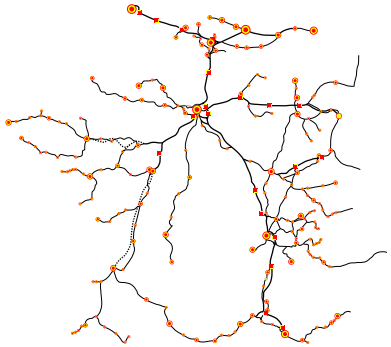
- k-cliques and $(k-1)$-grids have treewidth $k-1$

$\rightarrow$ Treelike: the treewidth is bounded by a constant

Courcelle's theorem

Treelike **data**



MSO query

(RER|metro)*
|(bus|tram)*

Courcelle's theorem

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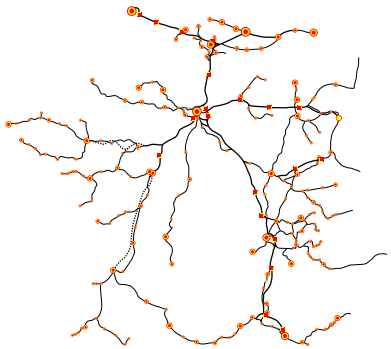

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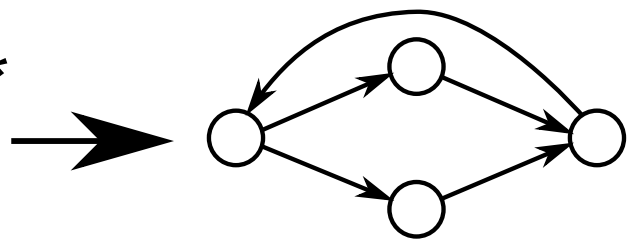
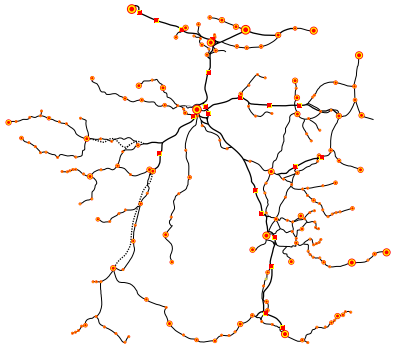


Tree **automaton**

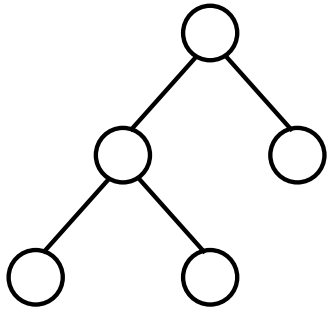
Courcelle's theorem

Treelike **data**



linear

Tree **encoding**



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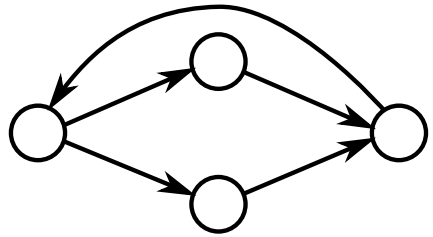

Courcelle's theorem

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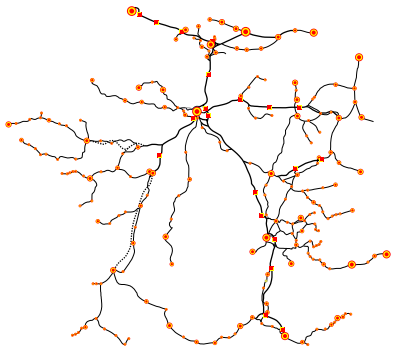

linear

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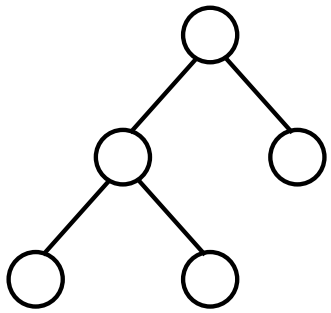


linear

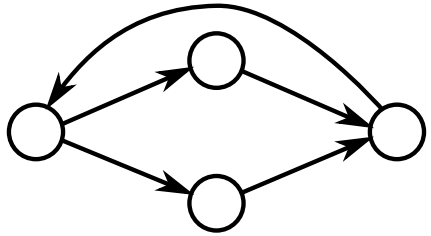
Query **answer**

TRUE

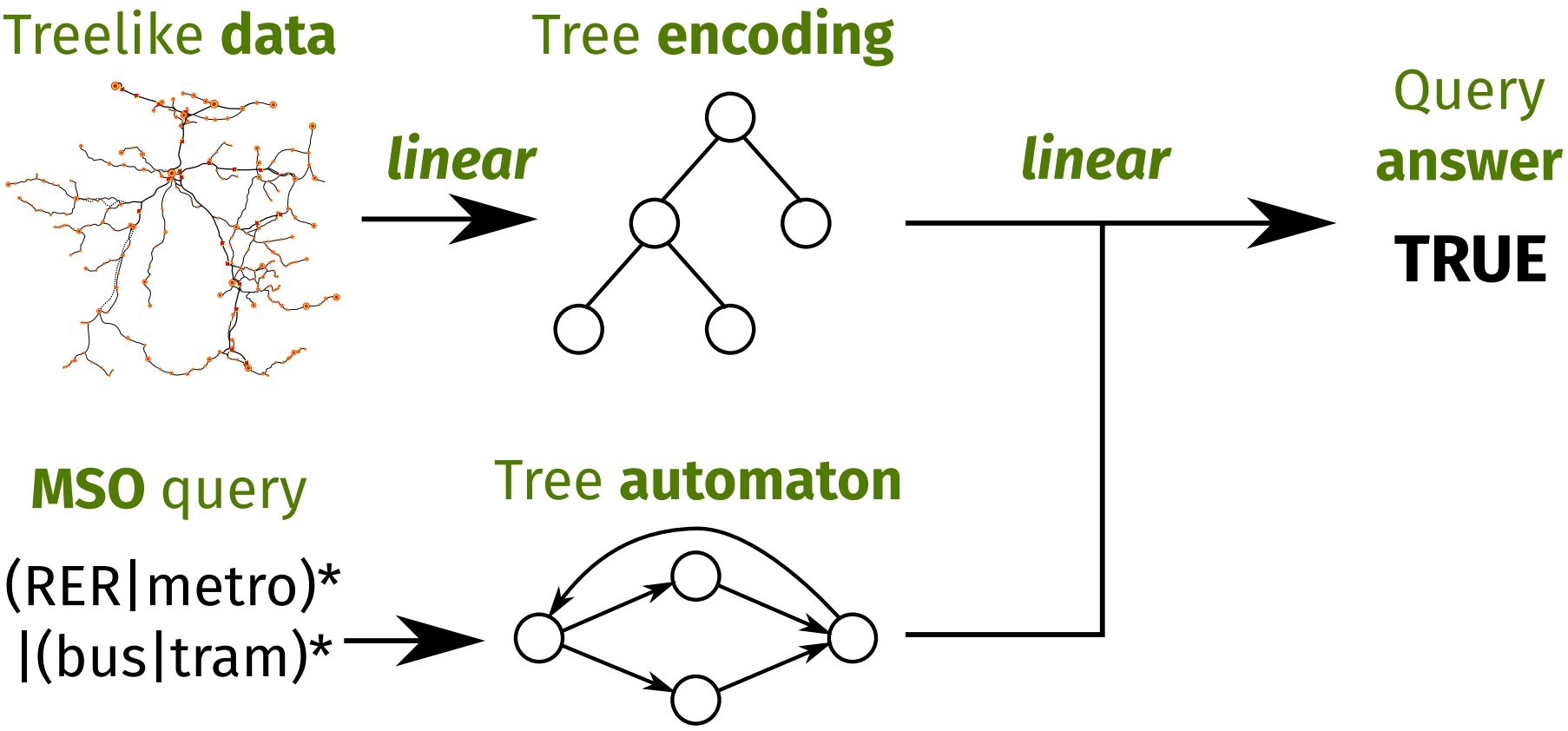
MSO query

(RER|metro)*
|(bus|tram)*

Tree **automaton**



Courcelle's theorem



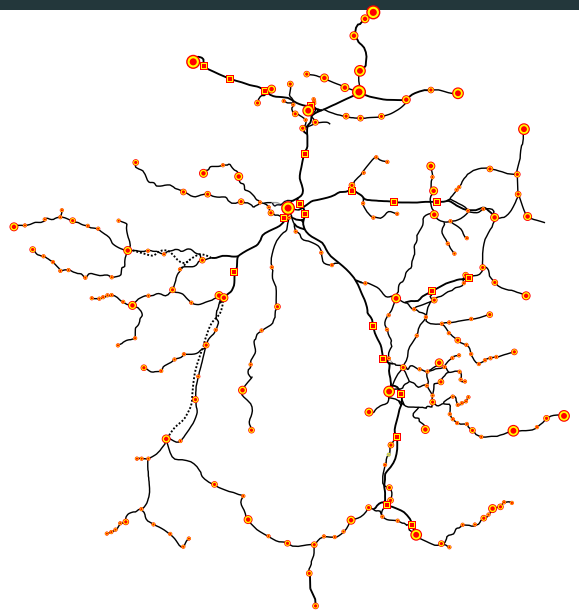
Theorem [Courcelle, 1990]

For any fixed Boolean MSO query Q *and* $k \in \mathbb{N}$,
given a database D *of treewidth* $\leq k$,
we can compute in ***linear time*** *in* D *whether* D *satisfies* Q

Table of contents

Probabilistic query evaluation on treelike data

- Database D with treewidth $\leq k$ for some constant k
- Probability of each fact of D to be actually present in the data (independently from other facts)



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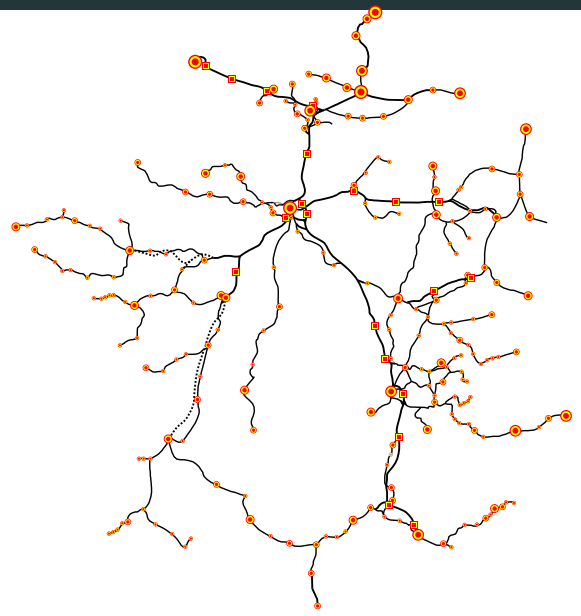

Query Q: a sentence in monadic second-order logic (MSO)

(Metro|RER)*
| (Bus|Tram)*

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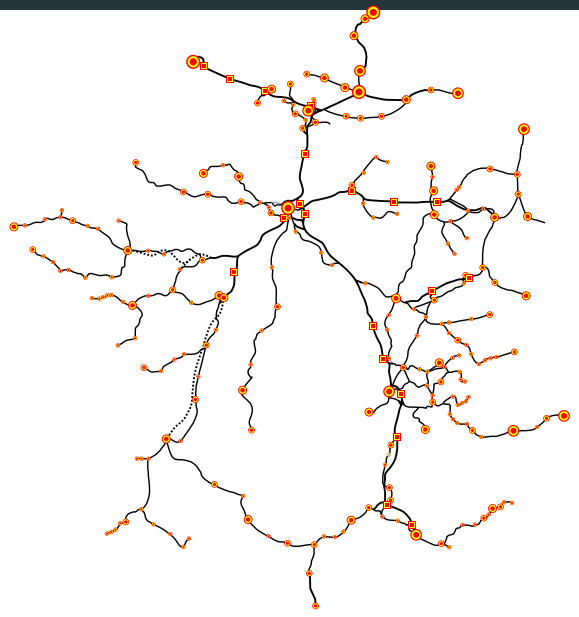

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Result: Probability that the database D satisfies query Q

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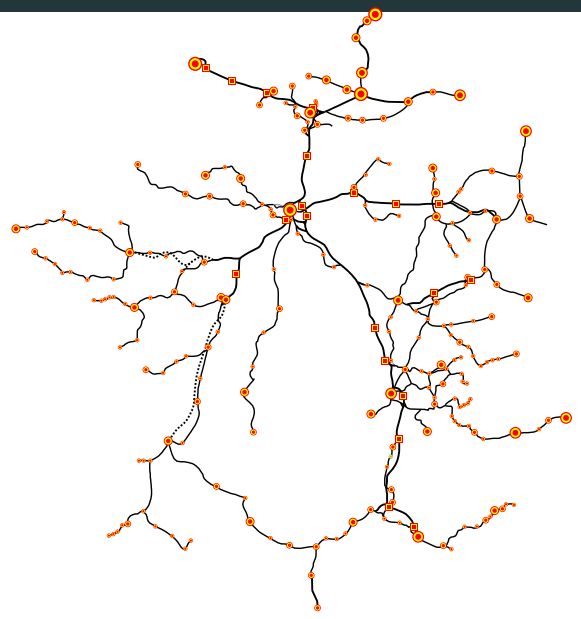


Query Q: a sentence in monadic second-order logic (MSO)

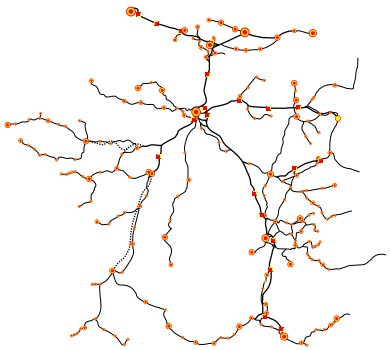
(Metro|RER)*
| (Bus|Tram)*

Result: Probability that the database D satisfies query Q

Computational complexity as a function of the database D
(the query Q is fixed)

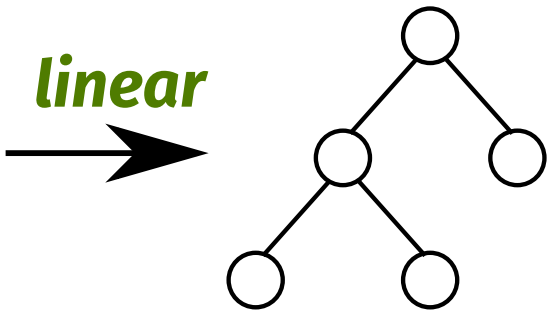
Roadmap

Treelike **data**



Tree **encoding**

linear



MSO query

(RER|metro)*
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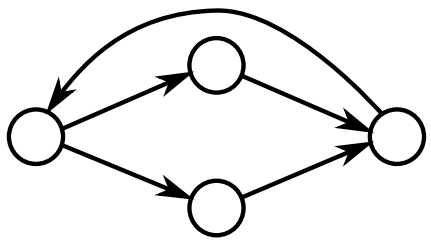
Tree **automaton**



Roadmap

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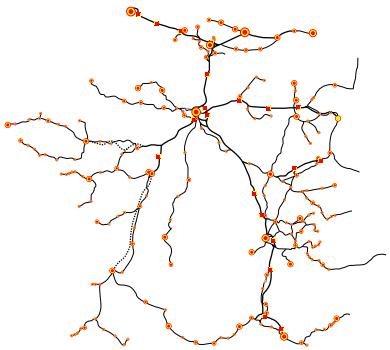

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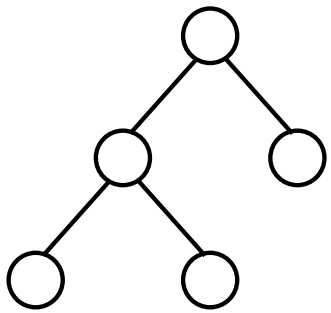
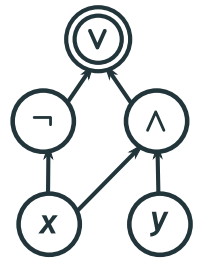


linear

Provenance circuit



MSO query

(RER|metro)*
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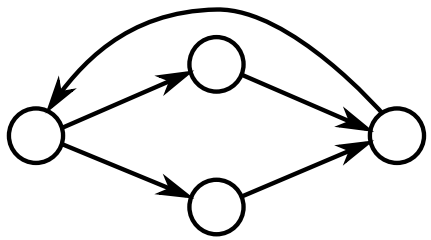
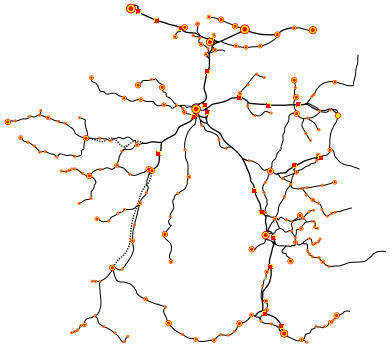


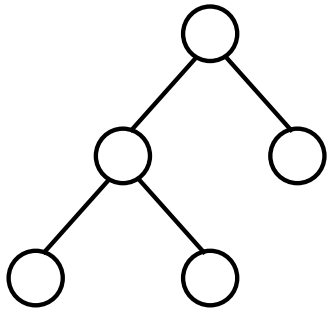
Roadmap

Probabilistic treelike **data**



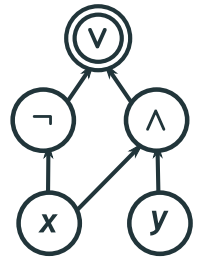
linear

Uncertain tree **encoding**



linear

Provenance circuit +probabilities



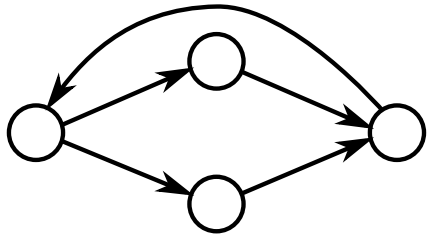
linear

95% Probability

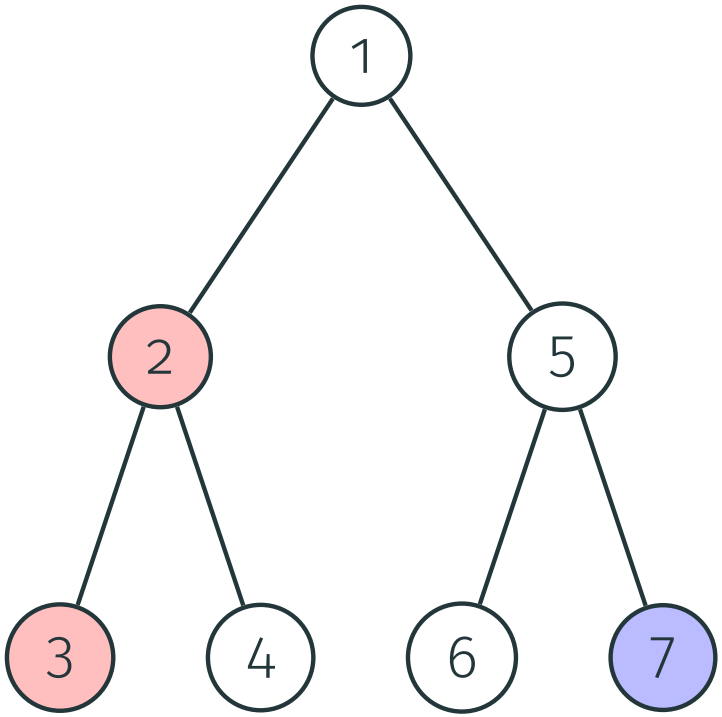
MSO query

(RER|metro)*
|(bus|tram)*

Tree **automaton**



Uncertain trees



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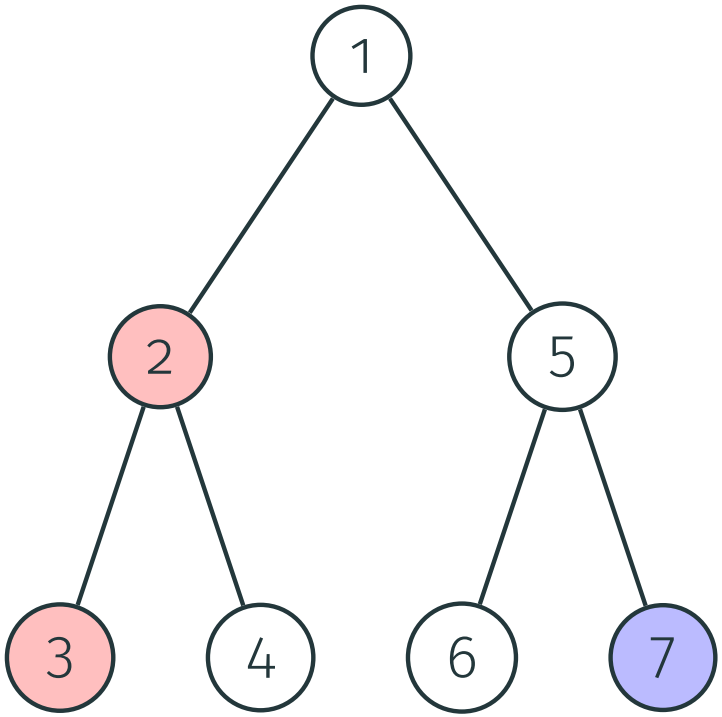
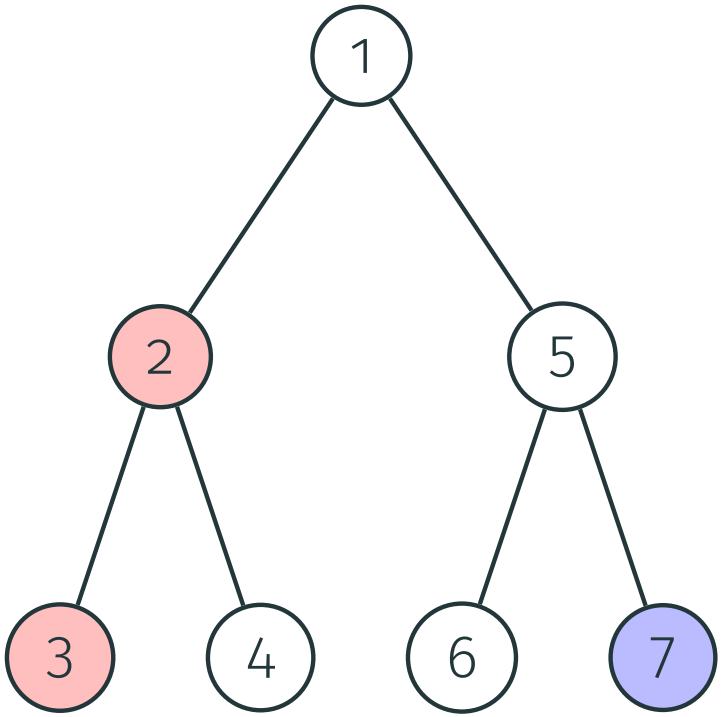


A **valuation** of a tree decides whether to **keep** (1) or **discard** (0) node labels

Uncertain trees



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Valuation: $\{2, 3, 7 \mapsto 1, \quad * \mapsto 0\}$

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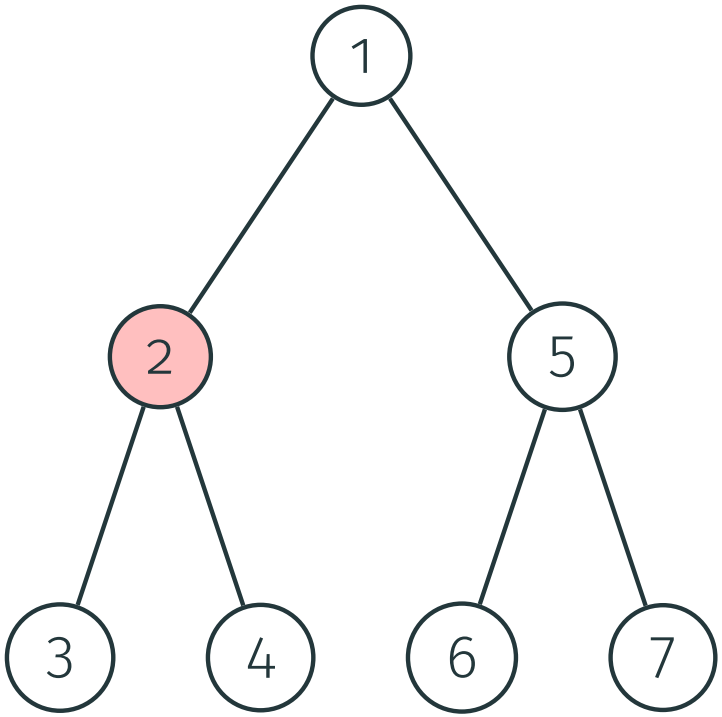

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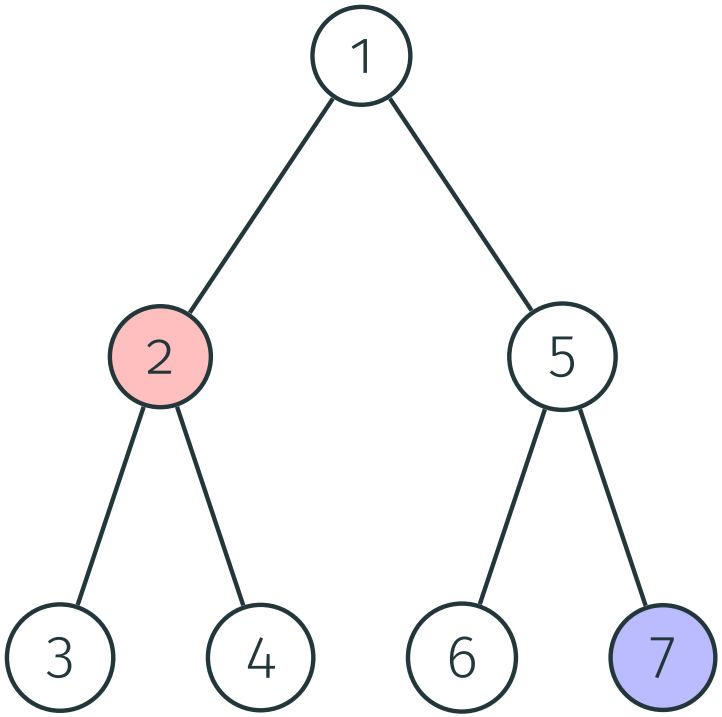

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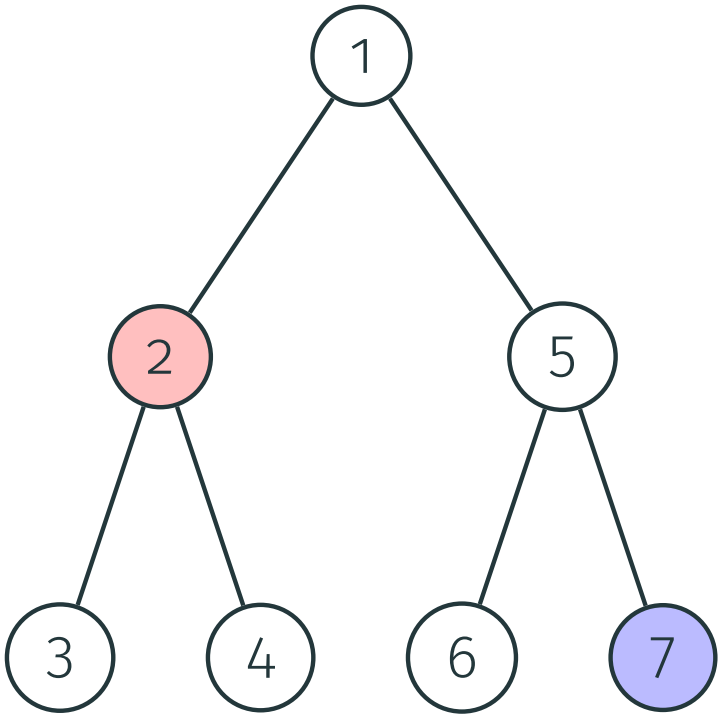

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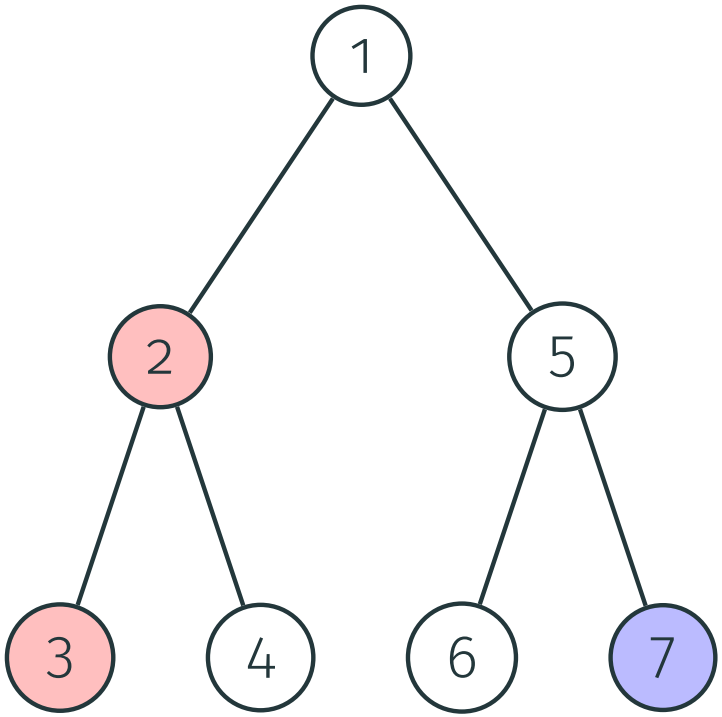
Uncertain trees



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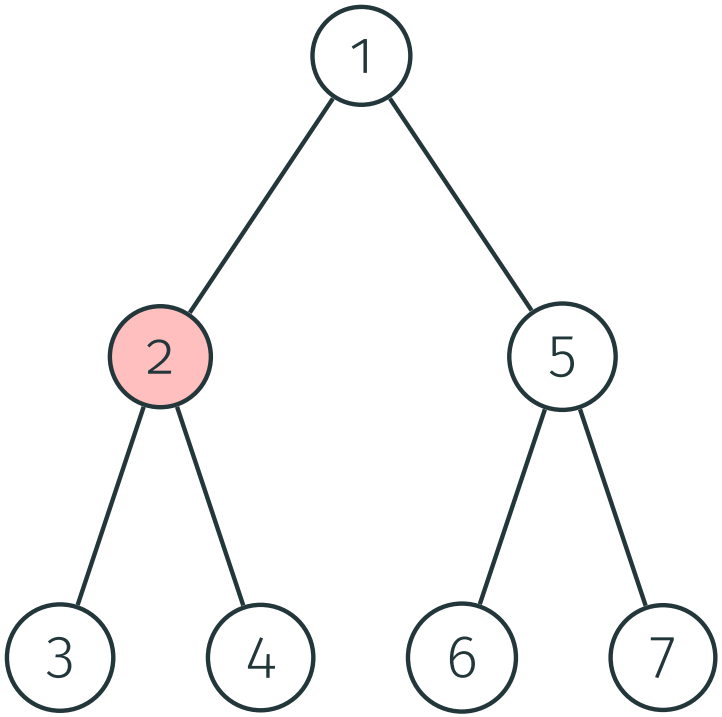

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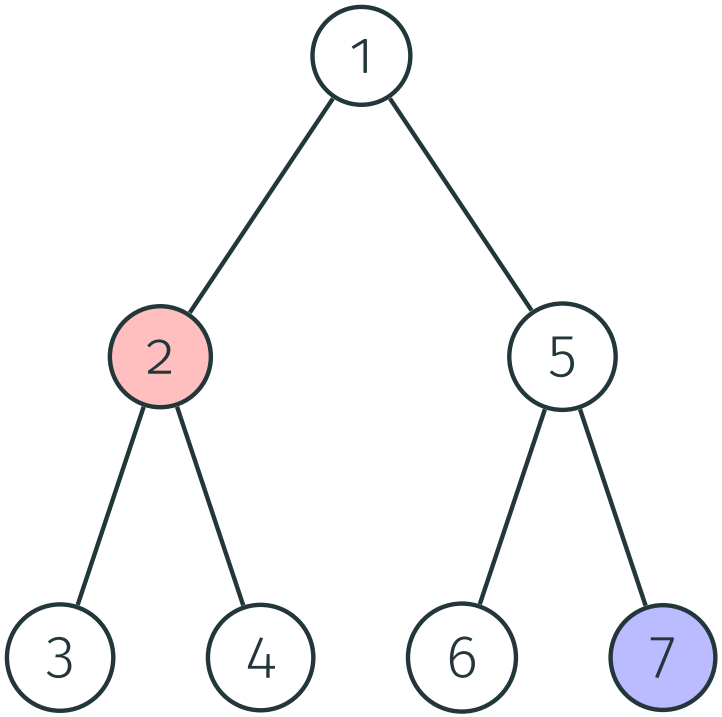

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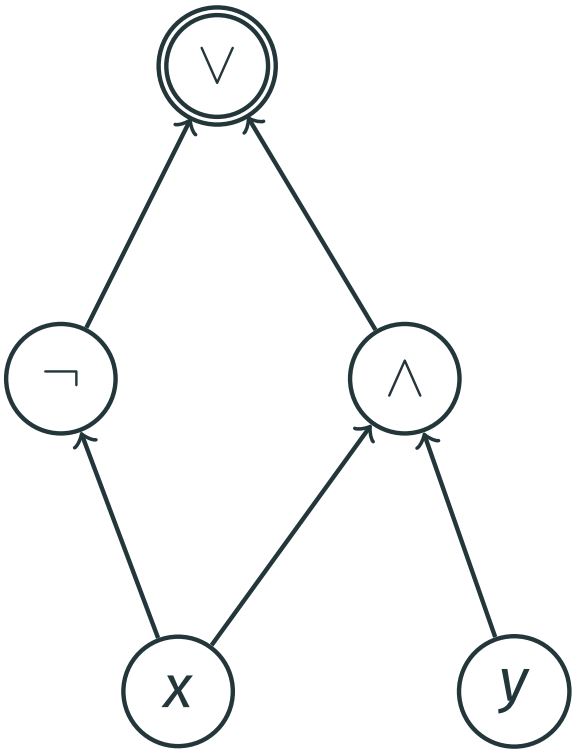
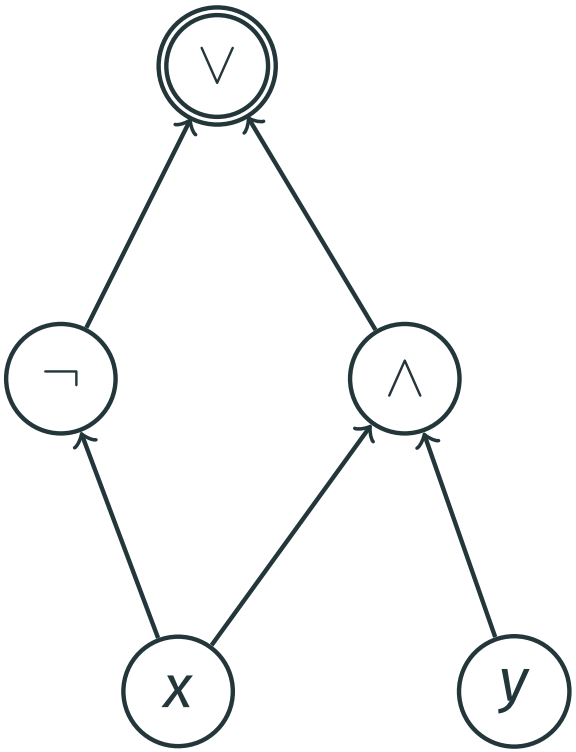
Boolean circuit



- Directed acyclic graph of gates

Boolean circuit



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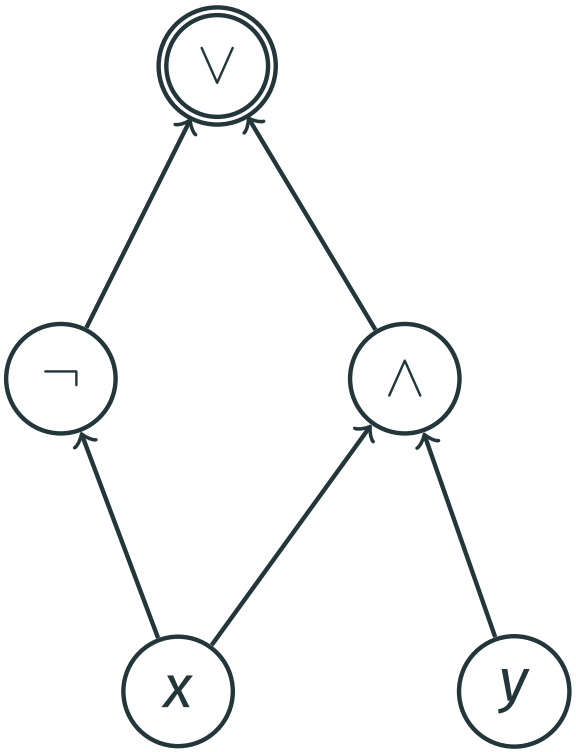
Boolean circuit



- Directed acyclic graph of gates
- Output gate: 
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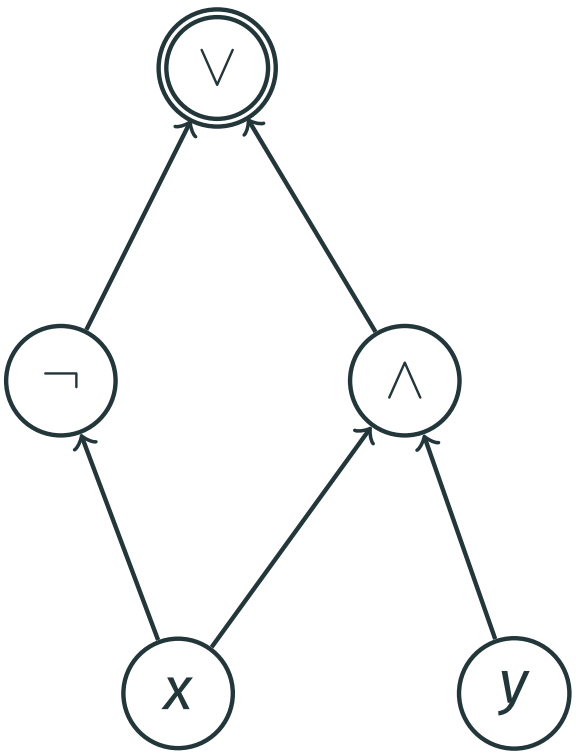
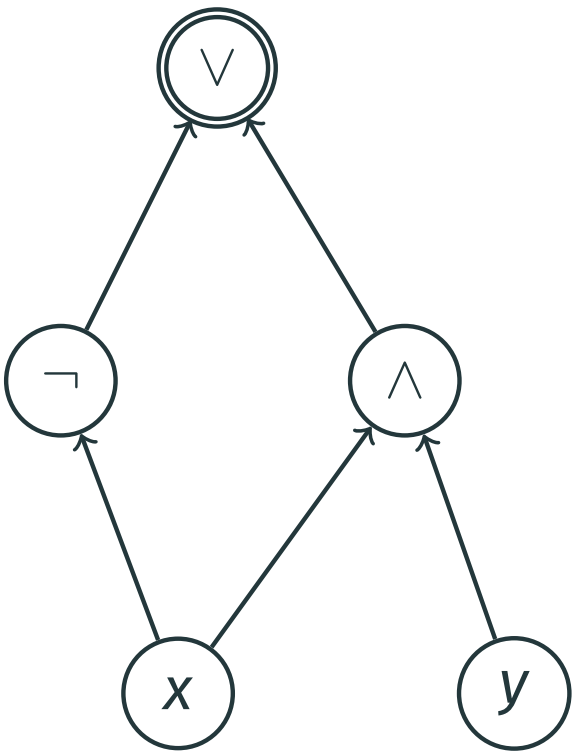
Boolean circuit



- Directed acyclic graph of gates
- Output gate: 
- Variable gates: 

- Internal gates: 

Boolean circuit



- Directed acyclic graph of gates
- Output gate: 
- Variable gates: 
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  Example: $\nu = \{x \mapsto 0,\ y \mapsto 1\}$...

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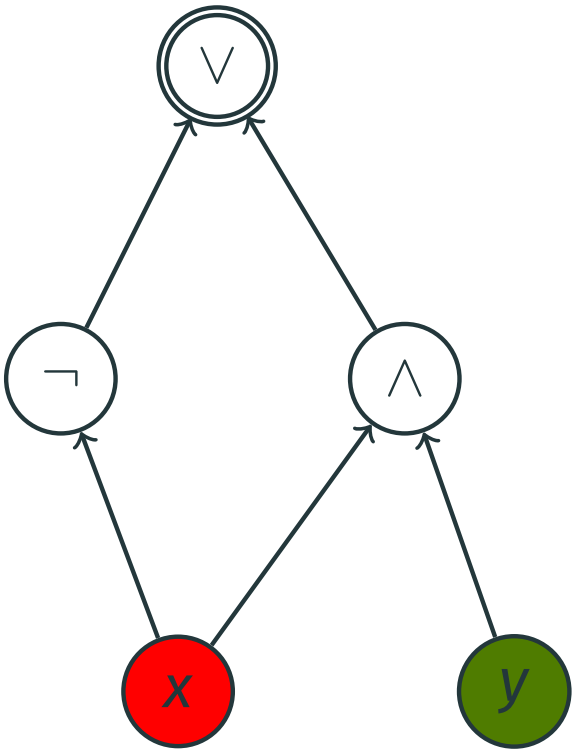

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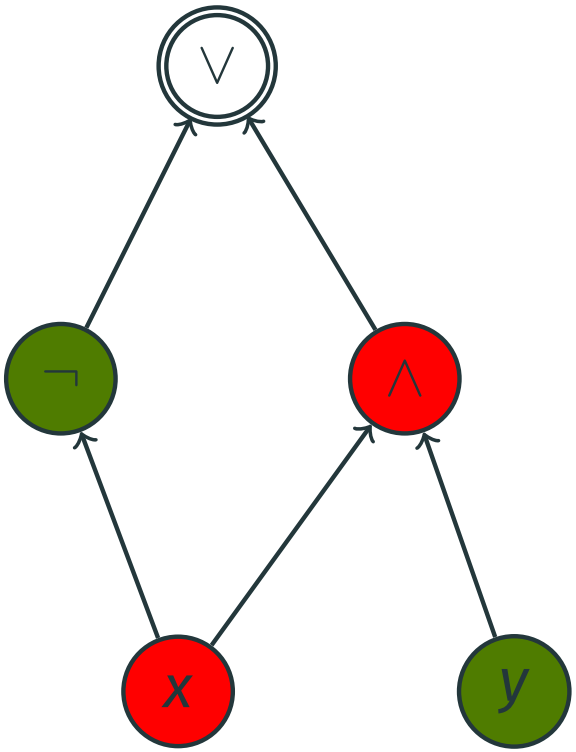

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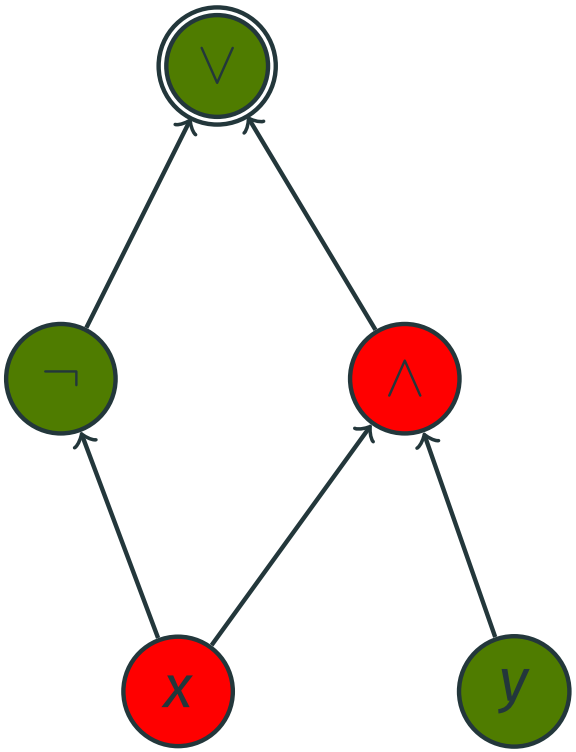

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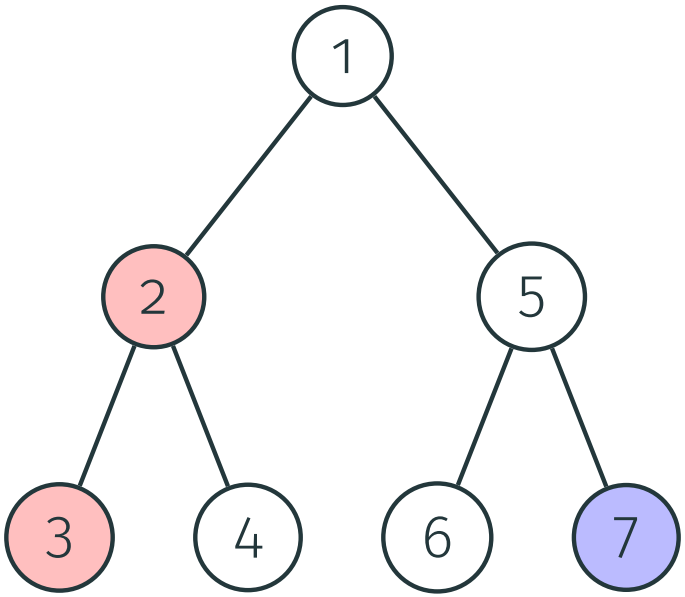
Provenance circuit



Query: *Is there both a pink and a blue node?*

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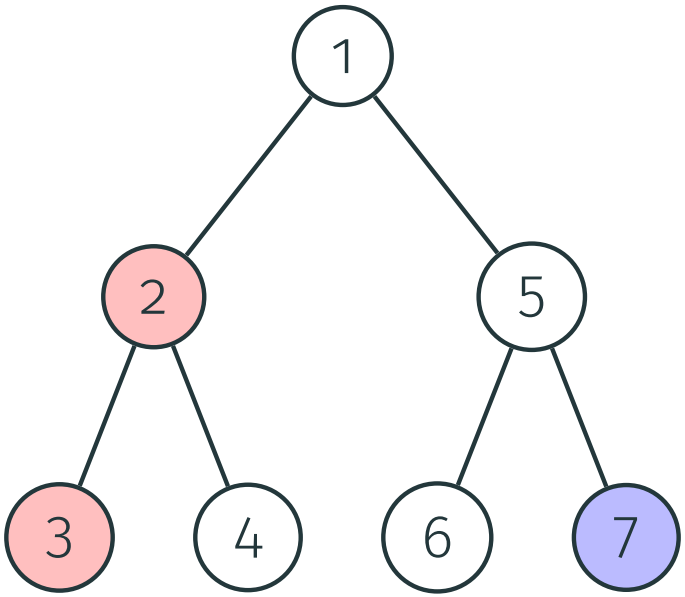

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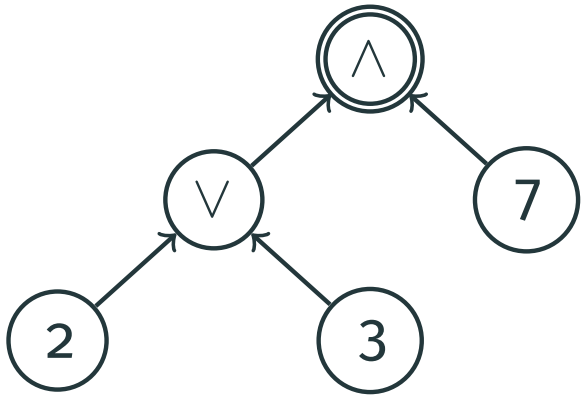

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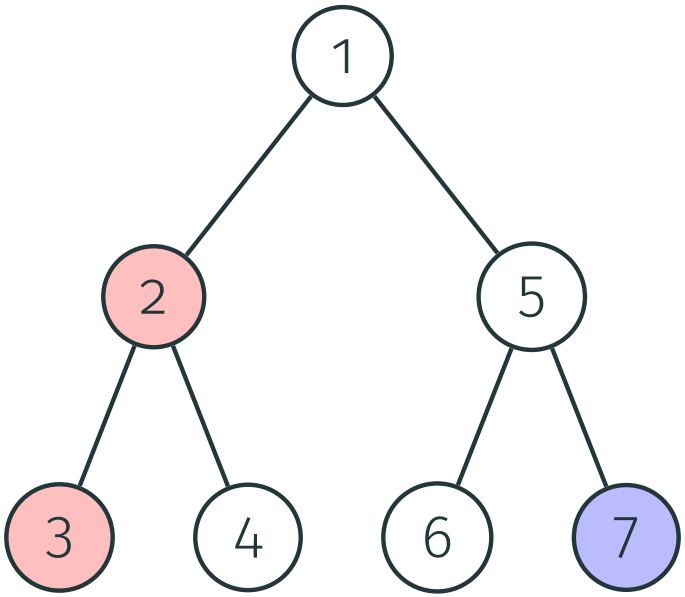

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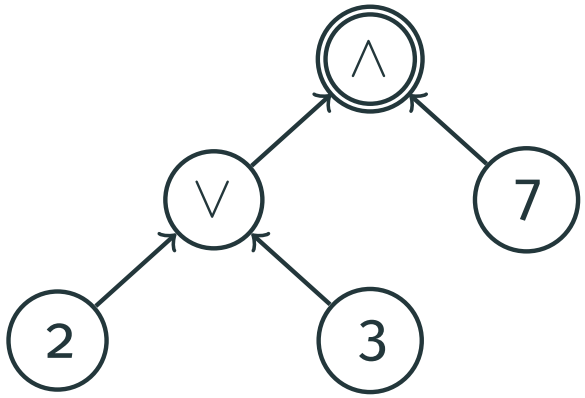

Formally:

- Tree automaton A, uncertain tree T, circuit C
- **Variable gates** of C: nodes of T
- **Condition:** Let ν be a valuation of T, then $\nu(C)$ iff A accepts $\nu(T)$

Building provenance circuits on trees

Theorem

For any bottom-up tree automaton A and input tree T,
we can build a provenance circuit of A on T in $O(|A| \times |T|)$

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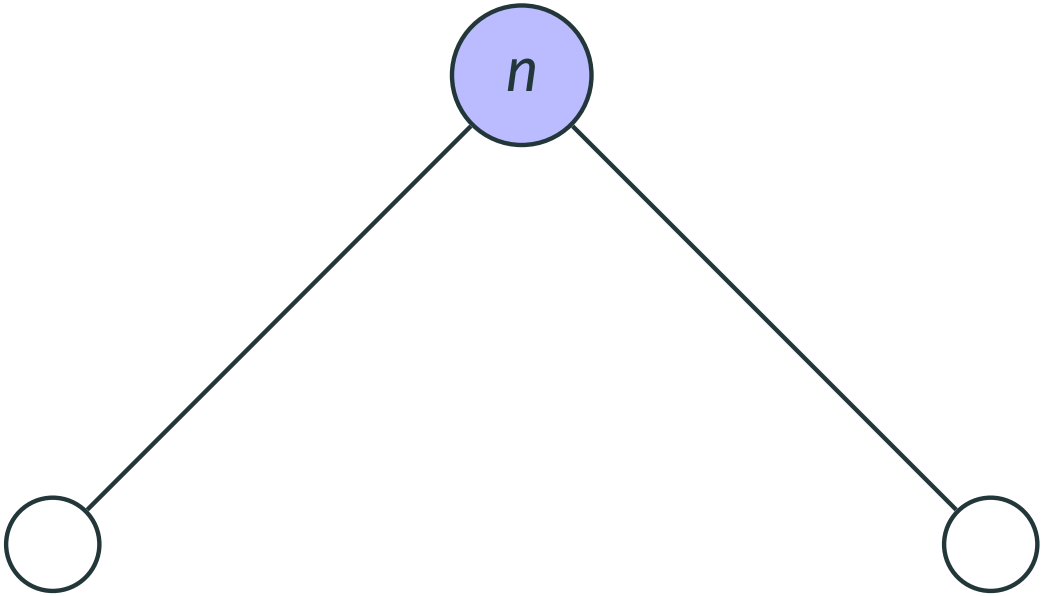

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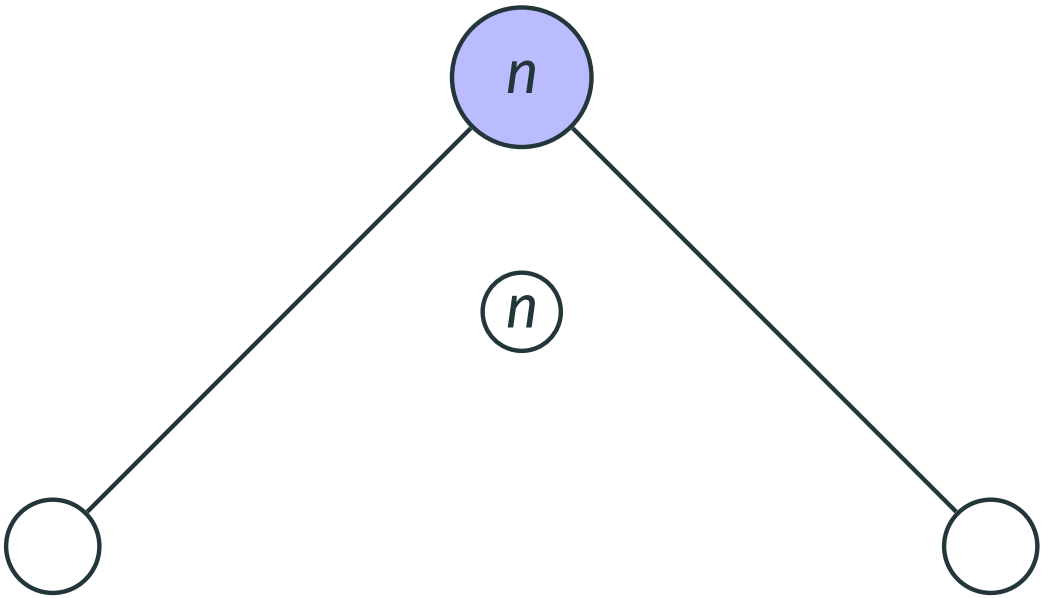

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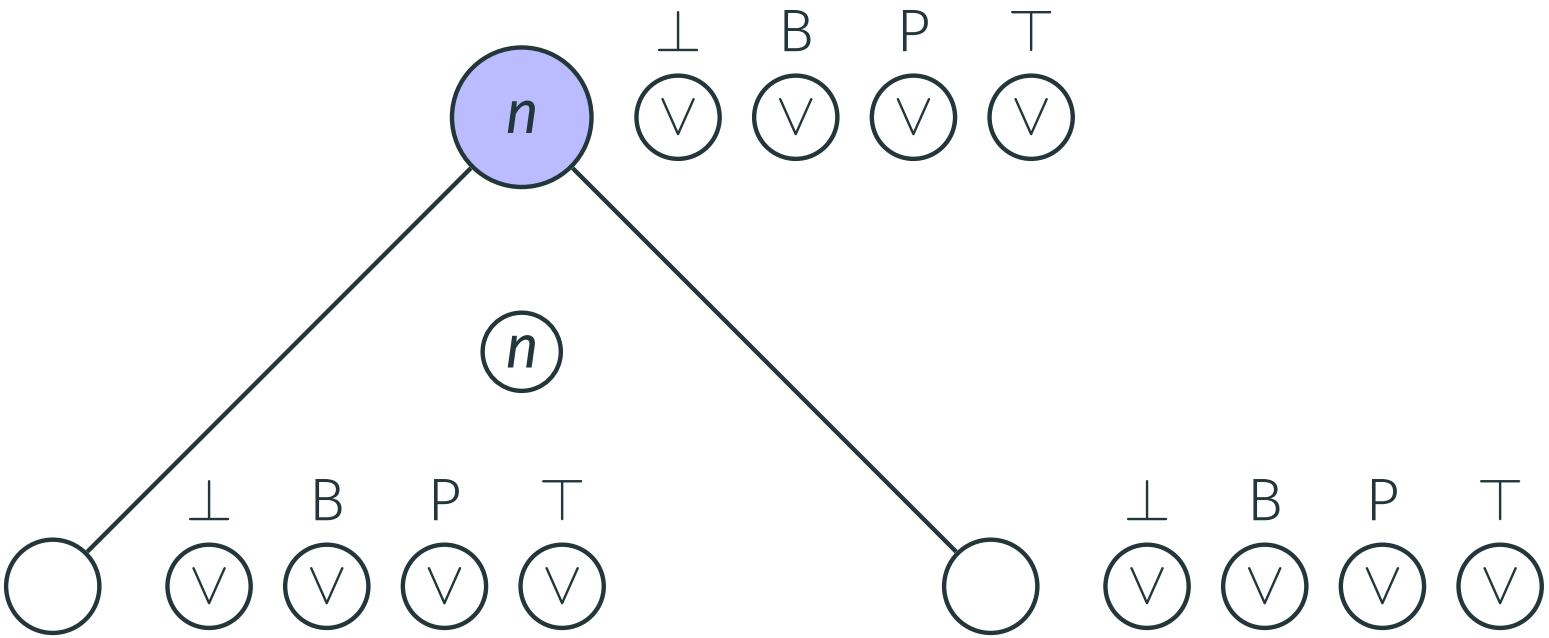

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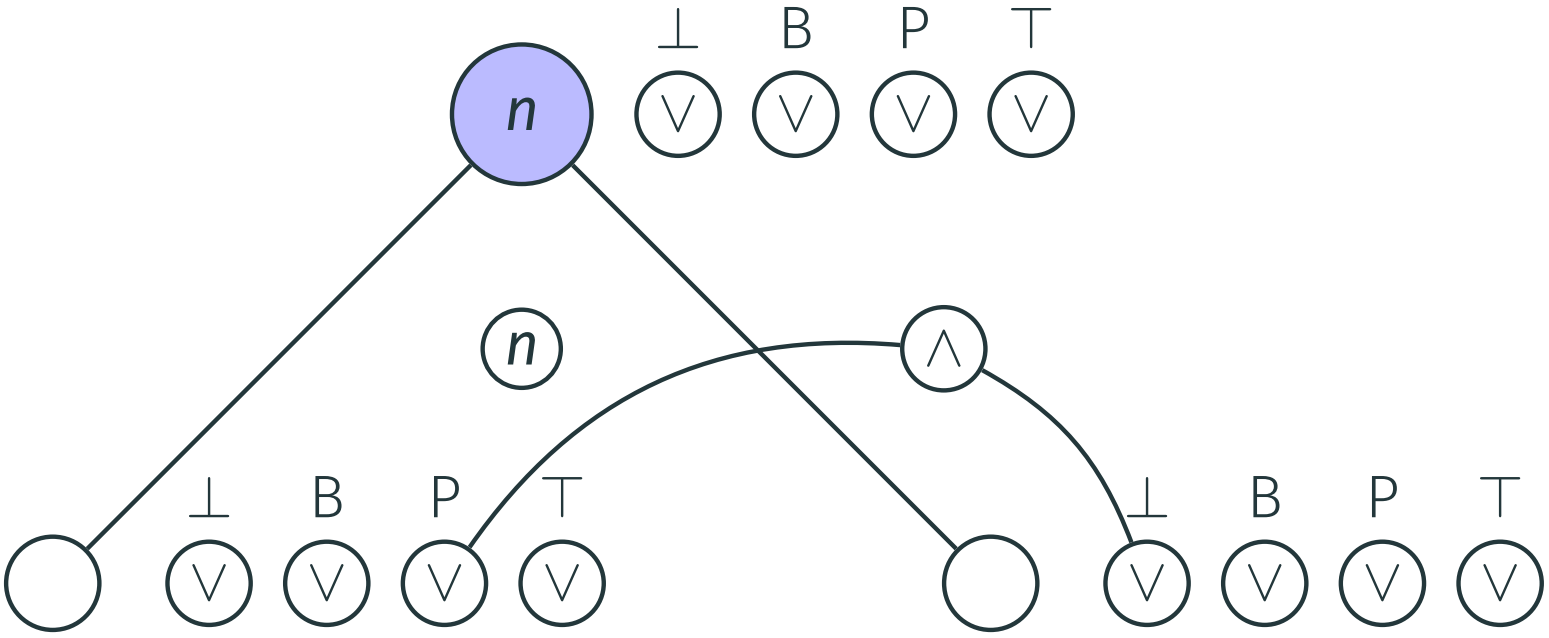

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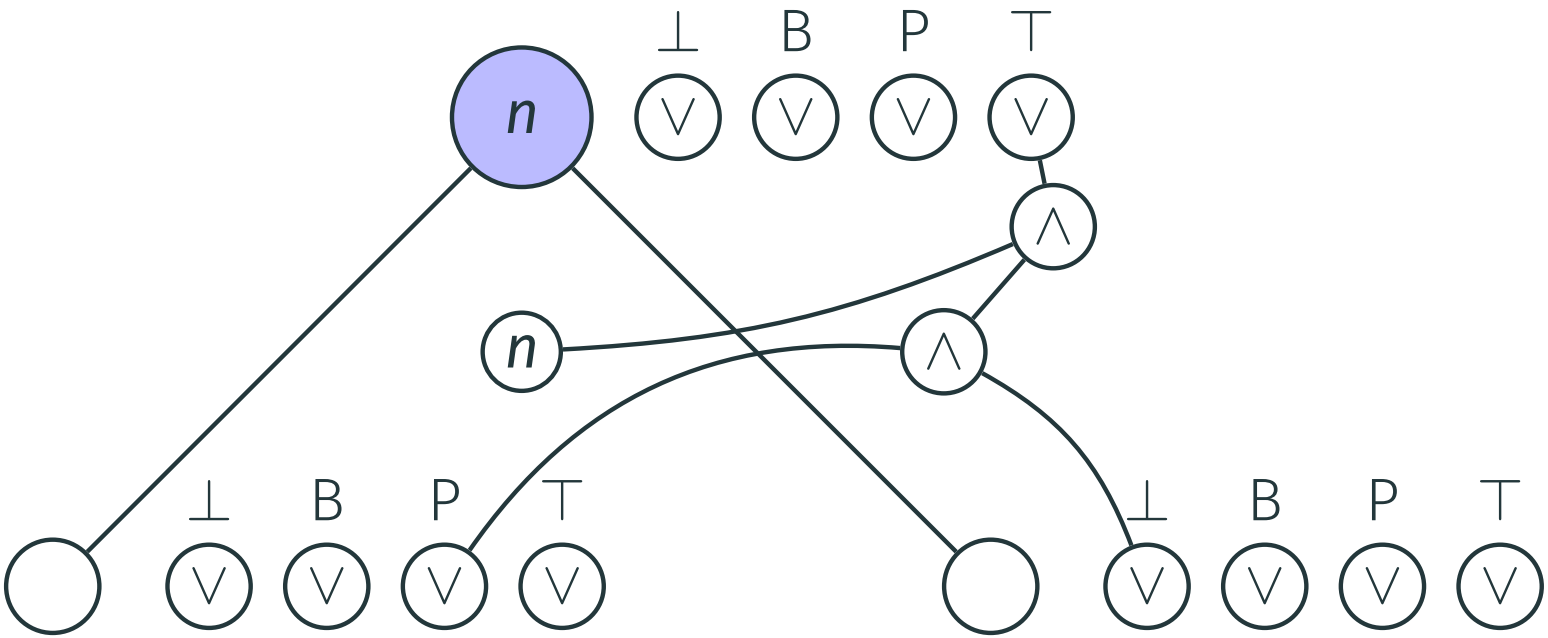

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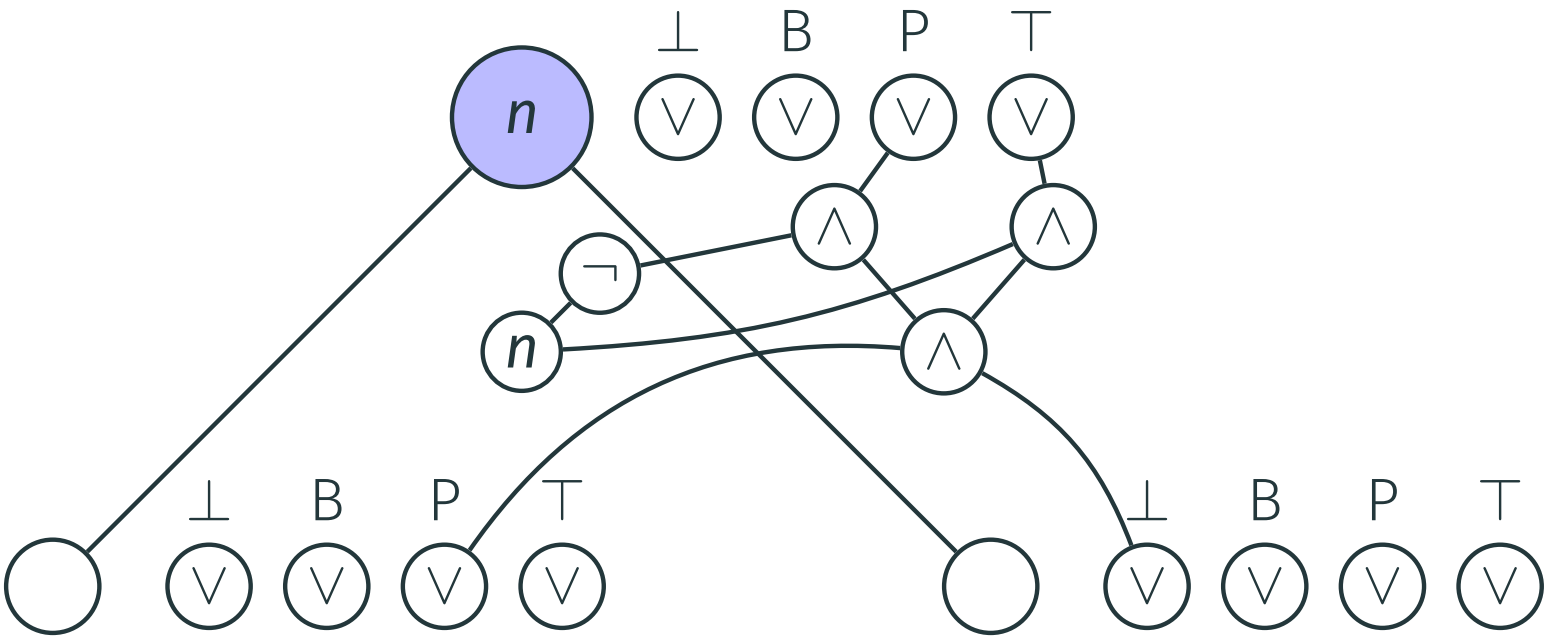
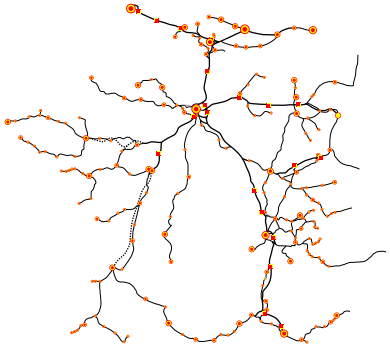


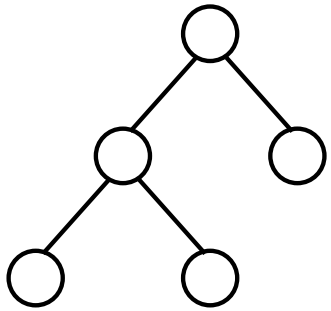
Probabilistic query evaluation

Probabilistic treelike **data**



linear

Uncertain tree **encoding**



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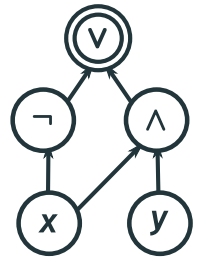
Provenance circuit +probabilities



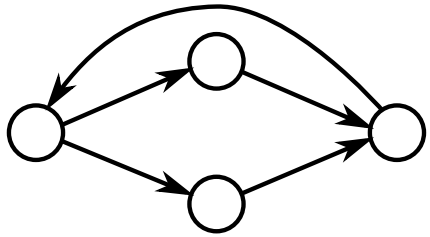
linear

95%
Probability

MSO query

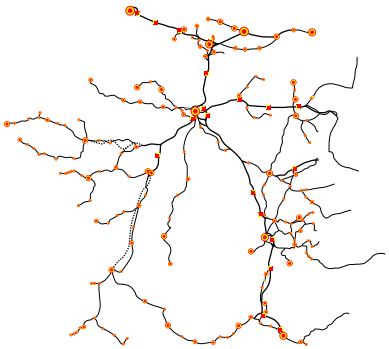
(RER|metro)*
|(bus|tram)*

Tree **automaton**



Details of the approach

Probabilistic treelike **data**



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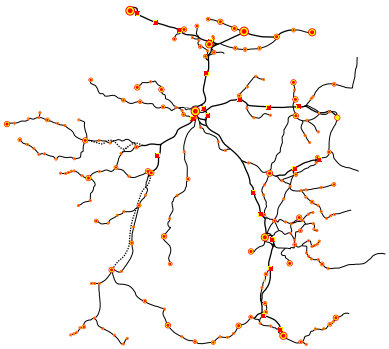

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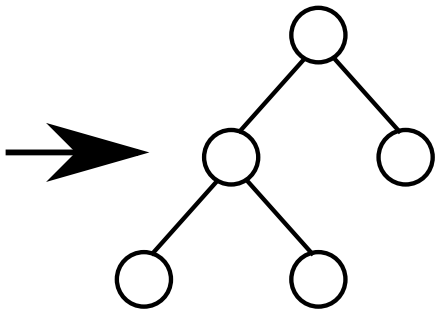

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*Each **node label** can **disappear** with the probability of the coded **fact***

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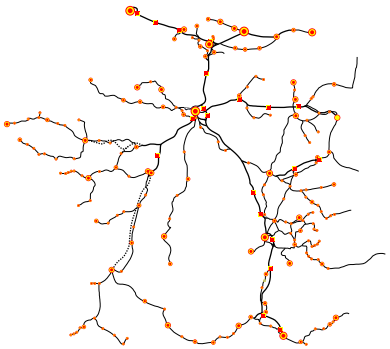

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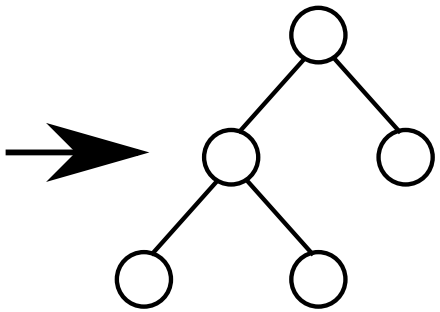

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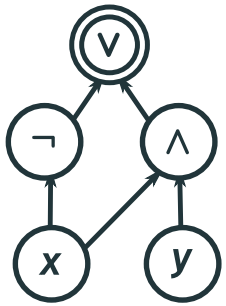
Provenance circuit +probabilities



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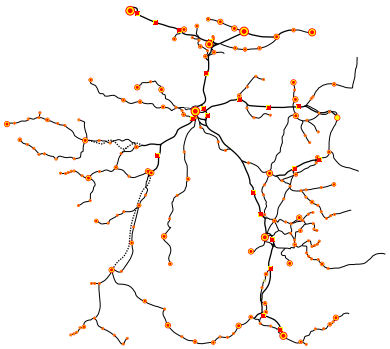

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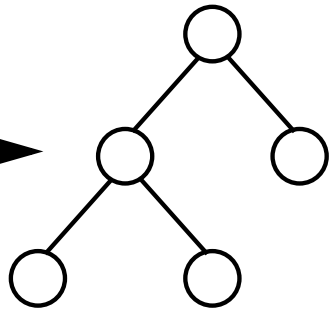

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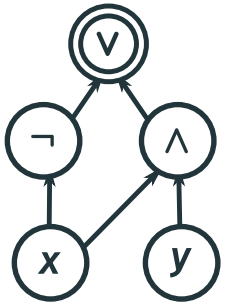

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Probability

95%

*Probability that the **circuit** evaluates to **true***

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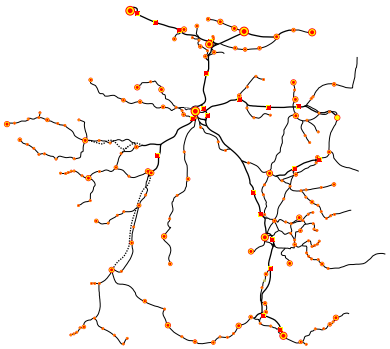

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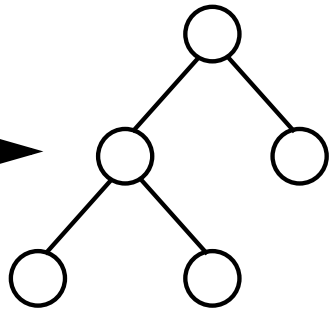

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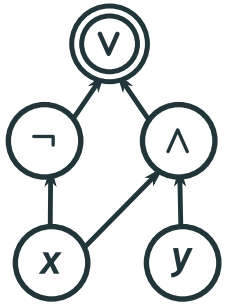

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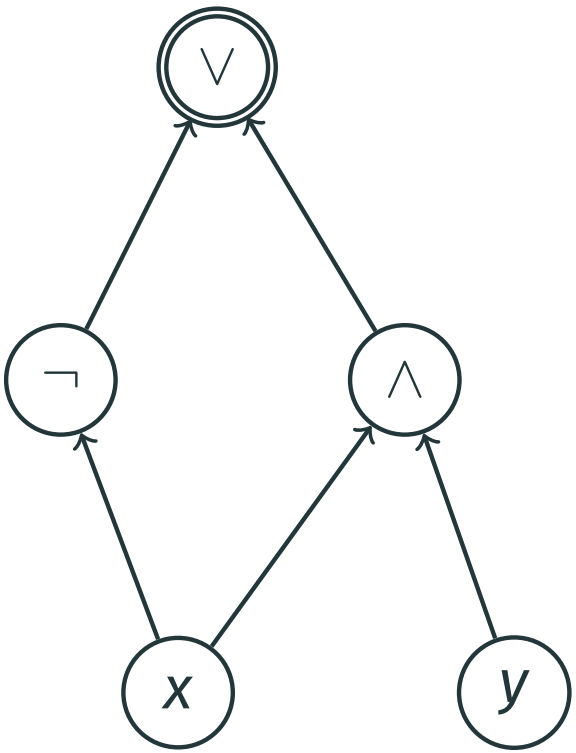
→ How to compute **efficiently** the probability of the circuit?

Computing the probability of a circuit

- We are given a circuit and a probability P for each variable



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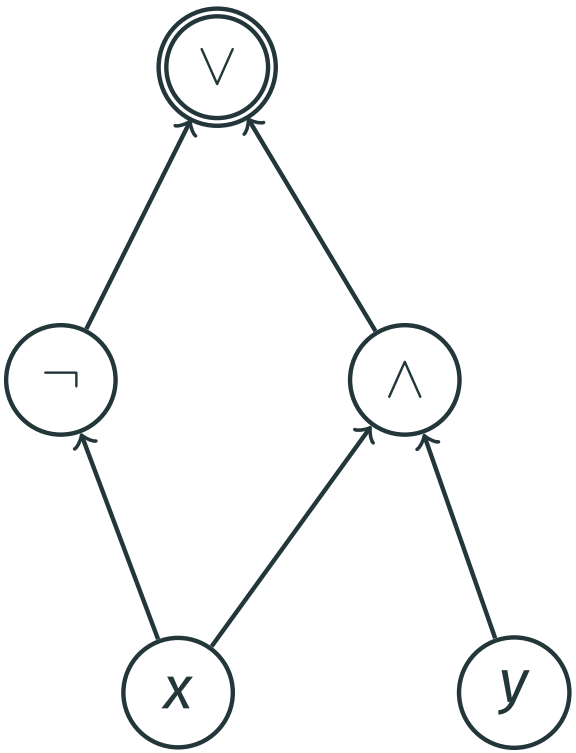

- $P(x) = 40\%$
- $P(y) = 50\%$

Computing the probability of a circuit

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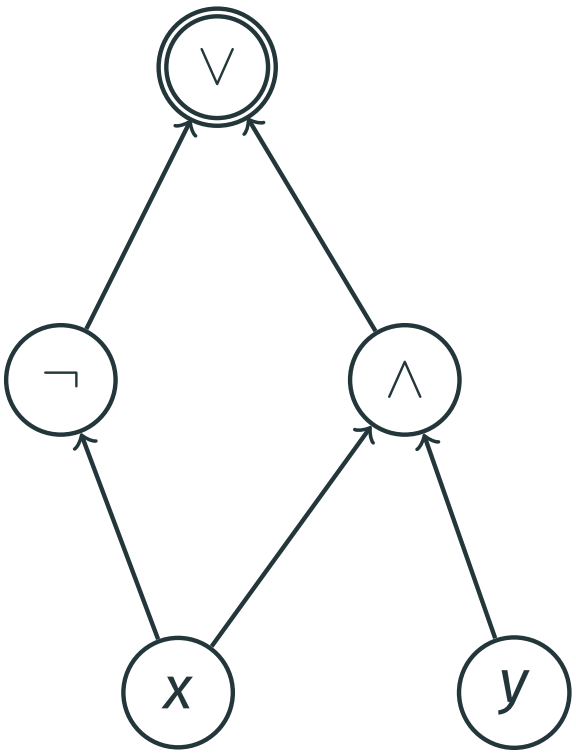

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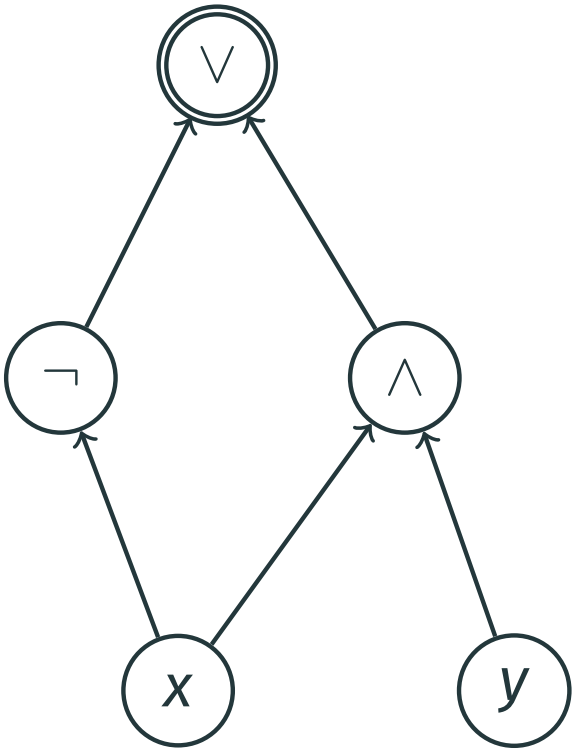

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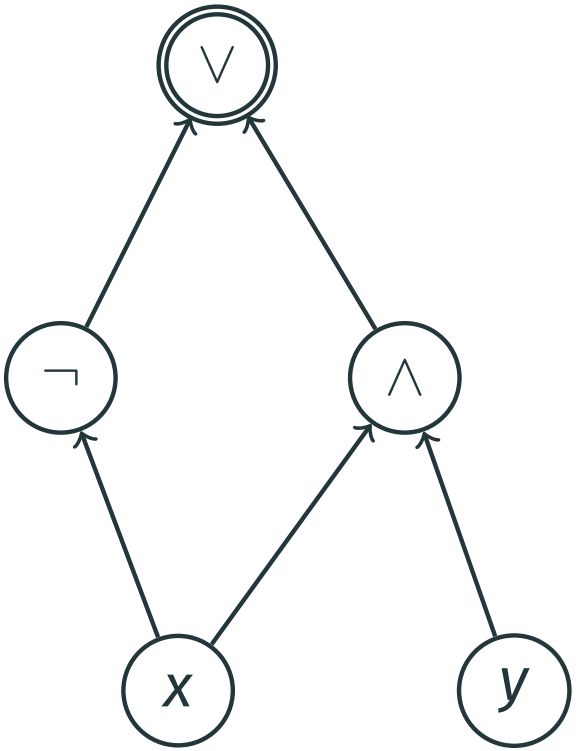

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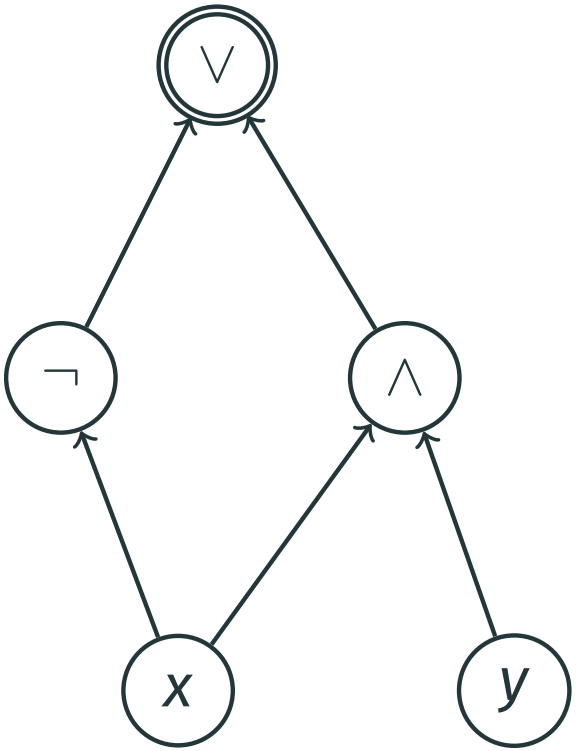

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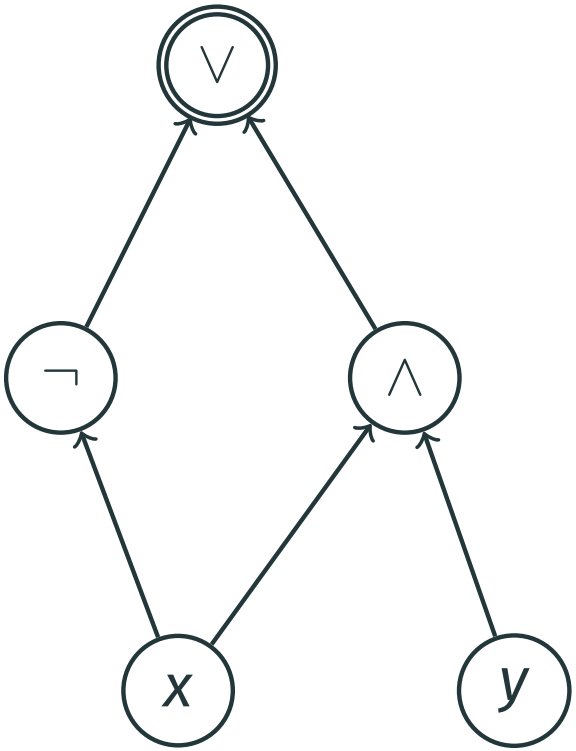

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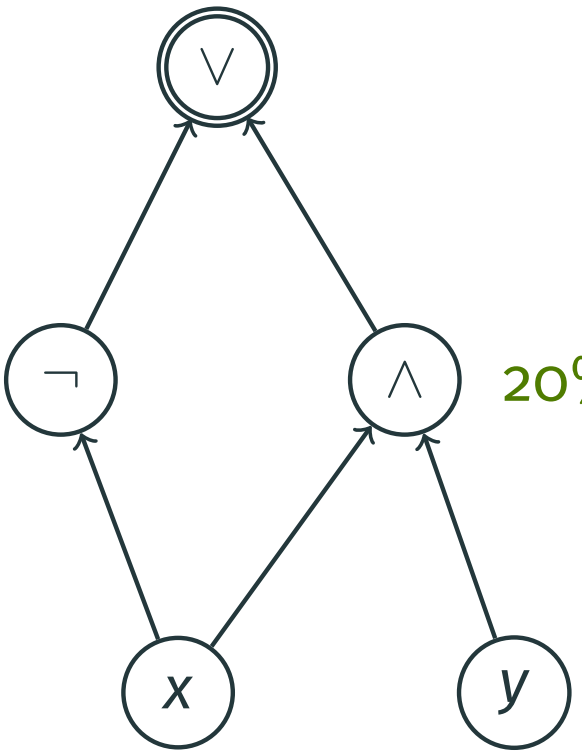

20%

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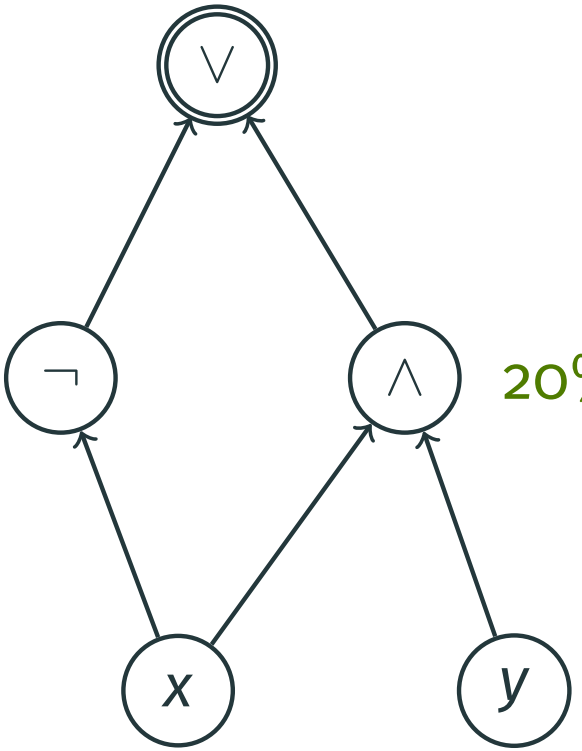

20%

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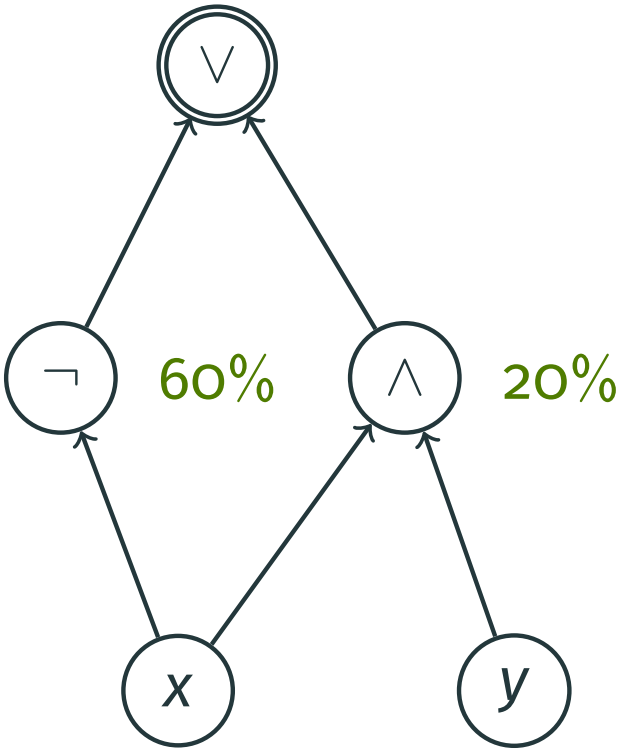

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- $P(x) = 40\%$
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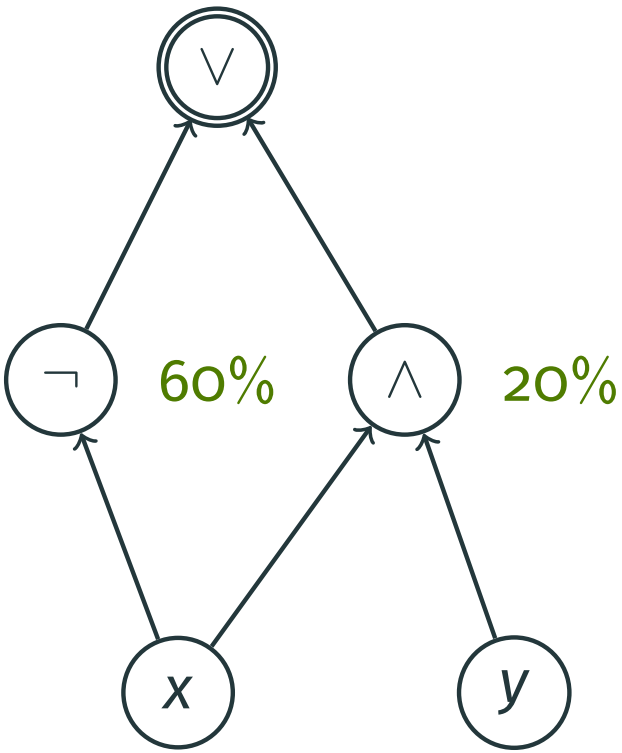

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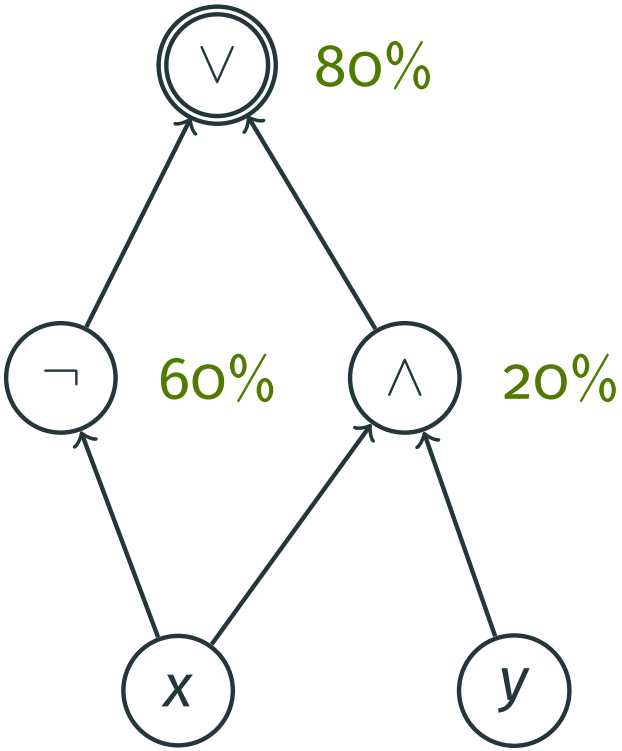

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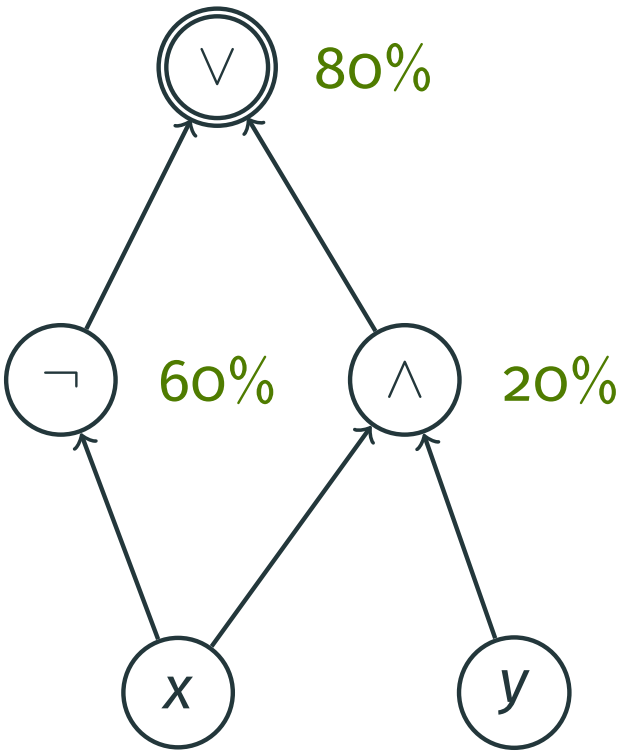

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- Let's focus on a **restricted class** of circuits that satisfies these conditions

d-DNNFs

The circuit is a d-DNNF...

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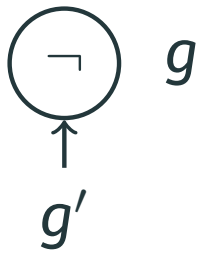

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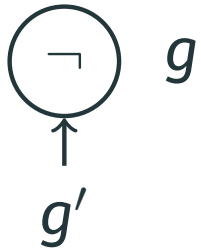

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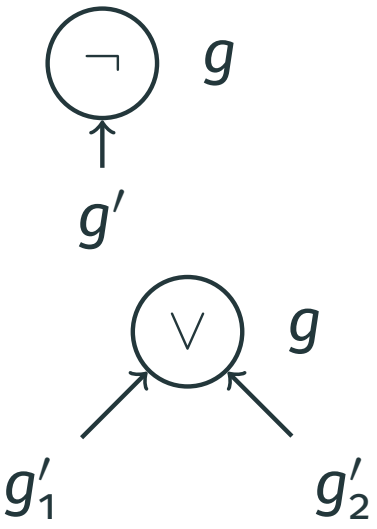

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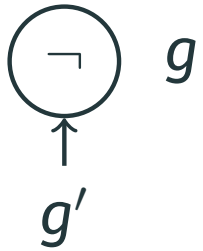

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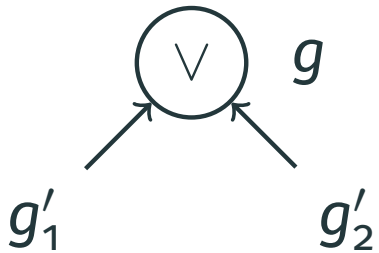

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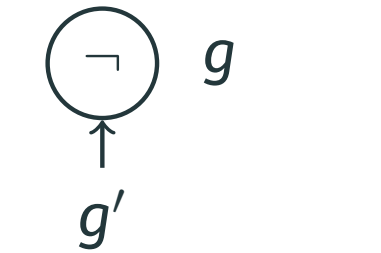

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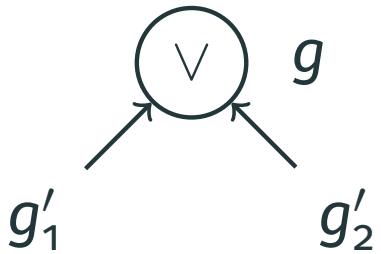

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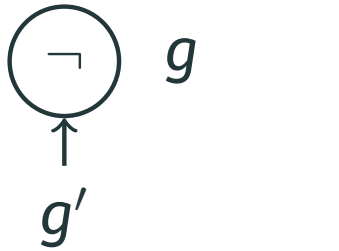

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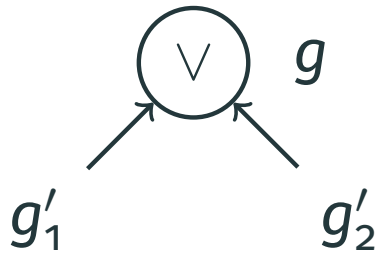

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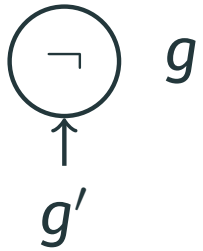

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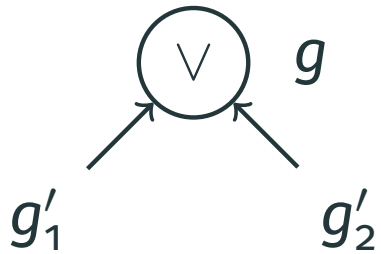

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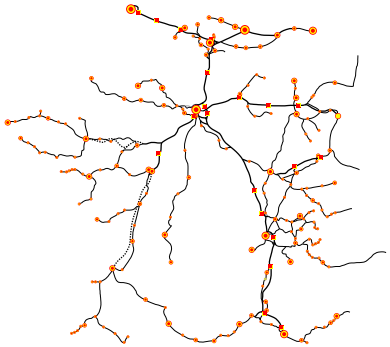
Lemma

*The **provenance circuit** computed in our construction is a **d-DNNF***

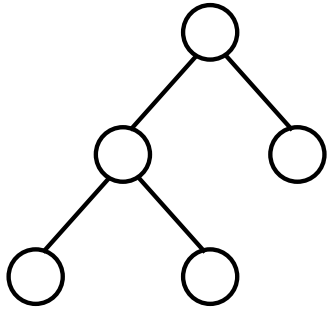
Final result

Probabilistic treelike **data**



linear

Uncertain tree **encoding**



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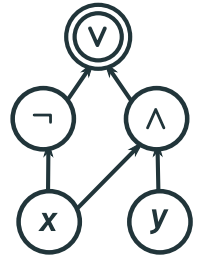
Provenance circuit +probabilities



MSO query

(RER|metro)*
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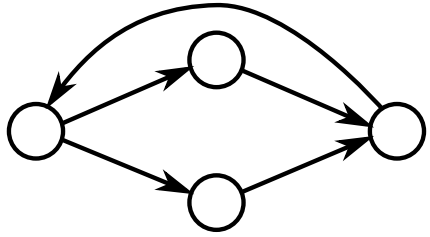
Tree **automaton**



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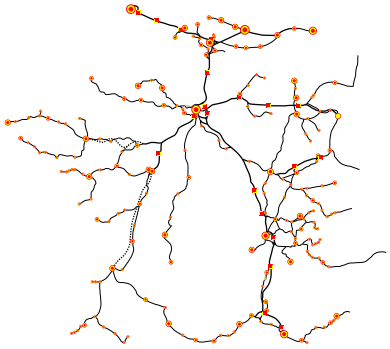

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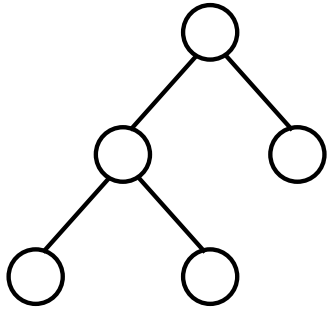

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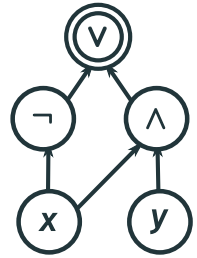

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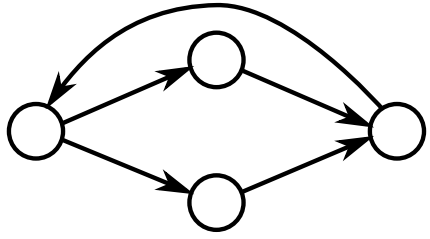

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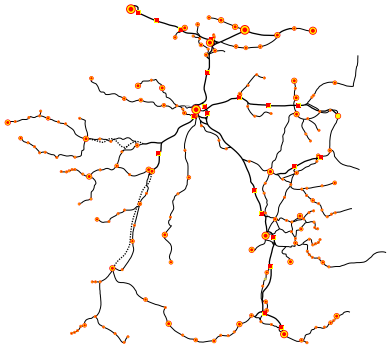

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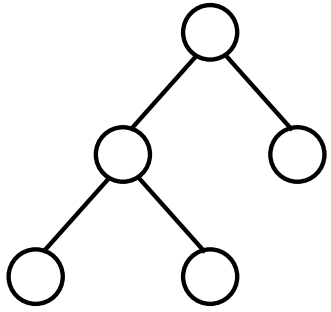

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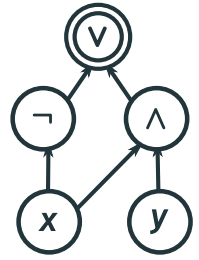

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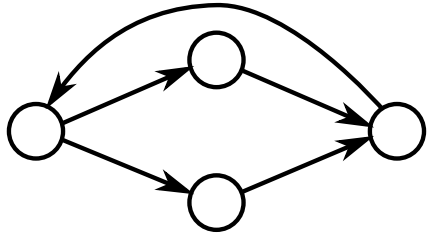

95%
Probability

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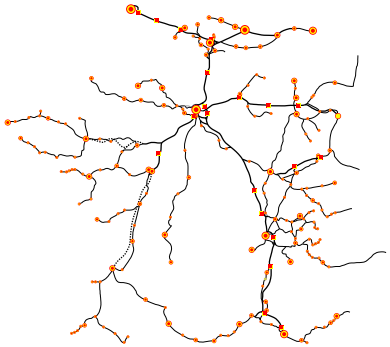

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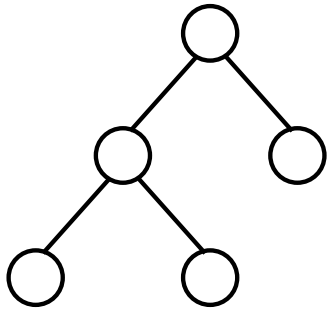

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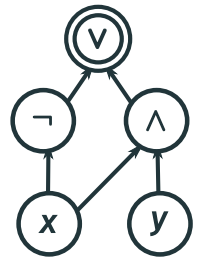

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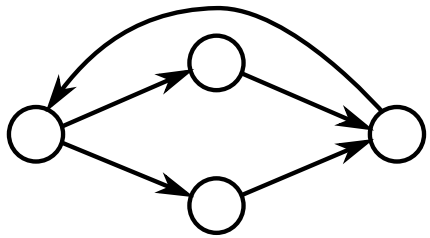

Theorem [Amarilli et al., 2015]

*For any **fixed** Boolean MSO query **Q** and **k** $\in \mathbb{N}$,*
*given a database **D** of **treewidth** $\leq$ **k** with **independent probabilities**,*
*we can compute in **linear time** the probability that **D** satisfies **Q***

Table of contents

Non-Boolean queries

- We have studied Boolean queries:

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$$Q(x, y) : P_{\color{pink}\bullet}(x) \wedge P_{\color{cyan}\bullet}(y)$$

- We can consider **each pair** (a, b) and test if $Q(a, b)$ is true
- Can we do **better**?

Circuits as factorized representations of query results

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$Q(X_1, X_2) : P_{\color{red}\bigcirc}(x) \wedge P_{\color{blue}\bigcirc}(y)$

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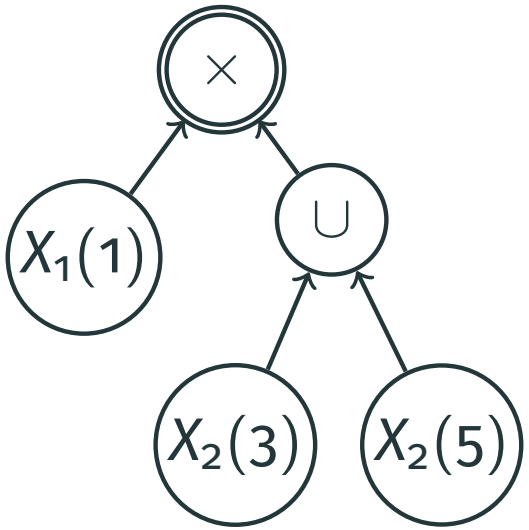

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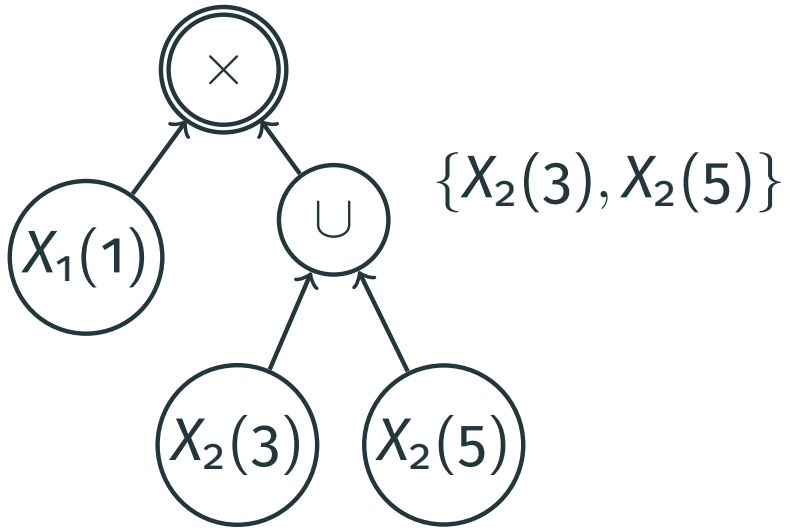

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$Q(X_1, X_2) : P_{\bigcirc}(x) \wedge P_{\bigcirc}(y)$

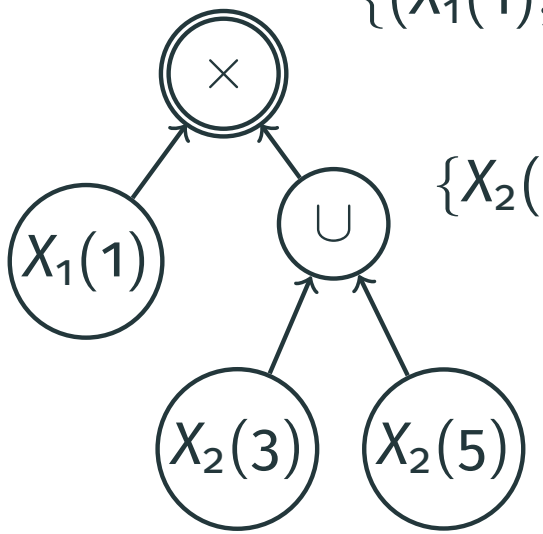
Database:

Results:

X_1	X_2
1	3
1	5

Provenance circuit:

$\{(X_1(1), X_2(3)), (X_1(1), X_2(5))\}$



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Conclusion and perspectives

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  - **Lower bounds:** probabilistic query evaluation is hard unless treewidth is bounded (modulo assumptions) [Amarilli et al., 2016]
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- Ongoing work (with my wonderful co-authors):
  - More efficient enumeration algorithms on **words**
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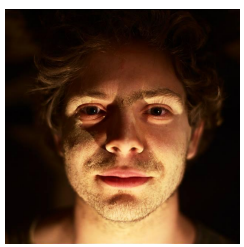
  - More expressive provenance: **cycluits** (circuits with cycles)
  - **Combined tractability** for probabilistic query evaluation


Pierre


Louis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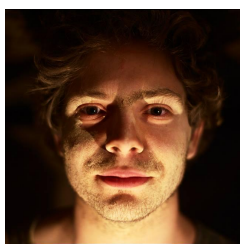
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Thanks for your attention!

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